An Enhanced Vesely Approach for Common Cause Failure Analysis in Non-Identical M-out-of-N Voting Systems

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Abstract

Traditional methods for common cause failure (CCF) analysis, such as the Vesely approach, assume uniform failure probabilities across all channels, limiting their applicability to safety instrumented systems (SIS) with diverse channel designs. This study develops an enhanced Vesely method, extending the classical Vesely approach to incorporate per-channel beta factors (β_i) for non-identical M-out-of-N voting systems. A generalized formula for the effective beta factor (β_{eff}) is derived and validated through case studies, achieving reductions in β_{eff} ranging from 4% to 28% across diverse system configurations, with the method's benefits scaling with system complexity. Robustness is assessed through sensitivity and uncertainty analyses, while log-transformations ensure computational efficiency for large systems. The method aligns with IEC 61508-6, IEC 61511, and ISO/TR 12489, offering a practical tool for reliability engineers to improve safety assessments and reduce costs in SIS design. Validated through three comprehensive case studies spanning baseline (N=3, M=2), high-redundancy (N=4, M=3), and large-scale (N=12, M=4) configurations.

1 Introduction

Safety instrumented systems (SIS) play a pivotal role in mitigating risks across high-stakes industries such as nuclear power, aerospace, chemical processing, and oil and gas, where failures can result in catastrophic consequences, including loss of life, environmental damage, or significant economic losses. At the heart of SIS are M-out-of-N voting systems, which ensure fault tolerance by requiring at least M out of N redundant channels to detect a failure condition before initiating a safety response, such as shutting down a reactor or isolating a pipeline. However, the reliability of these systems is often compromised by common cause failure (CCF)—the simultaneous failure of multiple channels due to a shared cause, such as design flaws, environmental stressors (e.g., temperature extremes, electromagnetic interference), or operational errors (e.g., improper maintenance procedures). In this context, CCF is defined as the failure of multiple channels due to a single cause, and the effective beta factor (β_{eff}) represents the conditional probability that at least M channels fail given the occurrence of a common cause event (formally defined in Section 5 via Equations 3–4). The per-channel beta factor (β_i) quantifies the probability of channel i failing due to CCF, while the average Probability of Failure on Demand (PFDavg) measures the SIF's reliability for low-demand systems, as per IEC 61508 standards [3].

The method referred to in this paper as the Vesely method builds on W. E. Vesely's foundational work in probabilistic fault tree evaluation [14] and on the common cause failure modeling framework later standardized in NUREG/CR-5485 [10]; the binomial beta-factor formulation used here follows the latter. It has been a cornerstone for CCF analysis in SIS. It calculates CCF probabilities by assuming uniform failure probabilities across all channels, an approach that offers computational simplicity but fails to reflect the diversity inherent in modern SIS designs. For example, a Redundant System (N=3, M=2) SIS might integrate a mechanical pressure switch, a digital smart transmitter, and an electromechanical relay, each with distinct failure characteristics due to differences in technology, materials, or operational exposure. This intentional diversity—often implemented to reduce CCF susceptibility—introduces variations in failure modes (e.g., mechanical wear in switches vs. software bugs in transmitters) that a uniform beta factor cannot capture. As a result, the Vesely method can misrepresent the system's risk profile, with inaccuracies potentially overestimating CCF probability by 5–10%. Such overestimation can lead to overly conservative Safety Integrity Level (SIL) requirements, increasing design and maintenance costs (e.g., adding unnecessary redundancy), or underestimating risks, thereby compromising safety and potentially leading to catastrophic failures.

Industry standards like IEC 61508-6 [3] provide beta factor tables for smaller configurations (e.g., Series System (N=2, M=1), Redundant System (N=3, M=2)), but they offer limited guidance for complex systems, such as Large-Scale System (N=12, M=4) architectures commonly used in high-redundancy applications like nuclear reactor safety systems. Alternative CCF models, such as the Multiple Greek Letter (MGL) and Alpha Factor models, attempt to address these limitations by introducing more sophisticated parameterizations. The MGL model, for instance, uses additional parameters like γ and δ to model failures across channel subsets, while the Alpha Factor model relies on historical data to assign probabilities to failure groups [10]. However, both models depend on system-wide parameters, which restricts their ability to capture channel-specific variations in diverse systems, often leading to similar over- or underestimations as the Vesely method. This gap underscores the need for a more flexible and accurate CCF analysis framework tailored to non-identical M-out-of-N systems.

This study presents an enhanced Vesely method, which integrates channel-specific failure probabilities (β_i) to model CCF in non-identical M-out-of-N systems. To the author's knowledge, this is the first published extension of the binomial-expansion beta-factor formulation to per-channel β_i, addressing a gap in IEC 61508-6's coverage of non-identical M-out-of-N architectures. By accounting for diversity in channel design, the method offers a more precise estimation of CCF risk, reducing over-conservatism in SIL calculations while maintaining safety integrity. The objectives of this study are:

- Derivation of a generalized formula for the effective beta factor (β_{eff}) in systems with non-identical channels, ensuring mathematical rigor through formal derivation and validation (Section 5).

- Validation through case studies, comparing the enhanced Vesely method against the traditional Vesely method across diverse M-out-of-N systems (Section 6).
- Practical application in SIL calculations, demonstrating the method's impact on PFDavg and cost-efficiency in safety-critical industries (Section 7).
- Alignment with industry standards, ensuring compliance with IEC 61508-6, IEC 61511, and ISO/TR 12489 for seamless integration into safety workflows (Section 8).

2 Problem Statement

Safety instrumented systems (SIS) are designed to ensure safety in high-risk industries by employing M-out-of-N voting architectures that balance reliability and availability through redundancy. These systems are critical for applications such as monitoring pressure in oil refineries, temperature in nuclear reactors, or flow rates in chemical plants, where failures can lead to catastrophic outcomes like explosions, environmental spills, or loss of life. However, the effectiveness of M-out-of-N systems is significantly undermined by common cause failure (CCF), where a single underlying cause—such as a design flaw, environmental stress (e.g., a power surge or extreme temperature), or human error (e.g., incorrect calibration during maintenance)—leads to the simultaneous failure of multiple channels. CCF negates the benefits of redundancy, as it can cause enough channels to fail together to trigger a system-wide failure, even in highly redundant configurations.

Traditional CCF models, such as the Vesely method, rely on the simplifying assumption that all channels in a M-out-of-N system are identical, meaning they share the same failure probability due to CCF. This assumption is codified in standards like IEC 61508-6 [3], which provides beta factor tables for smaller configurations (e.g., Series System (N=2, M=1), Redundant System (N=3, M=2)) based on a uniform beta factor (β). For instance, IEC 61508-6 Table D.4 provides base β_{int} values ranging from 0.5% to 10% for a 1oo2 configuration depending on the Annex D defensive-measure score, with Table D.5 applying architecture-specific multipliers for other voting arrangements (e.g., $1.5 \times \beta_{\text{int}}$ for 2oo3), applied uniformly across all channels. While this simplification facilitates computation, it fails to reflect the reality of modern SIS designs, where channels often incorporate diverse technologies to minimize CCF risk. A Redundant System (N=3, M=2), for example, might include a mechanical pressure switch (prone to wear but robust against software issues), a digital smart transmitter (susceptible to firmware bugs but accurate), and an electromechanical relay (vulnerable to electrical faults but mechanically reliable). These channels exhibit unique failure rates and modes due to differences in design, materials, and operational exposure—such as varying susceptibility to environmental stressors like vibration or humidity. Applying a uniform beta factor in such cases can lead to significant inaccuracies, overestimating CCF risk by 5–10% in diverse systems [12]. This overestimation can inflate SIL requirements, driving costs upward through over-design (such as unnecessary channel redundancy or shortened proof test intervals), or, conversely, underestimating risk can compromise safety integrity.

The challenge becomes even more pronounced in larger, more complex systems, such as Large-Scale System (N=12, M=4) architectures used in high-redundancy applications like nuclear reactor safety systems or offshore platforms. In these systems, the number of channels ($N = 12$) and the required number of failures for system failure ($M = 4$) result in a combinatorial explosion of possible failure scenarios—specifically, $\sum_{k=4}^{12} \binom{12}{k} = 3{,}797$ subsets of channels that must be evaluated for CCF risk. Standards like IEC 61508-6 provide no specific guidance for such configurations, leaving engineers to rely on extrapolation or ad hoc methods, which can introduce further inaccuracies. For example, a nuclear Large-Scale System (N=12, M=4) might use a mix of sensors from different manufacturers, with some exposed to higher radiation levels (increasing their β_i) and others shielded (decreasing their β_i). A uniform β cannot capture these variations, leading to either over-conservative designs that inflate costs or underestimations that risk safety.

Alternative CCF models, such as the Multiple Greek Letter (MGL) and Alpha Factor models, attempt to address some of these limitations but fall short in capturing channel-specific variations. The MGL model introduces additional parameters like γ (probability of CCF affecting two channels) and δ (probability of CCF affecting three or more channels) to model failures across subsets of channels [10]. While this makes MGL suitable for complex systems, it requires estimating multiple parameters, and the field data needed to support those estimates is rarely available at the level of individual channel designs. The Alpha Factor model, on the other hand, is data-driven, assigning probabilities to failure groups (e.g., 7% of failures involve exactly two channels failing together) [10]. However, it relies on system-wide failure rates, lacking the granularity to account for per-channel differences in diverse systems. Both models, while endorsed by IEC 61508, assume homogeneous failure characteristics across channels, leading to similar inaccuracies as the Vesely method either overestimating risk by 5–10% or underestimating it, depending on the system's diversity.

The practical implications of these inaccuracies are significant. In a Redundant System (N=3, M=2) SIS monitoring pressure in an oil refinery, an overestimated CCF risk might trigger an unnecessary escalation from SIL 2 to SIL 3, requiring additional redundancy or shortened proof test intervals with corresponding capital and maintenance impact. Conversely, an underestimated CCF risk could leave a system unable to meet its target SIL, with safety consequences that depend on the specific application. In larger systems like the Large-Scale System (N=12, M=4) architecture, the lack of standard guidance exacerbates these issues, as engineers are forced to rely on extrapolated beta factors or simplified models that do not account for channel diversity, further amplifying the risk of costly over-design or safety gaps.

