

Dynamic boundary element - finite element analysis of railway bridges considering soil-structure-track interaction *

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This paper investigates how soil-structure interaction affects the dynamic behavior of railway bridges. It uses a three-dimensional coupled finite element-boundary element modeling approach. This formulation applies to both the frequency and time domains, ensuring a consistent representation of wave propagation and radiation damping in the supporting subsoil. After outlining the underlying numerical framework, the focus is on the dynamic analysis of a single-span steel-girder railway bridge. The novel contribution of this study is in the comprehensive application of a three-dimensional coupled finite element-boundary element model of a steel railway bridge to assess the combined influence of subsoil and track modeling on both modal properties and transient response within a unified frequency-time domain numerical framework. The results demonstrate that soil-structure interaction and track can significantly affect the dynamic characteristics and response amplitudes of steel railway bridges. The findings underscore the importance of explicitly accounting for these effects when assessing the dynamic performance of railway bridges, particularly when evaluating serviceability limit states governed by vibration criteria.

Keywords: Bridge dynamics; Dynamic subsystem interaction; Boundary elements; Finite elements; Computational framework.

1. Introduction

Soon after the construction of the first high-speed railway lines in France, the destabilization of the ballast was observed on several bridges along the line. This phenomenon was attributed to excessive vibrations caused by resonance effects.¹ As a consequence, a maximum limit for the admissible bridge deck acceleration of 3.5 m/s^2 for ballasted railway bridges was introduced in the EN1990-A1.² To ensure compliance with this limit, reliable response predictions must be performed during the design phase using numerical or analytical models. In engineering practice,

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however, soil-structure interaction is often neglected and the track superstructure is frequently modeled in a simplified manner in order to limit computational effort.

Recent numerical investigations^{3–10} accompanied by experimental validations^{10,11} have demonstrated the significant influence of soil-structure interaction on the dynamic response of short-span railway bridges. In particular, portal-frame railway bridges have been investigated and it was shown that the effective modal damping ratios, accounting for soil-structure interaction, can largely exceed the admissible values prescribed in the EN1991-2¹² due to their embedding in the surrounding subsoil.^{3,10,11} Similar trends of increased damping due to soil-structure interaction have been reported for reinforced and prestressed simply-supported concrete bridges.^{4–7} These effects are especially relevant in resonance conditions induced by uniformly spaced axle loads moving at so-called *resonance speeds*, where damping has a decisive influence on the predicted structural response. Although soil-structure interaction effects have been extensively studied for portal-frame and concrete simply-supported railway bridges, numerical investigations focusing on steel bridges are limited. Nevertheless, studies in König, et al.^{8,9} reported similar effects, such as increased damping and shifted natural frequencies for steel bridges using simplified beam-based models that incorporate soil-structure interaction. Early¹ and recent¹³ experimental investigations on steel bridges also showed significantly higher damping ratios than the recommended design values of the EN1991-2.¹² Furthermore, the influence of the ballast stiffness and damping on steel bridges has been investigated experimentally¹⁴ and numerically¹⁵ using a simplified beam model. These studies indicate that the track substantially contributes to the overall system stiffness and damping.

Despite these findings, a clear gap remains in the numerical investigation of steel railway bridges using detailed, three-dimensional models that consistently account for soil-structure interaction and track-bridge interaction. Many existing studies rely on simplified *Euler-Bernoulli* beam formulations^{5–9,16} combined with substructure approaches in which the subsoil is represented by spring-dashpot elements. More sophisticated numerical approaches use finite elements to discretize parts of the subsoil volume,¹¹ or use boundary elements to discretize part of the subsoil surface.⁴ These approaches enable the consideration of foundation-soil-foundation interaction effects that are neglected in simpler models. In the case of subsoil discretization using finite elements, the subsoil volume must be truncated, and artificial, non-reflective boundary conditions must be introduced to prevent elastic wave reflections. Such absorbing boundary conditions are commonly modeled using infinite elements,¹⁷ or perfectly matched layers.¹⁸ These requirements increase modeling complexity and computational cost. In contrast, boundary element formulations inherently account for wave propagation towards infinity and therefore allow the subsoil surface to be discretized without the need for additional absorbing boundary conditions. This makes the method particularly well-suited for analyzing soil-foundation interaction problems.

