COMPARISON OF THE HYPERBOLIC AND ASAOKA OBSERVATIONAL METHODS OF MONITORING CONSOLIDATION WITH VERTICAL DRAINS[i]

Discussion by Sudhir Kumar Tewatia[ii]

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Introduction

The authors present an important paper and demonstrate that both the hyperbolic and Asaoka's methods require settlement data beyond the 60% consolidation stage in order to provide accurate estimates of the ultimate primary consolidation settlement, δ_{ult}, and the in situ consolidation coefficient, c_v. In order to determine the average degree of settlement, U, a reliable estimate of the ultimate primary settlement is necessary. The proposed methods (by authors) are based on theory with instantaneous loading. Since the methods attempt to fit field settlement data beyond the 60% consolidation stage, these are not sensitive to the initial loading condition as long as the loading period is small compared to the time required for 60% consolidation to occur. The results showed that using early settlement data in the Asaoka plot, say, from 0% to 30% consolidation, would give very low estimates of δ_{ult} and gross overestimates of c_v with improving estimates for larger time increments. Using the data range from 30% to 60% consolidation showed much better estimates, δ_{ult} is however still underestimated by about 10%, with c_v overestimated by about 30%. Only regression with settlement data beyond 60% consolidation determine the exact values for both δ_{ult} and c_v. Based on this study, it seems that both the hyperbolic and Asaoka methods can be applied effectively to monitor consolidation with vertical drains, provided that the bulk of settlement occurring is due mainly to one-dimensional primary consolidation, and not secondary consolidation at constant effective stress as in the case of the settlement of peat.

The authors are to be congratulated for their important contribution, particularly for their finding that c_v and δ_{ult} by any method depend upon the portion of the curve used for their determination. The writer would like to use this opportunity to suggest two more methods which can be used as observational methods of monitoring consolidation with vertical drains, and to give possible reasons for the authors' findings.

Possible Reasons/Explanation for Authors' Observations

Secondary consolidation is concurrent with primary consolidation (Hansen, 1961; Taylor and Merchant, 1940) and it essentially starts after 60% consolidation (Sridharan et al., 1995). Tewatia (1998) proposed a simple method for completely separating these from the experimental curve and showed that the presence of a distinct straight line in the initial portion of the $\delta - \sqrt{t}$ plot indicates that soil follows Terzaghi's theory in the straight line portion and the c_v in this portion is true c_v, which is higher than c_v by any method (Tewatia, 1998).

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[i] By S. A. Tan and S. H. Chew, Vol. 36, No. 3, September 1996, pp. 31-42.

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The domination of secondary compression over primary compression gradually increases with the progress of consolidation after this straight line portion and its effect is to decrease c_v and increase δ_{ult}. Table 8 shows the comparison of the values of true c_v and true δ_{ult} of a few typical soils with c_v and δ_{ult} by a few other methods. It is clear that the methods which use the early portion of the time-settlement curve give higher values of c_v and lower values of δ_{ult} because of less secondary compression effects, and the methods which use later portions of curve fitting give lower values of c_v and higher values of δ_{ult} because of higher secondary compression effects. Improved $\sqrt{t}$ method (Tewatia and Venkatachalam, 1997; Tewatia and Venkatachalam, 1998) also proves it. That is why in the initial stages of curve fitting, Asaoka's method gives a higher value of c_v and lesser value of δ_{ult}.

Table 8. The c_v (in cm^2/s) and δ_{ult} (in terms of dial gauge readings), by different methods, are shown for a few typical soils (Tewatia, 1998). w_l = liquid limit, w_p = plastic limit, *Robinson and Allam 1996

Method	Black cotton soil (w_l =69%, w_p =33%)	Bentonite + Sand mixture (w_l =100%, w_p =30%)	Bentonite (w_l =495%, w_p =49%)
True c_v & True δ_{ult}	2.78×10^{-4} & 250.0	5.68×10^{-5} & 377.0	2.91×10^{-5} & 615.4
Revised $\log(t)$*	1.84×10^{-4} & 303.5	4.91×10^{-5} & 399.8	1.51×10^{-5} & 817.8
Taylor	1.61×10^{-4} & 319.9	4.05×10^{-5} & 420.2	1.36×10^{-5} & 842.1
Casagrande	1.34×10^{-4} & 346.3	3.95×10^{-5} & 430.4	----------