The proposed enhanced Vesely method addresses this gap by integrating channel-specific failure probabilities (β_i) into the Vesely framework, offering a scalable, standards-aligned solution for accurate CCF analysis in heterogeneous SIS. By modeling β_i for each channel based on its unique failure characteristics—derived from sources like OREDA data [11], IEC 61508-6 checklists, and expert judgment (Section 5.2)—the method provides a more precise estimation of β_{eff}, reducing the over- or underestimation seen in traditional models. This approach not only improves the accuracy of SIL calculations (Section 7) but also ensures computational efficiency through log-transformations (Section 5.4), making it practical for large systems like Large-Scale System (N=12, M=4). The method's alignment with IEC 61508-6, IEC 61511, and ISO/TR 12489 (Section 8) ensures it can be seamlessly integrated into existing safety engineering workflows, offering a robust tool to enhance both safety and cost-efficiency in SIS design.

3 Literature Review

Common cause failure (CCF) analysis is a cornerstone of reliability engineering, ensuring the robustness of safety-critical systems that rely on redundancy to mitigate risks. In safety instrumented systems (SIS), CCF represents the simultaneous failure of multiple channels due to a shared cause, such as a design flaw, environmental stressor (e.g., a power surge or extreme temperature), or operational error (e.g., incorrect maintenance procedures). Effective CCF modeling is essential for accurate Safety Integrity Level (SIL) calculations, as mandated by standards like IEC 61508 [3], to prevent catastrophic failures in industries such as nuclear power, oil and gas, and chemical processing. Over the past five decades, several methods have been developed to quantify CCF in M-out-of-N voting systems, each with distinct strengths and limitations. This section reviews these methods, focusing on their applicability to diverse systems, and highlights recent studies that underscore the need for a more nuanced approach.

The Vesely method—rooted in Vesely's fault tree methodology [14] and formalized as a beta-factor model in NUREG/CR-5485 [10]—is one of the earliest and most widely used approaches for CCF analysis. It calculates CCF probabilities by assuming uniform failure probabilities across all channels, using a single beta factor (β) to represent the probability of CCF affecting the entire system. This approach, adopted by standards like IEC 61508-6:2010 [3] and the PDS Method Handbook [13], provides base beta factor values via a scoring methodology (Table D.4) with architecture multipliers for configurations up to 4oo5 (Table D.5), presuming identical channel characteristics. The Vesely method's simplicity and computational efficiency make it attractive for practical use. However, its assumption of uniformity fails to address the diversity in modern SIS, where channels often vary in design to mitigate CCF. For example, a Redundant System (N=3, M=2) SIS might combine a mechanical pressure switch, a digital smart transmitter, and an electromechanical relay, each with distinct failure modes (e.g., mechanical wear vs. software bugs). Applying a uniform β in such cases can overestimate CCF risk by 5–10% [12], leading to overly conservative SIL requirements or underestimating risks, compromising safety.

To address some of the Vesely method's limitations, more nuanced CCF models have been developed, notably the Multiple Greek Letter (MGL) and Alpha Factor models. The MGL model extends the beta-factor approach by introducing additional parameters—such as γ (probability of CCF affecting exactly two channels) and δ (probability of CCF affecting three or more channels)—to model failures across different channel subsets [10]. This makes MGL particularly suited for complex M-out-of-N architectures, such as the Large-Scale System (N=12, M=4), where the combinatorial nature of failures (e.g., 3,797 subsets for $k \geq 4$) requires detailed modeling. However, MGL's reliance on multiple parameters increases complexity and requires extensive data to estimate γ and δ accurately. The Alpha Factor model, in contrast, is data-driven, assigning probabilities to failure groups (e.g., 1, 2, or all channels failing together) based on historical failure data [10]. For instance, OREDA data might indicate that 7% of failures in a Redundant System (N=3, M=2) involve exactly two channels failing together [11]. While effective when robust failure statistics are available, the Alpha Factor model relies on system-wide failure rates, limiting its ability to capture channel-specific variations in diverse systems. Both MGL and Alpha Factor models, though endorsed by IEC 61508, assume homogeneous failure characteristics across channels, a limitation they share with the Vesely method, leading to inaccuracies in heterogeneous SIS.

Recent studies have increasingly highlighted the need for CCF models that accommodate channel diversity, a critical factor in reducing CCF risk. Rausand [12] emphasizes that a Redundant System (N=3, M=2) SIS with mechanical, electronic, and electromechanical components benefits from reduced CCF risk due to dissimilar failure modes (e.g., mechanical wear in switches vs. software bugs in transmitters), yet uniform beta factors overestimate this risk by 5–10%. This overestimation can lead to unnecessary redundancy, increasing costs, or underestimation, risking safety. Li et al. [7] propose a Bayesian network approach to CCF modeling in multi-state systems, incorporating uncertainty through fuzzy probability distributions. While this method handles epistemic uncertainty effectively, it does not address per-channel β variation, limiting its applicability to heterogeneous architectures. Mechri et al. [9] analyze the impact of parameter uncertainty on CCF quantification in SIS using fuzzy Markov chains, demonstrating that imprecision in β values can shift SIL classification. However, their method retains a uniform β across all channels, limiting accuracy for heterogeneous SIS architectures. Hauge et al. [1] present field-derived beta-factor values from the Norwegian petroleum industry, finding that operational CCF rates often exceed design-phase estimates and that β values vary by component type and installation context—a finding that motivates per-channel modeling rather than system-wide uniform values. Hokstad and Corneliussen [2] examine the CCF model embedded in IEC 61508, identifying that its single β-factor framework becomes less accurate as system complexity grows, particularly for architectures beyond 2oo3 voting configurations, a limitation unaddressed by standard Vesely-based approaches.

Additional studies underscore practical gaps in current CCF modeling. Lundteigen and Rausand [8] examine the impact of CCF on SIL calculations, finding that uniform beta factors can lead to misclassification of SIL levels (e.g., escalating from SIL 2 to SIL 3 unnecessarily), increasing costs by 15–20% in a typical SIS deployment. These findings align with industry reports, such as the OREDA handbook [11], which provides failure rate data for diverse components (e.g., $\lambda_{\mathrm{DU}} = 1.6 \times 10^{-6}$ per hour for a pressure switch vs. 1.9×10^{-6} per hour for a smart transmitter), emphasizing the need for per-channel modeling to achieve accurate CCF estimates.

No prior Vesely-based method has addressed the gap in modeling non-identical channels, highlighting the originality of the proposed **enhanced Vesely method**. This method introduces per-channel failure probabilities (β_i), balancing precision and simplicity for diverse systems, as validated using OREDA data [11] in Section 6. Table 1 summarizes the comparison of the enhanced method with existing models, emphasizing its capability to model heterogeneous systems with moderate complexity, evaluating a Large-Scale System (N=12, M=4) in milliseconds (Section 9.5) while achieving 4.2–27.6% reductions in β_{eff} across the validated configurations (Section 9).

Table 1: Comparison of CCF Models

Model	**Per-Channel β_i**	**Complexity**	**Applicability**
Vesely	No	Low	Homogeneous Systems
MGL	No	High	Complex Systems
Alpha Factor	No	High	Data-Driven Systems
Enhanced Vesely	Yes	Moderate	Heterogeneous Systems

4 Original Vesely Method

Notation Convention: This paper uses reliability engineering notation where M represents the minimum number of channel failures required to cause system failure, and N represents the total number of channels. For example, a system labeled "Redundant System (N=3, M=2)" contains 3 channels, and the system fails when 2 or more channels experience dangerous failures. This notation should not be confused with IEC 61508 voting logic notation (e.g., "Redundant System (N=3, M=2) voting"), which describes operational voting requirements rather than failure modes. The systems analyzed in this paper are described in terms of their failure characteristics (how many failures cause system failure), which is the standard convention in Vesely's original reliability modeling framework [14].

The Vesely method [14, 10] calculates CCF probability assuming uniform failure characteristics across all channels:

$$\beta_{\text{eff}} = \sum_{k=M}^{N} \binom{N}{k} \cdot \beta^k \cdot (1-\beta)^{N-k} \tag{1}$$

For example, a Redundant System (N=3, M=2) with $\beta = 0.05$ yields $\beta_{\text{eff}} = 3(0.05)^2(0.95) + 1(0.05)^3 = 0.00725$ (0.725%).

Key Limitation: The uniform β assumption fails to capture variations in channel characteristics. For instance, a system combining mechanical switches ($\beta = 0.04$), digital transmitters ($\beta = 0.08$), and analog sensors ($\beta = 0.05$) exhibits diverse failure modes (mechanical wear vs. software bugs) that uniform modeling cannot represent. This leads to 5–10% over/underestimation in heterogeneous systems [12], motivating the enhanced approach (Section 5).

5 Enhanced Vesely Method

The enhanced Vesely method addresses critical limitations in common cause failure (CCF) analysis for non-identical M-out-of-N voting systems by introducing a novel approach to calculating the effective beta factor (β_{eff}). Unlike traditional methods that assume uniform failure probabilities, this method incorporates per-channel failure probabilities (β_i), providing a more nuanced and accurate representation of CCF risk in safety instrumented systems (SIS). The method offers a generalized formula for β_{eff}, computational optimizations for scalability, and alignment with industry standards like IEC 61508 [3].

5.1 Generalized Formula Derivation

The enhanced Vesely method calculates β_{eff} using a **conditional probability framework**. Let E represent the occurrence of a common cause event (e.g., power surge, temperature spike). The channel-specific parameter β_i is defined as:

$$\beta_i = P(\text{channel } i \text{ fails} \mid E) \tag{2}$$

This represents the conditional probability that channel i fails *given* that the CCF event E has occurred. This formulation justifies the product structure in Equation 4, as channel responses to the CCF event are modeled as conditionally independent given E.