This paper examines the impact of soil-structure interaction on the dynamic

behavior of railway bridges via a comprehensive numerical study of a single-span steel-girder bridge. The adopted modeling approach combines finite element representations of the track-bridge-foundation system with a boundary element idealization of the supporting subsoil. This allows for a realistic description of wave propagation and radiation damping effects in the ground. This formulation can be used both in the frequency and time domains, allowing for the assessment of system dynamics based on the modal properties as well as under transient railway loading. Since the focus of this publication lies on the analysis of soil-structure interaction, complex train-bridge interaction¹⁹ is neglected and the train is considered by a moving single load model according to EN1991-2.¹²

The coupled model is used to examine how variations in subsoil stiffness affect the global dynamic characteristics of the bridge-track system. Frequency-domain analyses are carried out to determine natural frequencies, effective modal damping ratios, and associated mode shapes. These analyses emphasize the contribution of the track and surrounding soil to the overall stiffness and energy dissipation of the system. Subsequently, time-domain simulations of train passages are performed to quantify the impact of soil-structure interaction on bridge accelerations and displacements. The results underscore the significance of considering soil-structure interaction effects when assessing the dynamic performance of railway bridges, particularly with regard to serviceability-related response measures.

2. Dynamic coupled boundary element - finite element implementation

For the present study of the effect of soil-structure interaction on the dynamic response of railway bridges subjected to high-speed trains, the computational software **MDdyn** (Multi-Domain-Dynamics) based on three-dimensional boundary element²⁰ and finite element formulations²¹ has been developed. The frequency-domain formulation implemented in this software is used to identify the modal properties of the interacting system consisting of track, steel bridge, foundation, and soil. The time-domain formulation, also implemented, is employed to numerically predict the time-history response of the bridge due to high-speed traffic. The subsequent sections summarize the core aspects of the underlying numerical implementations.

2.1. Boundary element domain

Special attention is given to the implementation of boundary element procedures. Singular integrals inherent to the boundary element method are solved universally for the frequency- and time-domain formulations. Additional element subdivision is performed for time-domain computations. The following sections outline these considerations as they apply to the present implementation.

2.1.1. Frequency-domain formulation

The time-harmonic solutions for the boundary displacements $u_k(\mathbf{x}, \omega)$ and tractions $p_k(\mathbf{x}, \omega)$ are derived from the boundary integral formulation in the frequency domain according to²⁰

$$C_{lk}^{(i)}(\mathbf{x}^{(i)})u_k(\mathbf{x}^{(i)}, \omega) = \int_{\Gamma} \hat{u}_{lk}^*(\mathbf{r}, \omega)p_k(\mathbf{x}, \omega)d\Gamma - \oint_{\Gamma} \hat{p}_{lk}^*(\mathbf{r}, \omega)u_k(\mathbf{x}, \omega)d\Gamma \quad (2.1)$$

where $C_{lk}^{(i)}$ represents the so-called *free term*, which depends on the surface geometry at the collocation point $\mathbf{x}^{(i)}$. The full-space singular fundamental solutions of the displacement $\hat{u}_{lk}^*$ and the tractions $\hat{p}_{lk}^*$ ²⁰ depend on the distance vector $\mathbf{r} = \mathbf{x} - \mathbf{x}^{(i)}$ between collocation point $\mathbf{x}^{(i)}$ and observation point $\mathbf{x}$ at the boundary surface at the discrete frequency ω . Both surface integrals on the right hand side of Eq. (2.1) involve singular integrands for the case where the observation point approaches the collocation point ($\mathbf{x} \mapsto \mathbf{x}^{(i)}$) due to the singularities inherent in the fundamental solutions of displacements and tractions. The first integrand in Eq. (2.1) is limited to a *weak singularity* of $O(1/r)$, while the second integrand includes a *strong singularity* of $O(1/r^2)$ and the corresponding integral is formulated in the sense of a *Cauchy Principal Value* (CPV) integral, denoted by the horizontal dash of the integral symbol. Discretization of the surface geometry and numerical integration of the singular and CPV integrals, discussed further in section 2.1.4, yield the system of equations for the time-harmonic frequency-domain procedure,