Proposed Methods

Before Tewatia suggested settlement-velocity methods, all the methods determined c_v from the following relationship:

$$c_v = \frac{T_U}{t_U} H^2 \tag{15}$$

where T_U and t_U are time factor and experimental time respectively corresponding to a particular degree of consolidation, U, and H is the drainage path of the consolidating layer. This relationship can be used to determine c_v only when the loading is applied instantaneously because to know t_U, the clock time of load increment, Ct_0, should be known first as $t_U = (Ct_U - Ct_0)$; and to know Ct_0, the loading should be instantaneous. The writer is presenting two methods which do not use Eq. (15) to determine c_v and δ_{ult}.

1) The $\boldsymbol{\delta - v}$ Method (Tewatia, 1998): In Fig. 13, settlement, in terms of dial gauge reading, δ, is directly plotted against velocity of settlement, $v = (d\delta/dt)$. A tangent, at any point on the $\delta - v$ curve is drawn. The intercept of the tangent on the δ axis gives δ_{ult}. If $m = (d\delta/dv)$ is the slope of the tangent, then c_v, at this δ, can be obtained from the following relationship:

$$c_v = -\frac{4H^2}{\pi^2 m} \tag{16}$$

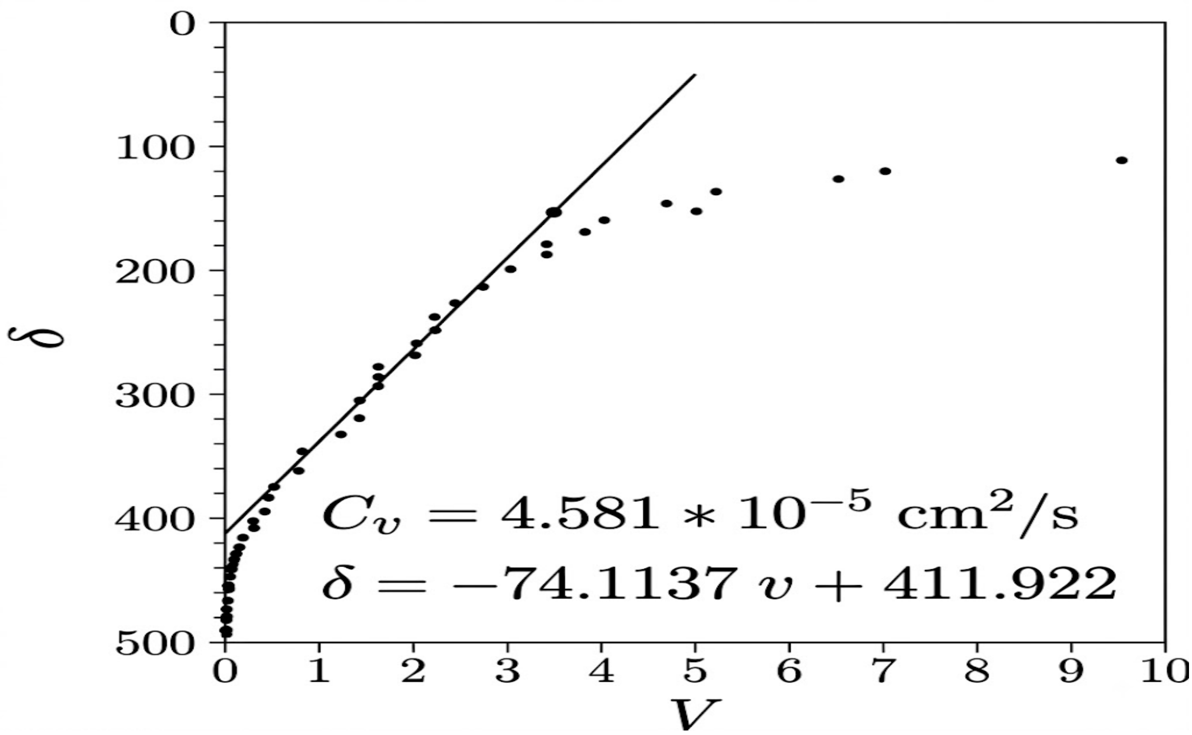

Fig. 13. Linear δ-v plot for Bentonite+Sand mixture (liquid limit 100%, plastic limit 30%). The δ is dial gauge reading which is set at zero before applying the next load increment; least count of dial gauge is 0.00025 cm. Velocity, v, is in dial gauge units per minute. Shown in figure is the equation of tangent and c_v at tangent point.