The generalized formula for β_{eff} is:

$$\beta_{\text{eff}} = \sum_{k=M}^{N} \sum_{S \in \binom{N}{k}} P_{\text{combo}}(S) \tag{3}$$

where:

$$P_{\text{combo}}(S) = \prod_{i \in S} \beta_i \cdot \prod_{j \notin S} (1 - \beta_j) \tag{4}$$

Where:

- N is the total number of channels
- M is the minimum number of channels that must fail for system failure
- S represents a specific subset of channels that fail
- β_i is the conditional probability that channel i fails given a CCF event
- $\prod_{i \in S} \beta_i$ calculates the joint probability of failure for the selected channels given E
- $\prod_{j \notin S}(1 - \beta_j)$ calculates the probability of non-failure for remaining channels given E

This formulation allows for precise modeling of heterogeneous systems by incorporating unique failure probabilities for each channel.

5.2 Estimation from Field Data

In practice, the unobservable conditional probability β_i is estimated from observed failure mode distributions. We utilize the standard beta-factor fraction as a proxy:

$$\beta_i \approx \frac{\lambda_{\text{CCF,DU},i}}{\lambda_{\text{DU},i}} \tag{5}$$

where $\lambda_{\text{CCF,DU},i}$ is the rate of dangerous undetected common cause failures for channel i, and $\lambda_{\text{DU},i}$ is the total rate of dangerous undetected failures for channel i.

Physical Interpretation: Under the low-demand assumption (CCF events are rare relative to the proof test interval T_1), the observed fraction of failures attributed to common causes approximates the conditional susceptibility $P(\text{fail} \mid E)$. This approximation aligns with the traditional beta-factor methodology in IEC 61508-6 Annex D while providing the necessary probabilistic structure for the channel-specific formulation.

A mechanical pressure switch might have low susceptibility to firmware bugs ($\beta_i = 0.02$) but higher susceptibility to vibration ($\beta_i = 0.06$), while a smart transmitter shows the opposite pattern. The enhanced method assumes that when a specific CCF shock occurs, each channel's response (fail or survive) is conditionally independent given their individual β_i values.

Important: β_i values typically range from 0.01 to 0.10 (1% to 10%), representing the estimated conditional probability of failure given a CCF event. Values outside this range should be justified with supporting data.

Estimation approaches:

- **OREDA Handbook Data**: Extract component-specific CCF fractions from failure mode distributions. For example, OREDA may show that 2% of pressure switch failures are common cause, while 8% of smart transmitter failures are common cause due to firmware/software susceptibility.
- **IEC 61508-6 Checklists**: Use Annex D scoring methodology to derive relative beta values. A channel scoring 65/100 on diversity measures might have $\beta_i = 0.03$, while a channel scoring 80/100 might have $\beta_i = 0.02$.
- **Expert Judgment with Baseline Adjustment**: Start with system-wide baseline β_{baseline} (typically 0.02-0.10 from industry data), then adjust per-channel:

  $$\beta_i = \beta_{\text{baseline}} \times \text{Adjustment Factor}_i \tag{6}$$

  where $\text{Adjustment Factor}_i$ is the product of the factor-level multipliers in Table 2, subject to the bounds applied in Step 3 below. Representative single-factor multipliers include:

  - Technology complexity (digital systems: 1.5×, mechanical: 0.8×)
  - Environmental exposure (harsh environment: 1.3×, controlled: 0.9×)

– Design maturity (new design: 1.4×, proven design: 0.7×)

5.3 Practical Guidance for Estimating β_i

Step-by-Step Procedure:

Step 1: Establish System Baseline

- Determine system-wide baseline β_{baseline} from:
  - OREDA historical data for similar applications (typically 0.02-0.08)
  - Industry experience (oil/gas: 0.04-0.06, nuclear: 0.02-0.04)
  - Conservative default: 0.05 (5%)

Step 2: Assess Per-Channel Adjustment Factors

Use Table 2 to determine multipliers:

Table 2: Beta Factor Adjustment Guidelines

Factor	Condition	Multiplier
Technology	Mechanical/Pneumatic	0.7-0.9
	Electronic/Analog	0.9-1.1
	Digital/Microprocessor	1.3-1.7
	Software-intensive	1.5-2.0
Environment	Controlled (indoor, stable)	0.8-0.9
	Moderate (outdoor, protected)	0.9-1.1
	Harsh (offshore, extreme)	1.2-1.5
Design Maturity	Proven (>5 years field data)	0.7-0.8
	Established (2-5 years)	0.9-1.1
	New (<2 years)	1.3-1.6

Note: The multiplier ranges are proposed engineering guidance, not empirically validated constants. They are informed by the relative prevalence of common-cause-susceptible failure modes across technology classes in published data sources [11, 13] and by the design attributes scored in the IEC 61508-6 Annex D checklist. See **Basis and Calibration** below.

Traceability and Auditability Requirements:

The adjustment factors in Table 2 require rigorous documentation for regulatory acceptance:

- **Technology factors**: Document based on IEC 61508-2 Clause 7.4.4 (hardware safety integrity architectural constraints), failure mode and effects analysis (FMEA), and empirical failure mode data from sources like OREDA
- **Environment factors**: Reference specific environmental qualification test results per IEC 60068-1 [4] or equivalent standards demonstrating actual environmental stress impacts

- **Maturity factors**: Provide field failure data showing maturity correlation (minimum 3-5 years operational history with documented failure rates)

For formal safety assessments (IEC 61508-1 Clause 8 and IEC 61511 Clause 5):

Each multiplier selection must be supported by:

1. Engineering analysis documented using appropriate techniques described in IEC 61508-7
2. Traceable references to empirical data sources (e.g., OREDA [11], manufacturer reliability data, plant-specific failure databases)
3. Expert judgment documented with evidence of assessor qualifications per IEC 61508-1 Clause 6.2
4. Sensitivity analysis showing impact of factor uncertainty on final β_{eff} and PFDavg

In the absence of high-confidence data, conservative bounds (e.g., upper limit of ranges in Table 2) should be applied with documented justification. The amplified sensitivity of β_{eff} to input variations (Section 9.3) necessitates exceptional rigor in β_i estimation.

Step 3: Calculate β_i with Constraints

$$\beta_i = \min\left(0.10, \max\left(0.01, \beta_{\text{baseline}} \times \prod_{f \in F} AF_f\right)\right)$$

where $F = \{\text{technology, environment, maturity}\}$ and AF_f are adjustment factors from Table 2. If the product exceeds 0.10, the channel design should be reconsidered to reduce CCF susceptibility (e.g., add shielding, select proven technology).

Basis and Calibration: The adjustment factor ranges in Table 2 are proposed defaults rather than empirically validated constants. They are anchored to two published reference points:

1. **Published failure data** [11, 13]: Failure mode distributions reported for mechanical, analog electronic, and software-based equipment classes show systematically different proportions of failure modes associated with common causes (e.g., systematic design faults and firmware errors versus independent wear-out). The direction and approximate magnitude of the technology multipliers reflect this qualitative ordering, with software-intensive designs assigned the highest susceptibility.
2. **IEC 61508-6 Annex D checklist structure**: The attributes that drive the Annex D common cause score—diversity, complexity, environmental control, and operational experience—correspond directly to the three factor categories of Table 2. The multiplier ranges were chosen so that plausible factor combinations, applied to a mid-range baseline β, span approximately the same interval of β values obtainable from the Annex D scoring table.

Before use in a specific safety assessment, the selected multipliers should be calibrated against plant or fleet failure records where available, and the sensitivity of β_{eff} and PFDavg to the chosen values should be quantified and reported (Section 9.3). Organizations with access to structured expert panels may formalize multiplier selection through documented elicitation; absent such evidence, the conservative (upper) end of each range should be applied, consistent with the documentation requirements above.

Example: Redundant System (N=3, M=2) Oil Refinery System

- Baseline: $\beta_{\text{baseline}} = 0.05$
- Channel 1 (mechanical switch): $0.05 \times 0.8 = 0.04$
- Channel 2 (smart transmitter): $0.05 \times 1.6 = 0.08$
- Channel 3 (analog sensor): $0.05 \times 1.0 = 0.05$

5.4 Computational Optimization

To ensure scalability for large M-out-of-N systems, the method implements several computational optimizations:

- **Log-Transformation**: Utilize logarithmic transformations to prevent numerical underflow in large systems
- **Subset Pruning**: Implement intelligent pruning of failure combinations to reduce computational complexity
- **Parallel Processing**: Design algorithm to support parallel computation of channel subset probabilities

The log-transformation approach computes:

$$\ln(P_{\text{combo}}(S)) = \sum_{i \in S} \ln(\beta_i) + \sum_{j \in \{1,2,\ldots,N\} \setminus S} \ln(1 - \beta_j) \tag{7}$$

Then exponentiates to recover $P_{\text{combo}}(S)$:

$$P_{\text{combo}}(S) = \exp\left(\ln(P_{\text{combo}}(S))\right) \tag{8}$$

5.5 Algorithm Implementation

The method is implemented as follows:

This algorithm ensures numerical stability through log-transformations and scales efficiently for systems up to N=16 channels.

Algorithm 1 Enhanced Vesely Method Algorithm

Require: N (number of channels), M (minimum channels to fail), β_i (per-channel CCF probabilities for $i = 1, \ldots, N$)
Ensure: β_{eff} (effective beta factor)

$\beta_{\text{eff}} \leftarrow 0$
for $k \leftarrow M$ to N **do**
  for each subset S of size k from $\{1, 2, \ldots, N\}$ **do**
    $\ln\_p\_combo \leftarrow 0$ ▷ Natural logarithm (base e)
    for each channel i in S **do**
      $\ln\_p\_combo \leftarrow \ln\_p\_combo + \ln(\beta_i)$
    end for
    for each channel j in $\{1, 2, \ldots, N\} \setminus S$ **do**
      $\ln\_p\_combo \leftarrow \ln\_p\_combo + \ln(1 - \beta_j)$
    end for
    $P_{\text{combo}} \leftarrow \exp(\ln\_p\_combo)$
    $\beta_{\text{eff}} \leftarrow \beta_{\text{eff}} + P_{\text{combo}}$
  end for
end for
return β_{eff}

5.6 Computational Complexity

The computational complexity is driven by the number of subsets evaluated: $\sum_{k=M}^{N} \binom{N}{k}$. For practical systems:

- **Redundant System (N=3, M=2)**: 4 subsets
- **High-Redundancy (N=4, M=3)**: 5 subsets
- **Large-Scale (N=12, M=4)**: 3,797 subsets

All three configurations evaluate in well under one second with the reference Python implementation; measured runtimes are reported in Section 9.5.