$$\mathbf{H}(\omega) \mathbf{u}_B(\omega) = \mathbf{G}(\omega) \mathbf{p}_B(\omega) \quad (2.2)$$

with the frequency-dependent system matrices $\mathbf{H}$ and $\mathbf{G}$ and the vectors of displacements and tractions $\mathbf{u}_B$ and $\mathbf{p}_B$.

2.1.2. Time-domain formulation

The time-domain boundary integral equation with zero initial conditions and body forces reads as²⁰

$$C_{lk}^{(i)}(\mathbf{x}^{(i)})u_k(\mathbf{x}^{(i)}, t) = \int_0^{t^+} \int_{\Gamma} u_{lk}^*(\mathbf{r}, t - \tau)p_k(\mathbf{x}, \tau)d\Gamma d\tau - \int_0^{t^+} \oint_{\Gamma} p_{lk}^*(\mathbf{r}, t - \tau)u_k(\mathbf{x}, \tau)d\Gamma d\tau \quad (2.3)$$

and is based on the convolution of the full-space time-domain singular fundamental solutions of displacements u_{lk}^* and tractions p_{lk}^* ²² and the boundary-surface tractions and displacements p_k and u_k . Similar to the frequency-domain boundary integral formulation of Eq. (2.1), the fundamental solutions in the time domain used in Eq. (2.3) are singular in $O(1/r)$ and $O(1/r^2)$, and the second spatial integral of Eq. (2.3) is formulated as a CPV integral. The integrations in Eq. (2.3)

are performed in two steps. First, the integration in time is performed analytically using piecewise linear interpolation functions for the displacements and piecewise constant interpolation functions for the tractions for the $n = 1, \dots, N_t$ time steps.²³ Numerical integration is then performed on the spacial integrals, considering the singular integration of section 2.1.4 and additional element subdivision to improve the integration of the wavefront between compression and shear waves. This yields the discretized transient boundary integral equation for the n -th step,²⁰

$$\mathbf{H}^{nn} \mathbf{u}_B^n = \mathbf{G}^{nn} \mathbf{p}_B^n + \sum_{m=1}^{n-1} [\mathbf{G}^{nm} \mathbf{p}_B^m - \mathbf{H}^{nm} \mathbf{u}_B^m] \quad (2.4)$$

where the system matrices $\mathbf{G}^{nm}$ and $\mathbf{H}^{nm}$ result from the spatio-temporal integration along the $(n - m)$ -th time interval.

2.1.3. Free term evaluation

The *free term* matrix $\mathbf{C}(\mathbf{x}^{(i)})$ at a collocation node $\mathbf{x}^{(i)}$ is identical for static, time-harmonic, and transient boundary element formulations and, for the transient case, is non-zero only for the time step $n = m$ when the full-space fundamental solutions derived from a *Dirac*-delta impulse are considered. Mantić²⁴ presents a generally applicable algorithm for computing the $\mathbf{C}$ matrix at a boundary node with an arbitrary number of tangent planes belonging to adjacent elements. In the present implementation, this algorithm evaluates the *free term* matrix. Because only a finite part of the soil half-space is modeled, the computation of the $\mathbf{C}$ matrix requires the assumption that the flat surfaces or straight edges extend beyond the edges of the truncated mesh edges, as described in detail by König.²⁵

2.1.4. Singular integration

Guiggiani and Gigante²⁶ show the numerical integration of the *strongly singular* CPV integral involving the elastostatic fundamental solution of tractions by performing element-wise integration of the elements containing the singular node $\mathbf{x}^{(i)}$. This procedure was originally developed for elastostatics, however, the similarities in the singularities of the time-harmonic fundamental solutions²⁰ and time-domain fundamental solutions²² motivate adopting the procedure of Guiggiani and Gigante²⁶ for the frequency- and time-domain formulations of this paper. The validity of this approach is discussed briefly below.