Here units of length of δ and v are kept the same so that the unit of m is the unit of time in v. For example: from the equation of tangent shown in the figure, the intercept of tangent on the δ axis (where $v = 0$) is 411.9 which is δ_{ult} at the tangent point. Tangent slope, $m = (d\delta/dv)$ is -74.114 minutes. Slope is negative because as settlement δ increases, v decreases. If $H = 0.709$ cm then from Eq. (16), at the tangent point, $c_v = 2.749 \times 10^{-3} cm^2$/minute $= 4.581 \times 10^{-5} cm^2/s$. It can be noticed from this figure that with the progress of consolidation, c_v decreases (as m increases) and δ_{ult} increases.

2) The $\boldsymbol{\delta - \log_{10}(v)}$ method (Tewatia, 1998): This is a semi-log $\delta - v$ plot which is a nearly symmetrical S-shaped curve having the point of inflection at 50% U (Fig. 14). Draw a tangent at any δ in the lower portion of the curve which is convex downwards. The slope of this tangent over one $\log_{10}$ cycle, s, is divided by $\log_e 10$ and added to δ to determine δ_{ult}. If v is the velocity of settlement at this point, then c_v at this δ can be determined as:

$$c_v = 0.9332 \frac{v}{s} H^2 \tag{17}$$

Methods 1 and 2 are applicable after 60% U.

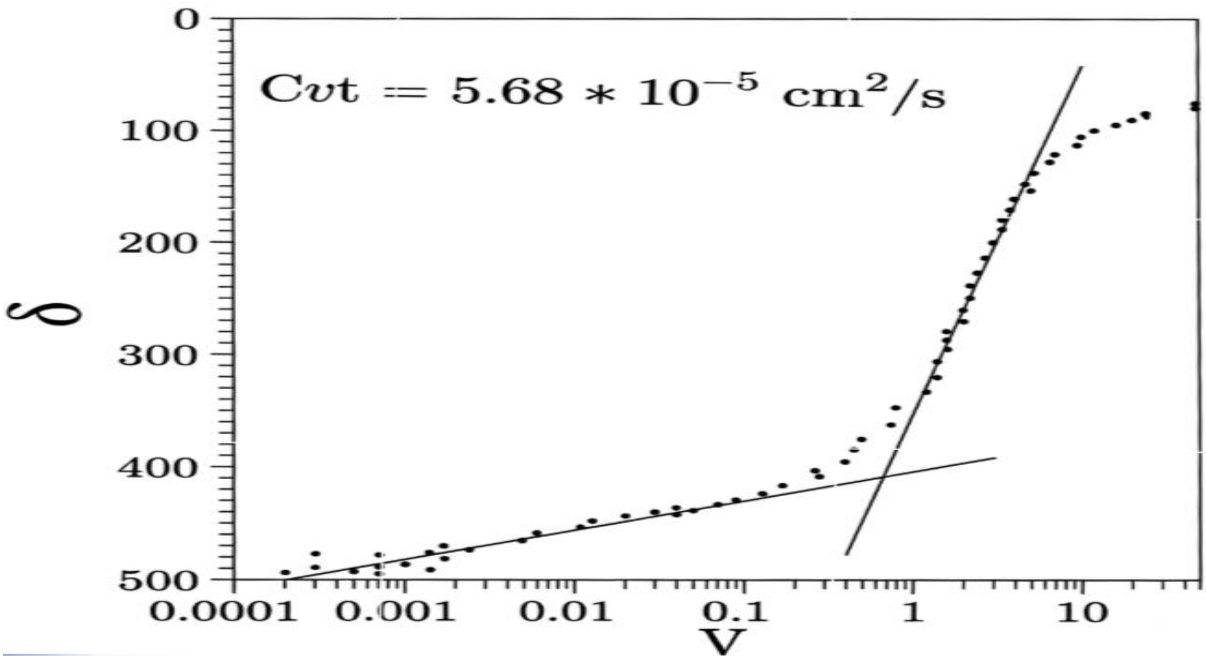

Fig. 14. Semi-log plot of same readings of $\delta - v$ of Fig. 13, for same soil. C_{vt} is true c_v.