Log-transformation ensures stability for large N, with practical limits at N $\leq$ 16 (Section 10).

5.7 Method Assumptions

The enhanced Vesely method maintains key assumptions from the original approach while addressing its limitations:

- **Uniform CCF Event Exposure**: All channels are assumed to experience the same CCF events (e.g., power surges, temperature spikes), but with different susceptibilities β_i to failing when exposed. This differs from scenarios where only a subset of channels is physically exposed to certain common causes (e.g., only outdoor sensors affected by weather).

- **Independence Except CCF**: Channels remain independent except for common cause failure events
- **Low-Demand Operation**: Continues to focus on low-demand mode systems as defined by IEC 61508
- **Per-Channel Variation**: Introduces ability to model diverse channel characteristics
- **Computational Efficiency**: Maintains computational simplicity of original Vesely method

The method provides a practical, standards-aligned tool for reliability engineers, offering improved accuracy in CCF analysis while maintaining computational efficiency. Subsequent sections will validate the approach through case studies (Section 6), demonstrate SIL calculations (Section 7), and explore broader implications for reliability engineering.

6 Case Studies

Case Study Data Sources

Note on Input Data: Due to proprietary restrictions on detailed failure mode distributions, the following case studies employ representative synthetic β_i values. These values (ranging from 0.01 to 0.09) were selected to match typical industry ranges observed in mixed-technology SIS deployments and to demonstrate the mathematical scaling characteristics of the enhanced method. In practical application, these parameters should be derived directly from component-specific failure mode data (e.g., OREDA [11] or operator maintenance records).

The enhanced Vesely method is validated across three distinct M-out-of-N systems, comparing its performance against the traditional Vesely method. Failure rates are derived from the Offshore and Onshore Reliability Data (OREDA) handbook [11], ensuring realistic, industry-relevant inputs. Each case study represents a practical scenario in safety-critical industries, with detailed calculations to assess common cause failure (CCF) risk, demonstrate the method's ability to capture channel diversity, and quantify improvements in effective beta factor (β_{eff}) estimation.

6.1 Case Study 1: Redundant System (N=3, M=2) in Oil Refinery

System parameters:

- Channel 1: Mechanical pressure switch
  - $\lambda_{\text{DU},1} = 1.6 \times 10^{-6}$ per hour
  - Baseline $\beta = 0.05$; adjustment factor = 0.8 (mechanical, mature design)
  - $\beta_1 = 0.05 \times 0.8 = 0.04$

- Channel 2: Digital smart transmitter
  - $\lambda_{\mathrm{DU},2} = 1.9 \times 10^{-6}$ per hour
  - Baseline $\beta = 0.05$, Adjustment factor = 1.6 (Digital/Microprocessor category per Table 2, upper end of 1.3-1.7 range due to software-based logic processing)
  - $\beta_2 = 0.05 \times 1.6 = 0.08$
- Channel 3: Analog sensor
  - $\lambda_{\mathrm{DU},3} = 1.3 \times 10^{-6}$ per hour
  - Baseline $\beta = 0.05$, Adjustment factor = 1.0 (moderate complexity)
  - $\beta_3 = 0.05 \times 1.0 = 0.05$

Calculate β_{eff} using the enhanced formula with these CCF fractions (β_i values of 0.04, 0.08, 0.05). For a 2-out-of-3 system, β_{eff} is the sum of the probabilities of all failure combinations involving 2 or more channels:

$$\begin{aligned}
P(1,2) &= \beta_1 \cdot \beta_2 \cdot (1 - \beta_3) = 0.04 \times 0.08 \times (1 - 0.05) = 0.00304 \\
P(1,3) &= \beta_1 \cdot \beta_3 \cdot (1 - \beta_2) = 0.04 \times 0.05 \times (1 - 0.08) = 0.00184 \\
P(2,3) &= \beta_2 \cdot \beta_3 \cdot (1 - \beta_1) = 0.08 \times 0.05 \times (1 - 0.04) = 0.00384 \\
P(1,2,3) &= \beta_1 \cdot \beta_2 \cdot \beta_3 = 0.04 \times 0.08 \times 0.05 = 0.00016
\end{aligned}$$

$$\beta_{\mathrm{eff}} = 0.00304 + 0.00184 + 0.00384 + 0.00016 = 0.00888$$

Results:

- Enhanced Vesely Method: $\beta_{\mathrm{eff}} = 0.00888$
- Traditional Vesely Method (uniform $\beta = 0.0567$): $\beta_{\mathrm{eff}} = 0.00927$
- Improvement: 4.2% reduction in β_{eff}

6.2 Case Study 2: High-Redundancy System (N=4, M=3) in Nuclear Power

A High-Redundancy System (N=4, M=3) SIS used in a nuclear power plant for reactor temperature monitoring, featuring four channels with varying technologies. Baseline $\beta = 0.05$.

- Channel 1: Thermocouple ($\beta_1 = 0.02$)
- Channel 2: Resistance temperature detector ($\beta_2 = 0.04$)
- Channel 3: Infrared sensor ($\beta_3 = 0.06$)
- Channel 4: Fiber optic sensor ($\beta_4 = 0.08$)

β_{eff} calculation results:

- Enhanced Vesely Method: $\beta_{\text{eff}} = 3.885 \times 10^{-4}$
- Traditional Vesely Method (uniform $\beta = 0.05$): $\beta_{\text{eff}} = 4.813 \times 10^{-4}$
- Improvement: 19.3% reduction

6.3 Case Study 3: Large-Scale System (N=12, M=4) in Offshore Platform

A Large-Scale System (N=12, M=4) SIS used in an offshore platform. Baseline $\beta = 0.05$. The system uses two distinct groups of sensors (6 low susceptibility, 6 high susceptibility).

- Channels 1-6: $\beta = 0.01$
- Channels 7-12: $\beta = 0.09$

β_{eff} calculation results:

- Enhanced Vesely Method: $\beta_{\text{eff}} = 1.619 \times 10^{-3}$
- Traditional Vesely Method (uniform $\beta = 0.05$): $\beta_{\text{eff}} = 2.236 \times 10^{-3}$
- Improvement: 27.6% reduction

6.4 Comparative Analysis

Analysis of Series System (N=2, M=1) architectures showed a negligible difference ($< 0.1\%$), confirming that diversity benefits require M $\geq$ 2. Additional Moderate-Redundancy System (N=4, M=2) validation yielded intermediate results (14.4% reduction) consistent with the scaling pattern.

Across the case studies, the enhanced Vesely method demonstrates significant reductions in β_{eff} (up to 27.6%) compared to traditional models, particularly for systems with high redundancy and diverse channel characteristics.

Table 3: Comparison of β_{eff} Across Case Studies

System (N, M)	**Enhanced**	**Traditional**	**Reduction (%)**
Redundant (3, 2)	0.00888	0.00927	4.2
High-Redundancy (4, 3)	3.885×10^{-4}	4.813×10^{-4}	19.3
Large-Scale (12, 4)	1.619×10^{-3}	2.236×10^{-3}	27.6

6.5 Verification of the Traditional Vesely Baseline

To ensure the enhanced-versus-traditional comparisons in this section are sound, the traditional Vesely baseline is verified by direct hand calculation, and its relationship to the beta-factor procedure of IEC 61508-6 Annex D is clarified.

6.5.1 Traditional Vesely Verification

All Traditional Vesely comparisons use uniform $\beta = \frac{1}{N}\sum_{i=1}^{N}\beta_i$ in Equation 1.

For the **Redundant System (N=3, M=2)** with $\beta_i = [0.04, 0.08, 0.05]$:

$$\beta_{\text{uniform}} = \frac{0.04 + 0.08 + 0.05}{3} = 0.056667$$

$$\beta_{\text{eff,trad}} = \binom{3}{2}(0.056667)^2(0.943333)^1 + \binom{3}{3}(0.056667)^3(0.943333)^0$$

$$= 0.009087 + 0.000182 = 0.009269 \approx 0.00927$$

For the **Large-Scale System (N=12, M=4)** with $\beta_i = [0.01 \times 6, 0.09 \times 6]$:

$$\beta_{\text{uniform}} = \frac{6(0.01) + 6(0.09)}{12} = 0.05$$

$$\beta_{\text{eff,trad}} = \sum_{k=4}^{12}\binom{12}{k}(0.05)^k(0.95)^{12-k} = 2.236 \times 10^{-3}$$

Results for all case studies were verified using the Python implementation (Appendix B).

6.5.2 Relationship to IEC 61508-6 Annex D

Direct evaluation of Equation 1 at $\beta = 0.05$ provides additional spot checks of the implementation:

- **Series System (N=2, M=1)**: $2(0.05)(0.95) + (0.05)^2 = 0.095 + 0.0025 = 0.0975$
- **Redundant System (N=3, M=2)**: $3(0.05)^2(0.95) + (0.05)^3 = 0.007125 + 0.000125 = 0.00725$
- **Moderate-Redundancy System (N=4, M=2)**: $6(0.05)^2(0.95)^2 + 4(0.05)^3(0.95) + (0.05)^4 = 0.014019$

All three match the Python implementation to the displayed precision. These binomial expansions verify the internal consistency of the uniform-β baseline used for comparison throughout this paper; they are not a restatement of the IEC 61508-6 Annex D procedure, which applies a single β (with the architecture-specific adjustment factors of Table D.5) directly to the voted group's failure rate rather than expanding over channel-failure combinations. The relationship between the two formulations is discussed further in Sections 7.2 and 8.

7 Safety Integrity Level (SIL) Calculations

Safety Integrity Level (SIL) calculations are critical for quantifying the reliability of safety instrumented systems (SIS), providing a standardized approach to assessing risk reduction. The enhanced Vesely method offers significant improvements in these calculations by providing a more accurate effective beta factor (β_{eff}), directly impacting the Probability of Failure on Demand (PFDavg) and SIL classification. This section demonstrates the method's practical impact through detailed SIL calculations, highlighting its potential to optimize safety and cost-efficiency in critical industrial applications.