To apply this method, the singular remainder inside the second integral of Eqs (2.1) and (2.3) must have the form,²⁶

$$\bar{P}_{lk} = \frac{1}{r^2} \left[n_k(\mathbf{x}) \frac{x_l - x_l^i}{r} - n_l(\mathbf{x}) \frac{x_k - x_k^i}{r} \right] \quad (2.5)$$

For the frequency-domain formulation of Eq. (2.1), this condition is inherently satisfied, as shown by Guiggiani.²⁷ In the time-domain formulation of Eq. (2.3),

the time integration is carried out before spatial integration. As demonstrated by König,²⁵ this yields an expression similar to that of Eq. (2.5). It reads as follows,

$$\overline{P}_{lk}^{nm} = \frac{1}{r^2} \left[D^{(I)} n_k(\mathbf{x}) \frac{x_l - x_l^i}{r} + A^{(I)} n_l(\mathbf{x}) \frac{x_k - x_k^i}{r} \right] \quad (2.6)$$

where nm again denotes the discrete time difference in the convolution. The factors $D^{(I)}$ and $A^{(I)}$ result from analytic time integration. As shown by König,²⁵ these factors reduce to $A^{(I)} = -D^{(I)}$ and are independent of $\mathbf{r}$ for the piecewise linear interpolation of displacements. Therefore, they can be moved in front of the spatial integral, resulting in the same singular remainder as Eq. (2.5).

2.1.5. Element subdivision for the transient formulation

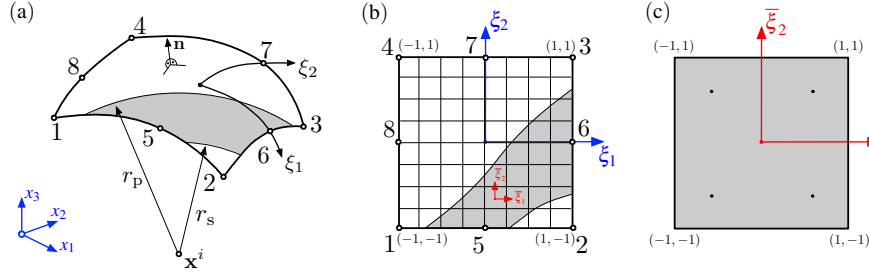


Fig. 1. Isoparametric 8-node element with highlighted integration region in (a) the global coordinate system, (b) the intrinsic coordinate system, with element subdivision ($N^{\text{sub}} = 8$), and (c) the subregion of integration with four *Gauss* points ($G^{\text{sub}} = 2$) (modified from Marrero and Domínguez²²)

In addition to the consideration of singular integrals, the transient time-domain boundary element formulation requires the integration of possible discontinuous integrals on an element at a given time step. This is due to the fact that there are only non-zero integral kernels between the two wave fronts of compression and shear waves (cf. Fig. 1(a), r_p and r_s at a given time step). To improve the numerical integration, the base element is further subdivided (cf. Fig. 1(b)). Integration is then performed sub-element by sub-element in the $\bar{\xi}_1, \bar{\xi}_2$ coordinate system of Fig. 1(c) using *Gaussian* integration, as for example done by Marrero and Domínguez.²²

Since the integration of only a part of the element surface is also relevant for the singular case where the collocation node $\mathbf{x}^{(i)}$ is on the element, element subdivision is also performed here. This is done for each triangular integration domain resulting from the formulation of polar integration of the singular cases using $\rho\vartheta$ -coordinates. Depending on the intrinsic coordinates $\boldsymbol{\eta}$, the three main cases depicted in Fig. 2 arise for quadrilateral elements with up to nine nodes. Each of the triangles (white and light blue) is subdivided into sub-regions of integration illustrated by the gray dashed lines, and the index g corresponds to the g -th *Gauss* point.