Merits of the Proposed Methods over Asaoka's and Hyperbolic Methods

1. If loading is instantaneous or the loading period is small compared to the time required for 60% consolidation to occur, and therefore, time, t, corresponding to this δ (where c_v is determined) is known, then T can be determined from the following standard relationship:

$$T = \frac{c_v t}{H^2} \tag{18}$$

and U, then, can be determined from the standard Terzaghi's $U - T$ relationship, obviously, without using δ_{ult}. Therefore, in these two methods U can be determined without determining δ_{ult} first. In Asaoka's and hyperbolic methods, we require δ_{ult} to be determined first to determine *U*.

2. As is seen from Asaoka's and these two methods that c_v and δ_{ult} vary with compression and are different at different stages of consolidation. Asaoka's method uses a considerable portion of the curve to determine these values and these values are different at every point in this considerable portion. Asaoka's method is based on Terzaghi's theory which assumes c_v is constant. Therefore it is not proper to apply Terzaghi's theory (and hence Asaoka's method) using a considerable portion in which c_v is not constant. But in these two settlement-velocity methods because c_v is determined using only a point (i.e. infinitely small portion) of the curve, and at any point c_v is constant, and hence Terzaghi's theory can be applied.

3. These methods are much simpler and faster than Asaoka's and hyperbolic methods and are not very sensitive to initial loading conditions because time, t, at some particular settlement, δ, at which c_v is determined (from Eqs. (16) or (17)), is not required to be known first to determine c_v. While in the methods used by the authors, c_v is determined from Eq. (15) and hence t is required. The t can be known only when the clock time of load increment is known i.e. load should be applied instantaneously, or the loading period should be small compared to the time required for 60% consolidation to occur. Once consolidation is in progress after 60% U, the $\delta - v$ curve can be plotted at any time without knowing the clock time of load increment and c_v, δ_{ult} etc. can be determined from the small-time data only, without knowing initial

loading conditions (i.e. δ_0, Ct_0 etc.). While in Asaoka's and hyperbolic methods, considerable data and clock time of load increment should be available/known.
4. Variation of c_v and δ_{ult} in Asaoka's and these methods may be attributed to secondary consolidation. For any soil, secondary consolidation essentially starts after 60% U. The effects of secondary consolidation on c_v, δ_{ult} etc. and its relative dominance over primary consolidation are better visible in the settlement-velocity methods as compared to Asaoka's and hyperbolic methods.

Determination of True c_v and True δ_{ult} (Tewatia, 1998)

To have an idea that to what extent c_v and δ_{ult} are affected by secondary consolidation, these may be compared with true c_v and true δ_{ult}. In the $\delta - \log_{10}(v)$ plot (S curve) where δ is settlement and v is the velocity $(d\delta/dt)$ of settlement, a tangent at the point of inflection $(U = 50\%)$ (Fig. 14) cuts the δ_0 line at which, on the S curve, corresponds to $U = 16.19\%$. True c_v can be evaluated as:

$$c_v = 0.2566 \frac{v_{16.19}}{s_{50}} H^2 \tag{19}$$

where, in the experimental δ versus $\log_{10}(d\delta/dt)$ plot, s_{50} is the slope of tangent, over one $\log_{10}$ cycle, at the point of inflection. For computing δ_0 a tangent is drawn at any point (δ, v) in the initial portion $(0 \leq U \leq 40\%)$ of the curve which is convex upwards. If s is the slope of the tangent, over one $\log_{10}$ cycle, then δ_0 can be found as:

$$\delta_0 = \delta - \frac{s}{\log_e 10} \tag{20}$$

and true δ_{ult} can be computed as:

$$\delta_{ult} = \delta_0 + \frac{s_{50}}{1.009} \approx \delta_0 + s_{50} \tag{21}$$

Equations (19) and (21) give true c_v and true δ_{ult} in the case of those soils which exhibit a straight line in the initial portion of the $\delta - \sqrt{t}$ curve.

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