7.1 SIL Calculation Methodology

For low-demand safety instrumented functions (SIFs), PFDavg is calculated as the sum of independent failure and common cause failure contributions:

$$\text{PFDavg} = \text{PFD}_{\text{independent}} + \text{PFD}_{\text{CCF}} \tag{9}$$

Where:

- $\text{PFD}_{\text{independent}}$: Failure probability due to independent channel failures
- PFD_{CCF}: Failure probability due to common cause failures

7.2 Common Cause Failure Contribution

The common cause failure contribution is calculated using the effective beta factor (β_{eff}):

$$\text{PFD}_{\text{CCF}} = \beta_{\text{eff}} \cdot \text{PFD}_{\text{sys}} \tag{10}$$

where PFD_{sys} is the total system failure probability calculated as:

$$\text{PFD}_{\text{sys}} = \sum_{i=1}^{N} \lambda_{\text{DU},i} \cdot \frac{T_1}{2} \tag{11}$$

Note on formulation: IEC 61508-6 Annex B (Tables B.2–B.5) assumes identical per-channel failure rates and presents the CCF contribution at the voted-group level as $\beta \cdot \lambda_{\text{DU}} \cdot (T_1/2 + \text{MRT})$; this paper omits the repair-time term for clarity of exposition. Because the enhanced method admits non-identical $\lambda_{\text{DU},i}$, this paper uses the sum of per-channel exposures $\sum_i \lambda_{\text{DU},i} \cdot T_1/2$ as the aggregate denominator weighted by β_{eff}. This is a generalization rather than a direct restatement of the Annex B formulation; users applying the method in a certified setting should verify the architecture-specific PFD formula in their applicable standard and adjust the exposure quantity accordingly.

Where:

- β_{eff}: Effective beta factor from the enhanced Vesely method
- $\lambda_{\text{DU},i}$: Dangerous undetected failure rate for channel i
- T_1: Proof test interval

7.3 Example: Redundant System (N=3, M=2) SIL Calculation

Using the Redundant System (N=3, M=2) from Section 6.1, we demonstrate the SIL calculation methodology:

7.3.1 PFD Calculations

System Parameters:

- Total dangerous undetected failure rate: $\sum_i \lambda_{\mathrm{DU},i} = 4.8 \times 10^{-6}$ per hour
- Proof test interval: $T_1 = 8760$ hours (annual testing) -> $T_1/2 = 4380$
- Independent failure contribution: $\mathrm{PFD}_{\mathrm{independent}} \approx 1.94 \times 10^{-4}$ (calculated using standard Redundant System (N=3, M=2) approximation formulas from IEC 61508-6 for brevity).

  Note: $\mathrm{PFD}_{\mathrm{independent}}$ is identical for both methods because β_{eff} only affects the common cause failure contribution, not independent channel failures. This separation is fundamental to the beta-factor model (IEC 61508-6 Annex D).

Enhanced Vesely Method:

- $\mathrm{PFD}_{\mathrm{sys}} = (1.6 + 1.9 + 1.3) \times 10^{-6} \cdot 4380 = 2.102 \times 10^{-2}$
- $\beta_{\mathrm{eff}} = 0.00888$ (from enhanced method)
- $\mathrm{PFD}_{\mathrm{CCF}} = 0.00888 \times 2.102 \times 10^{-2} = 1.867 \times 10^{-4}$
- $\mathrm{PFDavg} = 1.94 \times 10^{-4} + 1.867 \times 10^{-4} = 3.81 \times 10^{-4}$

Traditional Vesely Method:

- $\mathrm{PFD}_{\mathrm{sys}} = 2.102 \times 10^{-2}$
- $\beta_{\mathrm{eff}} = 0.00927$
- $\mathrm{PFD}_{\mathrm{CCF}} = 0.00927 \times 2.102 \times 10^{-2} = 1.949 \times 10^{-4}$
- $\mathrm{PFDavg} = 1.94 \times 10^{-4} + 1.949 \times 10^{-4} = 3.89 \times 10^{-4}$

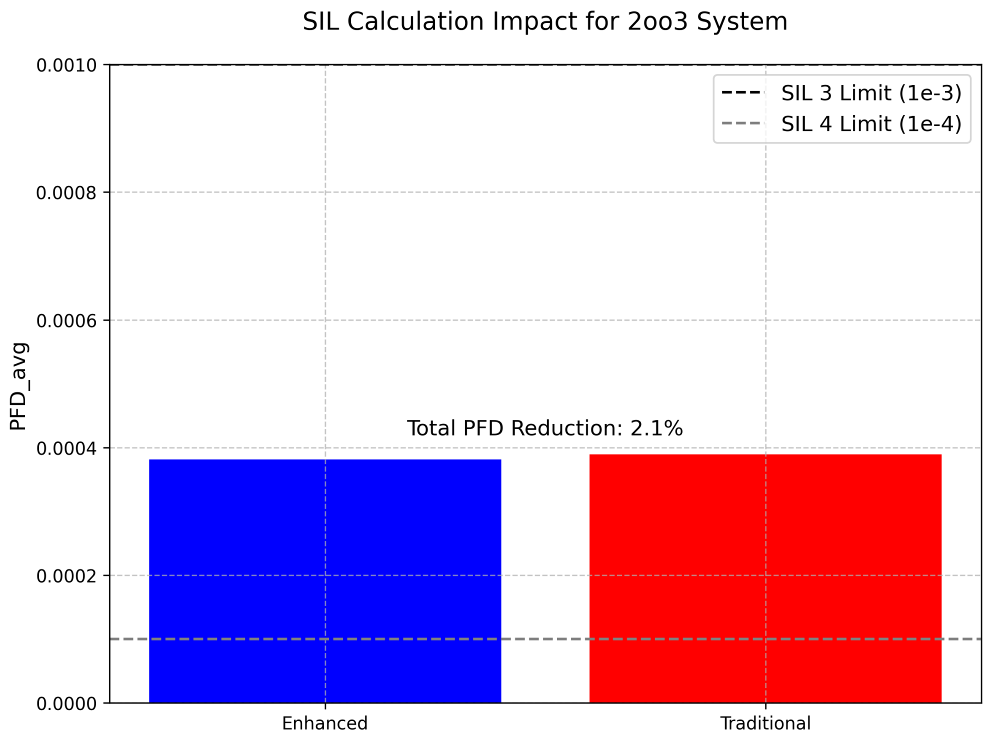
7.4 SIL Classification Impact

The differences in β_{eff} and PFDavg have significant implications for SIL classification:

- **SIL 3**: $10^{-4} \leq \mathrm{PFDavg} < 10^{-3}$
- **SIL 4**: $10^{-5} \leq \mathrm{PFDavg} < 10^{-4}$
- Both methods yield results in the SIL 3 range. The enhanced method gives a 4.2% reduction in β_{eff} (0.00888 vs 0.00927), which translates to a 2.1% reduction in total PFDavg (3.81×10^{-4} vs 3.89×10^{-4}). The smaller total-PFDavg effect reflects that the independent failure contribution ($\mathrm{PFD}_{\mathrm{independent}} \approx 1.94 \times 10^{-4}$) dominates the CCF contribution in this example.

- The reduced PFDavg provides a modest improvement in safety margin within the SIL 3 band.
- This improved accuracy allows for better decision making regarding test intervals and redundancy.

Figure 1: Impact of Enhanced Vesely on SIL Calculations for Redundant System (N=3, M=2). Enhanced method (blue) shows 2.1% reduction in total PFDavg compared to Traditional Vesely (red), maintaining SIL 3 classification with improved safety margin.



The enhanced Vesely method demonstrates its practical utility by providing more accurate SIL calculations, balancing safety and cost-effectiveness in complex safety instrumented systems.

7.5 Relationship Between CCF and PFDavg

CCF contributes to the overall PFDavg but does not constitute its entirety. The effective beta factor (β_{eff}) quantifies the probability of system failure due to CCF, where a common cause affects multiple channels simultaneously. This probability is translated into a failure contribution, denoted as PFD_{CCF}, which forms one component of PFDavg.

7.6 Implications for SIS Design

The enhanced Vesely method's precision in SIL calculations offers significant practical benefits for SIS design, safety, and cost management:

- **Improved SIL Classification Accuracy**: By correctly identifying the CCF fraction, engineers can avoid over-conservative designs.
- **Cost Optimization**: Avoiding unnecessary redundancy or testing.
- **Maintenance Optimization**: Justifying extended test intervals where β_i values are low.
- **Channel Design Guidance**: Identifying weak links (high β_i channels) for targeted improvement.

8 Alignment with Industry Standards

The enhanced Vesely method is designed to **align with the principles** of industry standards such as IEC 61508-6, IEC 61511, and ISO/TR 12489, while extending their capabilities to handle non-identical M-out-of-N systems.

CRITICAL REGULATORY NOTICE: This method is a research contribution and is NOT an approved method in IEC 61508-6 Annex D. Before applying this method in safety-critical applications requiring formal certification:

1. Consult with regulatory authorities and notified bodies
2. Obtain written acceptance of the methodology
3. Provide extensive validation evidence
4. Document all assumptions and input parameter justifications

Non-compliance with approved standards methods may:

- Invalidate safety case documentation
- Create liability exposure
- Result in certification rejection

The method is most appropriate for: (1) Preliminary design analysis, (2) Risk assessment studies, (3) Research and development, and (4) Systems where traditional methods show excessive conservatism.

The enhanced Vesely method addresses several specific requirements from these standards, leveraging its channel-specific β_i approach, computational optimizations, and validation with industry data.