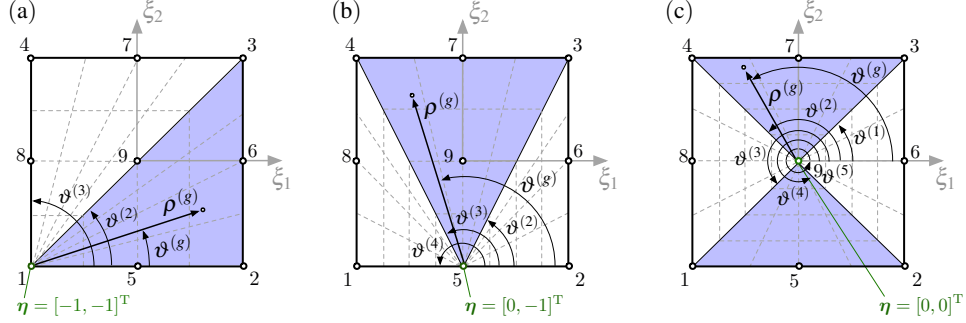


Fig. 2. Singular integration including element subdivision for the singular node at (a) a corner node, (b) an edge node, (c) the center node (modified from Guiggiani and Gigante²⁶).

2.2. Finite element domain

With a focus on the boundary element procedures, the software **MDdyn** imports the geometry and system matrices of the finite element model of the structure and track from the commercial finite element software **ABAQUS**,¹⁷ allowing for the use of a wide range of well-established finite element types.

2.2.1. Frequency-domain formulation

The time-harmonic finite element subsystem equations in the frequency domain read as follows²⁸

$$(\mathbf{K} + i\omega\mathbf{C} - \omega^2\mathbf{M})\mathbf{u}_F(\omega) = \mathbf{f}_F(\omega) \quad (2.7)$$

or simply

$$\mathbf{D}(\omega)\mathbf{u}_F(\omega) = \mathbf{f}_F(\omega) \quad (2.8)$$

In Eq. (2.7), the matrices $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ represent the mass, damping and stiffness matrices of the linear finite element model. The vector $\mathbf{u}_F$ contains the vector of nodal displacements and rotations and $\mathbf{f}_F$ is the vector of nodal forces and moments.

2.2.2. Time-domain formulation

In general, the equations of motion of the discretized finite element domain read²¹

$$\mathbf{M}\ddot{\mathbf{u}}_F(t) + \mathbf{C}\dot{\mathbf{u}}_F(t) + \mathbf{K}\mathbf{u}_F(t) = \mathbf{f}_F(t) \quad (2.9)$$

with the mass, damping, and stiffness matrices $\mathbf{M}_F$, $\mathbf{C}_F$, $\mathbf{K}_F$, the vector of nodal forces and moments $\mathbf{f}_F$, and the vector of nodal displacements and rotations $\mathbf{u}_F$. Equation (2.9) can be written as²¹

$$\mathbf{D}\mathbf{u}_F^n = \mathbf{f}_{\text{eff}}^n \quad (2.10)$$

after application of the *Newmark- β* algorithm²¹ in the n -th time step. Here, $\mathbf{D}$ represents the effective dynamic stiffness matrix and $\mathbf{f}_{\text{eff}}^n$ is the effective load vector of the n -th step.