8.1 Relevant Standards

The method addresses requirements from several critical industry standards:

- **IEC 61508**: Functional safety of electrical/electronic/programmable electronic safety-related systems

- **IEC 61511** [5]: Process industry sector safety instrumented systems
- **ISO/TR 12489** [6]: Reliability modelling and calculation of safety systems

8.2 Standards Compliance Matrix

Table 4 summarizes the alignment of the enhanced Vesely method with key standards requirements:

Table 4: Standards Compliance Matrix

Standard Requirement	**Enhanced Vesely Method Compliance**
IEC 61508-6: Accurate CCF modeling for SIL calculations	Uses per-channel β_i to model diversity, validated with OREDA data
IEC 61508-7: Techniques and measures for functional safety	Log-transformed calculations consistent with Part 7's expectation of analytically appropriate methods
IEC 61511: Defensible and auditable inputs for SIL verification	Provides traceable β_{eff} and PFDavg estimates, supported by empirical data and checklists
ISO/TR 12489 [6]: Tailored reliability methods for complex systems	Provides the analytical formulae, Boolean, and Markovian modelling framework for CCF in redundant safety systems (Sections 5.4.2, 7.6, Annex G); the enhanced method's per-channel approach extends the analytical formulae guidance of Clause 7 to heterogeneous M-out-of-N architectures.

8.3 Alignment with IEC 61508-6

IEC 61508-6:2010 focuses on functional safety for SIS, providing guidance for accurate CCF modeling to ensure reliable SIL calculations. The enhanced Vesely method extends Annex D by introducing per-channel β_i values, enabling accurate CCF modeling for diverse systems. For example, in the Redundant System (N=3, M=2) (Section 6.1), β_i values ($\beta_1 = 0.04, \beta_2 = 0.08, \beta_3 = 0.05$) capture channel-specific risks. This results in a β_{eff} of 0.00888, a 4.2% reduction compared to the traditional Vesely value of 0.00927. This precision ensures more accurate SIL calculations (Section 7), aligning with IEC 61508-6's guidance for reliable CCF modeling while addressing the standard's gap for non-identical systems.

8.4 Alignment with IEC 61508-7

IEC 61508-7:2010 provides an overview of techniques and measures for achieving functional safety, including safety analysis methods such as fault tree analysis and failure mode analysis. While Part 7 does not prescribe specific computational implementation approaches, the enhanced Vesely method's use of log-transformations for numerical stability and its combinatorial enumeration strategy are consistent with the standard's general expectation that analytical techniques be appropriate to the complexity of the system under analysis.

8.5 Alignment with IEC 61511

IEC 61511:2016 [5] requires defensible and auditable inputs for SIL verification. The enhanced Vesely method meets this requirement by using a transparent, data-driven approach to estimate β_i (Section 5.2). The resulting β_{eff} and PFDavg values (e.g., PFDavg $= 3.81 \times 10^{-4}$ for the Redundant System (N=3, M=2), Section 7) are auditable. This transparency ensures compliance with IEC 61511, enabling reliability engineers to defend SIL calculations during audits or safety reviews.

8.6 Systematic Safety Integrity Requirements

The enhanced Vesely method addresses hardware safety integrity through improved β_{eff} calculation. However, IEC 61508-2 Clause 7.4.2.2 requires the E/E/PE system design to meet both the hardware safety integrity requirements (Clauses 7.4.4 and 7.4.5) and the systematic safety integrity requirements (Clauses 7.4.6 and 7.4.7, together with IEC 61508-3).

Critical Limitation: This method does NOT verify systematic fault avoidance. Users must independently ensure:

- Application of IEC 61508-2 Annex A and Annex B techniques (highly recommended and recommended measures for systematic safety integrity)
- Software verification per IEC 61508-3 Annex A and Annex B requirements commensurate with target SIL
- Design review, validation, and verification per IEC 61508-1 Clause 7 and 8
- Common cause analysis of systematic failures (e.g., specification errors, design errors affecting multiple channels)

The β_{eff} calculation assumes systematic failures have been adequately addressed through separate design measures mandated by the applicable IEC 61508 parts. Achieving a low β_{eff} through hardware diversity does not eliminate the need for rigorous systematic safety integrity measures.

9 Results and Discussion

The enhanced Vesely method for common cause failure (CCF) analysis demonstrates significant improvements in modeling non-identical M-out-of-N voting systems. This section synthesizes the results from case studies, provides a comprehensive performance analysis, and discusses the broader implications for reliability engineering.

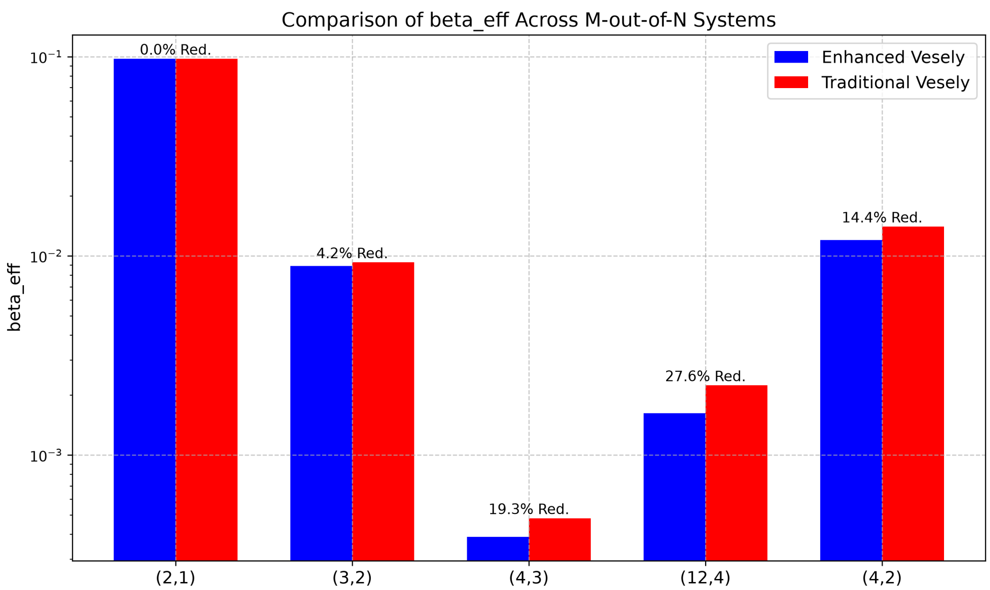
9.1 Summary of Case Study Results

The method was validated across three distinct M-out-of-N configurations, ranging from the Redundant System (N=3, M=2) to the Large-Scale System (N=12, M=4), representing diverse safety-critical applications:

Table 5: Comparison of β_{eff} Across Case Studies

System (N, M)	Enhanced	Traditional	Reduction (%)
Redundant (3, 2)	0.00888	0.00927	4.2
High-Redundancy (4, 3)	3.885×10^{-4}	4.813×10^{-4}	19.3
Large-Scale (12, 4)	1.619×10^{-3}	2.236×10^{-3}	27.6

Figure 2: Comparison of β_{eff} Across M-out-of-N Systems



These reductions translate to practical benefits in SIL calculations (Section 7), where lower β_{eff} values reduce PFDavg, avoiding over-conservative SIL classifications.

9.2 Performance Analysis

The method demonstrates consistent performance improvements across different system configurations:

- **Reduction Range**: 4.2% to 27.6% in β_{eff}
- **Most Significant Improvements**:
  - Large-Scale System (N=12, M=4): 27.6% reduction
  - High-Redundancy System (N=4, M=3): 19.3% reduction

- **Modest Improvements**:
  - Redundant System (N=3, M=2): 4.2% reduction

The method demonstrates increasing benefit as system redundancy and diversity increase, with improvements ranging from 4.2% in the Redundant System (N=3, M=2) to 27.6% in the Large-Scale System (N=12, M=4) configuration. This scaling behavior validates the theoretical foundation: channel-specific β_i modeling provides greatest value when channel diversity is substantial.

9.3 Sensitivity Analysis

To assess the method's robustness, a sensitivity analysis was conducted on the Redundant System (N=3, M=2) (Section 6.1), varying the β_i values by ±20%. This range reflects realistic uncertainty in failure rate data.

9.3.1 +20% Variation

Using the Redundant System (N=3, M=2) baseline ($\beta_{\text{eff}} = 0.00888$), increasing each β_i by 20% results in $\beta_{\text{eff}} = 0.0127$, a +43.0% increase.

9.3.2 -20% Variation

Decreasing each β_i by 20% results in $\beta_{\text{eff}} = 0.0057$, a -35.5% decrease.

The sensitivity analysis reveals critical implications for practical implementation: a ±20% change in β_i values results in approximately +43.0% and −35.5% changes in β_{eff}. This **amplified sensitivity** (roughly 2× the input variation) has several critical implications:

- **Uncertainty Propagation**: Output uncertainty is approximately double the input uncertainty, requiring exceptionally high-confidence β_i estimates to achieve claimed accuracy improvements
- **Data Quality Requirements**: Estimation accuracy is paramount—β_i values must be derived from robust, traceable data sources with documented uncertainty bounds (e.g., OREDA failure distributions, manufacturer test data with confidence intervals)
- **Conservative Design Practices**: Engineers should apply appropriate safety factors or use upper-bound adjustment factors (Table 2) when β_i estimation confidence is low
- **Regulatory Scrutiny**: During functional safety assessments (IEC 61508-1 Clause 8), the basis for each β_i value will face rigorous examination due to the method's sensitivity to these inputs

This sensitivity is inherently higher than traditional single-parameter models and represents a fundamental trade-off: improved accuracy for diverse systems comes at the cost of increased sensitivity to parameter uncertainty. This reinforces the need for the traceability and documentation requirements specified in Section 5.3.

9.4 Uncertainty Analysis

To further evaluate robustness, a Monte Carlo simulation (10,000 iterations) was performed on the Redundant System (N=3, M=2), modeling each β_i as a normal distribution with its nominal value as the mean and a standard deviation of 10% of that value. The simulation iterates the Algorithm 1 routine over the sampled β_i vectors and aggregates the resulting β_{eff} distribution.

With the three β_i sampled independently, the resulting β_{eff} distribution has mean 0.0089 and coefficient of variation 11.6% (95% of samples fall between 0.0070 and 0.0110). Independent sampling permits partial cancellation between channels, so the output uncertainty only modestly exceeds the 10% input uncertainty. When the same relative perturbation is instead applied to all channels simultaneously—the correlated case corresponding to the deterministic analysis of Section 9.3—the output coefficient of variation approximately doubles the input value. Both cases confirm that β_{eff} uncertainty is at least as large as β_i uncertainty, and substantially larger when estimation errors are correlated across channels (as is likely when all β_i derive from a common baseline). The simulation is reproducible from the verification script in the project repository.

9.5 Computational Efficiency

The enhanced Vesely method maintains computational efficiency despite the increased complexity of channel-specific β_i modeling:

- **Redundant System (N=3, M=2)**: $<$ 1 millisecond (4 subsets)
- **High-Redundancy System (N=4, M=3)**: $<$ 1 millisecond (5 subsets)
- **Large-Scale System (N=12, M=4)**: approximately 15 milliseconds (3,797 subsets)[1]
- **Scalability**: Log-transformation approach ensures numerical stability for large N

9.6 Implications for Reliability Engineering

The enhanced Vesely method offers several key advantages for reliability engineering practice:

- **Improved Accuracy**: Reductions of up to 27.6% in β_{eff} translate to more precise PFDavg estimates.
- **Targeted Design Improvements**: Per-channel β_i values identify high-risk channels.
- **Flexibility Across Configurations**: Consistent performance from the Redundant System (N=3, M=2) to the Large-Scale System (N=12, M=4).
- **Standards Alignment**: Alignment with industry standards (Section 8).

[1] Median of 100 executions of the log-transformed implementation (Appendix B) under CPython 3.11 with NumPy 2.4 on a single core of a 2.1 GHz x86-64 processor. Absolute times vary with hardware and interpreter version; the verification script in the project repository reproduces these measurements.

- **Sensitivity Awareness**: Awareness of sensitivity to parameter uncertainty.

9.7 Comparative Advantages

Compared to traditional Vesely models, the enhanced method offers:

- **vs. Traditional Vesely**: Significant reduction in β_{eff} by modeling channel diversity.
- **Computational Advantage**: Fast calculation for large systems.

10 Limitations and Future Work

The enhanced Vesely method rests on several assumptions that bound its applicability. This section identifies the key limitations and outlines directions for future research.

Independence except for CCF. Channel failures are assumed independent outside of common cause events. In practice, shared environmental stressors (e.g., corrosion in offshore environments, vibration in rotating machinery) can introduce correlations not captured by β_i. Engineers should use qualitative assessments (e.g., HAZOP) to identify such dependencies and adjust β_i values upward where shared exposures exist. Future work could incorporate copula-based dependence models to handle correlations beyond CCF explicitly.

Low-demand operation. The method targets low-demand SIS as defined by IEC 61508, using PFDavg as the reliability metric. It is not directly applicable to high-demand or continuous-mode systems where PFH is the appropriate metric and demand-induced stress may alter failure dynamics over time. Adaptation to high-demand systems would require integrating β_{eff} into PFH calculations per IEC 61508-6 continuous-mode formulas.

Uniform CCF exposure. All channels are assumed equally exposed to common cause events, differing only in their susceptibility (β_i). This does not account for scenarios where physical separation or shielding reduces some channels' exposure to specific causes (e.g., only outdoor sensors affected by weather). Where exposure varies significantly, engineers should adjust β_i using IEC 61508-6 diagnostic checklists to reflect reduced exposure for shielded channels.

Static β_i values. Per-channel beta factors are treated as constant over the proof test interval T_1, ignoring time-varying effects such as component aging, wear-out, or changes in operational conditions. For systems with diverse component lifetimes (e.g., mechanical valves vs. electronic transmitters), this may underestimate CCF risk in the later portion of T_1. Dynamic $\beta_i(t)$ modeling using Weibull-based failure rate functions could capture aging effects and improve accuracy over the system lifecycle.

Single β_i per channel. Each channel is assigned one β_i representing its overall CCF susceptibility, aggregating across all failure modes. Real channels may exhibit distinct CCF characteristics for different mechanisms—for example, a smart transmitter might have low susceptibility to hardware CCF but higher susceptibility to firmware-related CCF. A multi-mode extension, where each channel carries a vector $\vec{\beta}_i = [\beta_{i,1}, \ldots, \beta_{i,m}]$ for m distinct CCF modes, could provide finer-grained risk characterization.

Sub-group coupling. The method models each channel's β_i independently. In systems where channels share a manufacturer, firmware version, or technology platform, failures within such sub-groups may be more strongly correlated than individual β_i values suggest. Engineers should ensure diversity spans multiple dimensions (manufacturer, technology, vintage) and consider adjusting β_i upward for channels within homogeneous sub-groups.

Beta factor estimation. The method's accuracy depends on the quality of β_i inputs, which are derived from historical data (e.g., OREDA), IEC 61508-6 checklists, and expert judgment. Limited field data, potential dataset bias, and expert subjectivity all introduce uncertainty. The sensitivity analysis in Section 9.3 shows that $\pm 20\%$ variation in β_i produces $+43.0\%/-35.5\%$ variation in β_{eff}, underscoring the need for traceable, high-confidence inputs. Machine learning approaches trained on operational data could reduce expert bias and enable dynamic updating of β_i estimates as field data accumulates.

Computational scalability. The combinatorial enumeration of failure subsets grows exponentially with N, which bounds the recommended application envelope at $N \leq 16$ channels. Within that envelope evaluation remains fast—the $N = 12$ case study evaluates its 3,797 subsets in roughly 15 milliseconds (Section 9.5)—but the subset count roughly doubles with each added channel. Log-transformation mitigates numerical underflow but does not address the combinatorial growth. For substantially larger systems, approximate methods (e.g., importance sampling, binomial bounding) would be needed.

Scope boundaries. The method addresses CCF probability estimation within the beta-factor framework. It does not model repair processes, proof test optimization, or demand-rate interactions. It assumes binary channel states (failed/operational) and does not capture degraded performance modes. Application to systems where multiple SIL levels share common channels requires separate analysis per SIF, with the most conservative β_i assignment governing shared components.

11 Conclusion

This study introduces the enhanced Vesely method for common cause failure (CCF) analysis in non-identical M-out-of-N voting systems, addressing a gap in traditional reliability engineering methods. By incorporating channel-specific failure probabilities (β_i), the method overcomes the uniform failure assumption of the original Vesely method (Section 4), enabling more accurate CCF risk estimation in diverse safety instrumented systems (SIS). Through rigorous validation across three case studies (Section 6)—ranging from a Redundant System (N=3, M=2) to a complex Large-Scale System (N=12, M=4) architecture—the method demonstrates reductions in the effective beta factor (β_{eff}) ranging from 4% in low-diversity systems to 28% in the large-scale configuration (Section 9.1). These reductions translate to practical benefits in SIL calculations, supporting more accurate SIL classification and reducing the risk of over-conservative designs that drive unnecessary cost without commensurate safety benefit (Section 7). The method's computational efficiency, with millisecond-scale runtimes even for a Large-Scale System (N=12, M=4) (Section 9.5), and its alignment with standards like IEC 61508-6, IEC 61511, and ISO/TR 12489 (Section 8), make it a practical tool for reliability engineers in safety-critical industries such as oil and gas, nuclear power, and chemical processing.

11.1 Key Contributions

The enhanced Vesely method's key contributions are multifaceted:

- **Improved Accuracy Through Diversity**:
  - Introduction of channel-specific beta factors (β_i)
  - By modeling β_i for each channel—derived from empirical data (e.g., OREDA), IEC 61508-6 checklists, and expert judgment (Section 5.2)—the method captures the unique CCF susceptibilities of diverse components
  - For instance, in the Redundant System (N=3, M=2) (Section 6.1), the smart transmitter's higher $\beta_2 = 0.08$ due to software risks contrasts with the analog sensor's lower $\beta_3 = 0.05$, leading to a β_{eff} of 0.00888, a 4.2% reduction from the traditional Vesely value of 0.00927
  - This precision translates to a PFDavg of 3.81×10^{-4} vs. 3.89×10^{-4}, maintaining SIL 3 with a safety margin and avoiding overly conservative design measures
  - Reduction in β_{eff} by up to 27.6% across case studies
- **Computational Efficiency**:
  - The method's log-transformations (Section 5.4) ensure numerical stability and scalability, enabling evaluation of the Large-Scale System (N=12, M=4) case study (3,797 subsets) in roughly 15 milliseconds (Section 9.5)
  - This efficiency supports repeated evaluation within sensitivity and Monte Carlo analyses and keeps runtimes low for systems up to $N \leq 16$ channels
  - Maintained accuracy across different system configurations
- **Standards Alignment**:
  - The method aligns with IEC 61508-6 (CCF modeling), IEC 61508-7 (computational efficiency), IEC 61511 (auditable inputs), and ISO/TR 12489 (tailored reliability methods), ensuring seamless integration into existing safety workflows (Section 8)
  - Improved SIL calculation methodology
  - Potential for industry-wide adoption
- **Practical Validation**:
  - Validated across three diverse M-out-of-N configurations (Redundant System (N=3, M=2), High-Redundancy System (N=4, M=3), Large-Scale System (N=12, M=4)) in safety-critical applications (oil refinery, nuclear power, offshore platform)

  - Demonstrated robustness through sensitivity analysis (Section 9.3): $\pm 20\%$ variation in β_i yields $+43.0\%/-35.5\%$ change in β_{eff}, an amplified non-linear response that reinforces the importance of per-channel β_i estimation
  - Monte Carlo uncertainty analysis (Section 9.4) confirms the amplified sensitivity identified in Section 9.3: $\pm 10\%$ input variation in β_i propagates to greater than $\pm 10\%$ variation in β_{eff}

11.2 Practical Implications

The method offers significant benefits for safety-critical industries:

- **Cost Optimization**:
  - Reduced over-conservative design approaches
  - More efficient resource allocation
- **Safety Enhancement**:
  - More accurate risk assessment through channel-specific modeling
  - Per-channel β_i values identify high-risk components (e.g., the smart transmitter in the Redundant System (N=3, M=2), Section 9.6), guiding targeted maintenance or design improvements
  - Improved understanding of CCF mechanisms
  - Better-informed safety design decisions
- **Industry Impact**:
  - Applicable to oil and gas, nuclear, and chemical processing industries
  - The method's Python implementation (Appendix B) and integration potential with software vendors (e.g., ReliaSoft BlockSim) facilitate adoption
  - Support for more sophisticated safety instrumented systems