2.3. Boundary element - finite element coupling procedures

The coupling procedures in the frequency and time domains consider conforming meshes of the interface between the finite element domain and the boundary element domain. Therefore, compatibility – i.e., equality of nodal displacements of the coinciding boundary element and finite element nodes at the common interface (subscript "c") – is prescribed by

$$\mathbf{u}_{\text{Bc}} = \mathbf{u}_{\text{Fc}} \quad (2.11)$$

Furthermore, the equality of forces must be satisfied, requiring that the nodal forces evaluated at the coinciding nodes must be of equal magnitude and in opposite direction,

$$\mathbf{f}_{\text{Bc}} = -\mathbf{f}_{\text{Fc}} \quad (2.12)$$

Taking these conditions into account, a *direct coupling* procedure is used to condense the degrees of freedom of boundary elements and finite elements into a global system of equations for the coupling of the equations in the frequency domain, i.e., Eqs (2.2) and (2.8).^{29, 30}

For the coupling of the time-domain formulations, a *direct* approach that enforces compatibility and equilibrium at each time step could also be used.^{30, 31} This, however, requires the same time step for the finite element and boundary element domains, a condition that is not needed in more complex *iterative coupling* approaches. Since it is often advantageous to use different time steps for the boundary element and finite element domains, such an *iterative coupling* procedure in the form of the algorithm of von Estorff and Hagen³² is used for the coupling of the equations in the time domain (Eqs (2.4) and (2.10)). The detailed formulation of the iterative coupling of the time-domain formulations can be found in von Estorff and Hagen.³² However, applying this coupling procedure requires the definition of suitable parameters, consisting of the four convergence tolerances ε_{u} , ε_{f} , ε_{v} , ε_{g} , relaxation parameters ω_{u} and ω_{f} , and the interpolation parameter ϑ .³² Several computations were conducted using this iterative scheme. The parameters were chosen according to Table 1, and this led to good convergence and quality of the solution.

Table 1.
Tolerances and parameters applied for
the iterative coupling according to.³²

$\varepsilon_{\text{u}} = \varepsilon_{\text{v}}$	$\varepsilon_{\text{f}} = \varepsilon_{\text{g}}$	ω_{u}	ω_{f}	ϑ
10^{-4}	10^{-6}	0.8	0.8	0.2

In the following, the presented software is used to investigate the influence of track-structure and soil-structure interaction on the modal properties of a bridge structure, since its dynamic response excited by high-speed traffic depends to a large extent on these properties. In particular, a single-span steel-girder bridge with a span length of 16.8 m is considered. Figure 3(a) shows the complete system of track and bridge discretized with finite elements (Ω_{FE}), coupled with the sub-soil, discretized with boundary elements (Ω_{BE}). The geometry of the cross-section illustrated in Fig. 3(b) is taken from Salcher and Adam.³³ For further reference, the dimensions are listed in Table 7 of the appendix.

The bearing at the foundation located on the left in Fig. 3(a) is realized as pinned, coupling all displacements of the steel bridge with the bearing seat of the foundation. Relative displacements of the steel bridge and the bearing seat in Fig. 3 are possible only in the longitudinal direction of the bridge. Since the vertical displacements between the bridge structure and the foundation are coupled at the bearing, this model accounts for stiff bearings, such as steel bearings, and does not consider bearings with lower stiffness, such as elastomeric bearings. The material parameters of the ballast are taken from literature^{11,34} and listed together with the ballast dimensions in Table 8. Further parameters of the track are specified in Table 10. The dimensions and discretization of the concrete foundation are shown

in Fig. 4 and the corresponding material parameters are listed in Table 9.

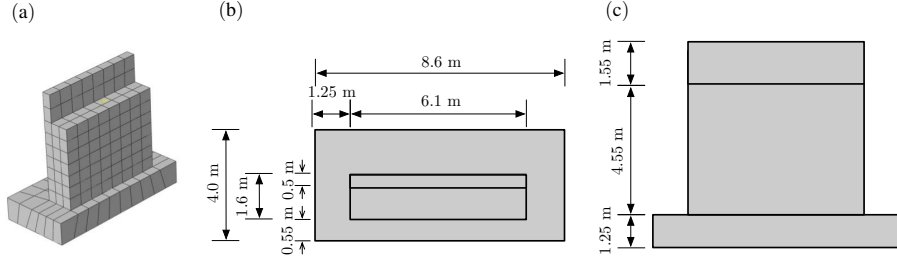


Fig. 4. Foundation of the bridge structure. (a) Meshed geometry. (b) Top view. (c) Front view.

Quadratic 20-node finite continuum elements (C3D20) are used for the discretization of the ballast, the foundation and the sleepers. The structure of the bridge is modeled with 3- and 4-node linear shell elements (S3 and S4) and 3-node quadratic beam elements (B32) are used for the rails. Using this discretization, the finite element domain Ω_{FE} has a total of $N_{FE} = 94,077$ active degrees of freedom. The boundary element domain Ω_{BE} of the subsoil, discretized with 8-node quadratic quadrilateral boundary elements, has a total of $N_{BE} = 10,077$ degrees of freedom with subsoil material parameters listed in Table 2.

First, soil–structure interaction is neglected in section 3.1, and the foundation and subsoil are therefore assumed to be rigid. Only the domain Ω_{FE} with fixed boundary conditions at the interface nodes is considered. Two variations of the structural model are considered herein. In "Model 1", only the mass of the track, which is uniformly distributed on the deck plate of the steel-girder bridge, is taken into account. "Model 2" includes an FE model of the full track subsystem (see "Model 1" and "Model 2" in Fig. 5).

Subsequently, the model is expanded in section 3.2 to include the subsoil, which is discretized using the proposed boundary element formulation. Again, two model variants are considered: in "Model 3" (Fig. 5), only the track mass added uniformly to the deck plate is included, whereas "Model 4" contains the complete track model (see Fig. 3(a) and "Model 4" in Fig. 5). Here, the *Young's* modulus of the soil E_{so} is varied from 25 to 700 MPa ($\Delta E_{so} = 25$ MPa) for soils of very low to high stiffness. The corresponding shear and compression wave velocities of the subsoil, c_s and c_p , are listed in Table 3. In practice, soils with $E_{so} \lesssim 100$ MPa (or $c_{so} \lesssim 150$ m/s) typically require pile foundations. However, this numerical study of spread footings also includes such low-stiffness soils to understand their effect on structural dynamics.

Table 2. Soil parameters.

Quantity	Variable	Unit	Value
Density	ρ_{so}	kg/m ³	2000
Poisson's ratio	ν_{so}	-	0.25
Young's modulus	E_{so}	MPa	25 ÷ 700

Table 3. Upper and lower bounds of the shear and compression wave velocity corresponding to the soil parameters in Table 2.

Quantity	Variable	Unit	Value
Shear wave velocity	c_s	m/s	70.7 ÷ 374.2
Compression wave velocity	c_p	m/s	122.5 ÷ 648.1

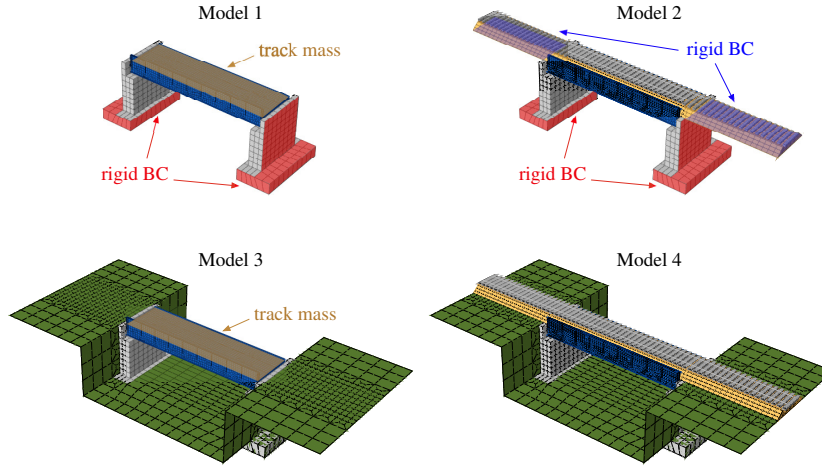


Fig. 5. Model 1: Structural model with track mass added to the deck plate and fixed boundary conditions at the foundations. Model 2: Structural model including the full track subsystem, with fixed boundary conditions at the foundations and at the base of the ballast before and after the bridge. Model 3: Soil-structure model with track mass added to the deck plate. Model 4: Soil-structure model including the full track subsystem.