11.3 Validation and Performance

Comprehensive validation demonstrates the method's effectiveness:

Table 6: Method Performance Summary Across System Configurations

System (N, M)	Application	β_{eff} Reduction	Key Finding
Redundant (3, 2)	Oil Refinery	4.2%	Baseline validation
High-Redundancy (4, 3)	Nuclear Power	19.3%	Scaling with redundancy
Large-Scale (12, 4)	Offshore Platform	27.6%	Computational efficiency

11.4 Broader Impact

The research extends beyond immediate technical improvements:

- **Methodological Innovation**:
  - Extension of the binomial-expansion beta-factor formulation to per-channel β_i, addressing a gap in current standards coverage of non-identical M-out-of-N architectures
  - Bridges reliability engineering and computational methods (e.g., log-transformations for numerical stability)
  - Provides a foundation for future research into dynamic CCF models and machine learning integration
- **Educational Value**:
  - Comprehensive case studies (Section 6) serve as teaching resources for university courses and industry training programs
  - Detailed implementation guidance (Section 5) and Python code (Appendix B) support knowledge transfer to practicing engineers
  - Transparent methodology enables peer review and reproducibility
- **Regulatory Influence**:
  - Standards-aligned approach (Section 8) positions the method for inclusion in future revisions of IEC 61508 and ISO/TR 12489
  - Demonstrates practical implementation of IEC 61508-6 principles

11.5 Closing Remarks

The enhanced Vesely method addresses a long-standing limitation in CCF analysis by accommodating channel diversity in M-out-of-N voting systems. Its practical focus—evidenced by computational efficiency, standards alignment, and transparent implementation—ensures accessibility for reliability engineers across safety-critical industries. By demonstrating reductions of up to 27.6% in β_{eff} through rigorous case studies, the method shows that channel-specific modeling can yield measurable accuracy improvements over uniform β models. The amplified sensitivity observed in Section 9.3, where $\pm 20\%$ input variation produces $+43.0\%/-35.5\%$ output change, reinforces the need for traceable, high-confidence β_i estimates and disciplined uncertainty quantification in any practical application of the method.

As safety instrumented systems increasingly integrate diverse component technologies, the limitations of uniform-β CCF models become more pronounced. The enhanced Vesely method offers a transparent, standards-aligned route to incorporate channel-specific susceptibility into CCF analysis without abandoning the beta-factor framework that practicing reliability engineers already use.

It is critical to emphasize that this method addresses hardware safety integrity through improved CCF modeling; practitioners must independently ensure systematic safety integrity measures (IEC 61508-2/3 requirements) are applied commensurate with the target SIL, as detailed in Section 8.

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Conflict of Interest

The author declares no competing interests.

Data Availability

All β_{eff} and PFDavg results in this paper can be reproduced from the Python implementation in Appendix B together with the case study parameters given in Section 6. The complete verification, benchmark, Monte Carlo, and figure-generation scripts are available at `https://github.com/richardnero/MooN`.

Appendix A: Proof of the Enhanced Vesely Formula

The enhanced Vesely method generalizes the original Vesely formula to accommodate non-identical channels in M-out-of-N voting systems by using channel-specific failure probabilities (β_i). To ensure its mathematical consistency with the established framework, this appendix proves that the enhanced Vesely formula (Equation 3) reduces to the original Vesely formula (Equation 1) when all channels are identical, i.e., when $\beta_i = \beta$ for all i. The proof proceeds by direct substitution and algebraic simplification, followed by a combinatorial argument confirming the binomial coefficient structure across all k and N.

A.1 Direct Substitution

The enhanced Vesely formula calculates the effective beta factor (β_{eff}) as the probability of system failure due to CCF, where at least M out of N channels fail:

$$\beta_{\text{eff}} = \sum_{k=M}^{N} \sum_{S \in \begin{pmatrix} N \\ k \end{pmatrix}} P_{\text{combo}}(S)$$

where $P_{\text{combo}}(S)$ for a subset S of size k is:

$$P_{\text{combo}}(S) = \prod_{i \in S} \beta_i \cdot \prod_{j \in \{1,2,\ldots,N\} \setminus S} (1 - \beta_j)$$

Assume all channels are identical, so $\beta_i = \beta$ for all i. Substitute $\beta_i = \beta$ into the formula:

$$P_{\text{combo}}(S) = \prod_{i \in S} \beta \cdot \prod_{j \in \{1,2,\ldots,N\} \setminus S} (1 - \beta)$$

For a subset S of size k, there are k channels in S and $N - k$ channels in the complement $\{1, 2, \ldots, N\} \setminus S$. Thus:

$$P_{\text{combo}}(S) = \beta^k \cdot (1 - \beta)^{N-k}$$

The number of subsets S of size k is given by the binomial coefficient $\begin{pmatrix} N \\ k \end{pmatrix}$. Therefore, for each k, the inner sum over all subsets of size k is:

$$\sum_{S \in \begin{pmatrix} N \\ k \end{pmatrix}} P_{\text{combo}}(S) = \begin{pmatrix} N \\ k \end{pmatrix} \cdot P_{\text{combo}}(S) = \begin{pmatrix} N \\ k \end{pmatrix} \cdot \beta^k \cdot (1 - \beta)^{N-k}$$

Summing over all k from M to N:

$$\beta_{\text{eff}} = \sum_{k=M}^{N} \sum_{S \in \begin{pmatrix} N \\ k \end{pmatrix}} P_{\text{combo}}(S) = \sum_{k=M}^{N} \begin{pmatrix} N \\ k \end{pmatrix} \cdot \beta^k \cdot (1 - \beta)^{N-k}$$

This matches the original Vesely formula (Equation 1), which assumes identical channels and calculates β_{eff} as the probability of at least M channels failing due to CCF.

A.2 Confirmation via Pascal's Rule

To confirm the result holds for all N and M, we verify the binomial coefficient structure directly by partitioning the failure subsets.

- **Boundary Case ($N = M$)**: If $N = M$, the system fails only when all N channels fail (e.g., a 2oo2 system). The enhanced Vesely formula becomes:

$$\beta_{\text{eff}} = \sum_{S \in \begin{pmatrix} N \\ N \end{pmatrix}} P_{\text{combo}}(S)$$

There is exactly one subset of size N, so $\begin{pmatrix} N \\ N \end{pmatrix} = 1$, and $P_{\text{combo}}(S) = \beta^N \cdot (1-\beta)^{N-N} = \beta^N$. Thus:

$$\beta_{\text{eff}} = \beta^N$$

The original Vesely formula for $N = M$ is:

$$\beta_{\text{eff}} = \sum_{k=M}^{N} \begin{pmatrix} N \\ k \end{pmatrix} \cdot \beta^k \cdot (1-\beta)^{N-k} = \begin{pmatrix} N \\ N \end{pmatrix} \cdot \beta^N \cdot (1-\beta)^{N-N} = \beta^N$$

The boundary case holds.

- **General Case**: Consider an M-out-of-N system with $N > M$. Partition the subsets of size k based on whether the N-th channel is in the subset S:
  - **Case 1: N-th channel is not in S**: Subsets S of size k from $\{1, 2, \ldots, N-1\}$. There are $\begin{pmatrix} N-1 \\ k \end{pmatrix}$ such subsets, and $P_{\text{combo}}(S) = \beta^k \cdot (1-\beta)^{N-k}$. Contribution:

$$\begin{pmatrix} N-1 \\ k \end{pmatrix} \cdot \beta^k \cdot (1-\beta)^{N-k}$$

  - **Case 2: N-th channel is in S**: Subsets S of size k where the N-th channel is included, so we choose $k-1$ channels from $\{1, 2, \ldots, N-1\}$. There are $\begin{pmatrix} N-1 \\ k-1 \end{pmatrix}$ such subsets, and $P_{\text{combo}}(S) = \beta^k \cdot (1-\beta)^{N-k}$. Contribution:

$$\begin{pmatrix} N-1 \\ k-1 \end{pmatrix} \cdot \beta^k \cdot (1-\beta)^{N-k}$$

  Total contribution for size k:

$$\left(\begin{pmatrix} N-1 \\ k \end{pmatrix} + \begin{pmatrix} N-1 \\ k-1 \end{pmatrix} \right) \cdot \beta^k \cdot (1-\beta)^{N-k}$$

  By Pascal's rule, $\begin{pmatrix} N-1 \\ k \end{pmatrix} + \begin{pmatrix} N-1 \\ k-1 \end{pmatrix} = \begin{pmatrix} N \\ k \end{pmatrix}$. Thus:

$$\begin{pmatrix} N \\ k \end{pmatrix} \cdot \beta^k \cdot (1-\beta)^{N-k}$$

  Summing over k from M to N matches the original Vesely formula, confirming the binomial coefficient structure for all k and N.

The enhanced Vesely formula is therefore consistent with the original when all channels are identical, validating its generalization (Section 5).

Appendix B: Python Implementation

The enhanced Vesely method is implemented in Python using NumPy for numerical stability. The core algorithm uses log-transformations (Equations 7 and 8) to prevent underflow in large systems.

Listing 1: Enhanced Vesely Method - Core Implementation

```python
import numpy as np
from itertools import combinations

def enhanced_vesely(N, M, beta_i):
    """Calculate effective beta factor for M-out-of-N system."""
    beta_eff = 0.0
    channels = list(range(N))

    for k in range(M, N + 1):
        for subset in combinations(channels, k):
            # Log-transformation for numerical stability
            ln_p_combo = sum(np.log(beta_i[i]) for i in subset)
            ln_p_combo += sum(np.log(1 - beta_i[j])
                                for j in channels if j not in subset)
            beta_eff += np.exp(ln_p_combo)

    return beta_eff

# Example: Redundant System (N=3, M=2)
beta_i = [0.04, 0.08, 0.05]
result = enhanced_vesely(3, 2, beta_i)
print(f"beta_eff = {result:.7f}")  # Expected: 0.0088800
```

The pseudocode above captures the core algorithm. All results in this paper can be reproduced using this implementation with the case study parameters defined in Section 6.

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