3.1. Models without soil-structure interaction

The structural model neglecting soil-structure interaction is analyzed in two phases. Firstly, the stand-alone finite element model of the steel-girder bridge on a rigid foundation and rigid subsoil is considered, where the mass of the track components is simply added uniformly to the deck plate of the bridge ("Model 1" in Fig. 5). After determining the relevant modal parameters of this system, the full structural model of the coupled track-bridge system on rigid foundation is analyzed ("Model 2" in Fig. 5). Structural damping of the bridge, track and foundation is considered with *Rayleigh* damping, where the element damping matrix $\mathbf{C}^{(e)}$ is assumed to be proportional to the element mass matrix $\mathbf{M}^{(e)}$ and the element stiffness matrix

$\mathbf{K}^{(e)}$,³⁵

$$\mathbf{C}^{(e)} = \alpha_c \mathbf{M}^{(e)} + \beta_c \mathbf{K}^{(e)} \quad (3.1)$$

Alternatively, it can be expressed in terms of the k -th modal damping ratio,³⁵

$$\zeta_k^{(R)} = \frac{\alpha_c}{4\pi f_k} + \pi \beta_c f_k \quad (3.2)$$

The *Rayleigh* coefficients α_c and β_c are chosen based on a damping ratio of 0.5% for steel and a damping ratio of 2.0% for concrete and ballast components at two specific frequencies f_1 and f_2 .

For this example problem, $f_1 = 9.96$ Hz, corresponding to the fundamental frequency of the rigidly supported bridge including the mass of the track (see Fig. 6(a)), and $f_2 = 100$ Hz, characterizing the maximum frequency of interest, are substituted into Eq. (3.2) to determine α_c and β_c . The corresponding coefficients are listed in Table 4.

Table 4. <i>Rayleigh</i> damping parameters for the structural components of the FE domain.			
Material	$\zeta_1^{(R)} = \zeta_2^{(R)}$	α_c	β_c
Steel	0.5%	0.5592	$1.45 \cdot 10^{-5}$
Concrete, ballast	2.0%	2.2765	$5.7896 \cdot 10^{-5}$

Natural frequencies and effective damping ratios of the system can be derived from the homogeneous form of Eq. (2.9), rearranged in state space,³⁶

$$\mathbf{A} \dot{\mathbf{z}}_h(t) + \mathbf{B} \mathbf{z}_h(t) = \mathbf{0} \quad (3.3)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & -\mathbf{M} \end{bmatrix}, \quad \mathbf{z}_h(t) = \begin{bmatrix} \mathbf{u}_F \\ \dot{\mathbf{u}}_F \end{bmatrix} \quad (3.4)$$

Taking $\mathbf{z}_h(t) = \phi_k e^{s_k t}$ for the homogeneous solution, the eigenvalue problem³⁶

$$(s_k \mathbf{A} + \mathbf{B}) \phi_k = \mathbf{0} \quad (3.5)$$

is obtained, which yields the eigenvalues in the form of complex conjugate pairs, often referred to as poles,³⁶

$$s_k = -\zeta_k \omega_k + i \omega_k \sqrt{1 - \zeta_k^2} \quad (3.6)$$

and the corresponding complex mode shapes ϕ_k , where $k = 1, \dots, N_{FE}$. The natural frequencies f_k and the effective damping ratios ζ_k follow from Eq. (3.6),³⁶

$$f_k = \frac{|s_k|}{2\pi}, \quad \zeta_k = -\frac{\Re(s_k)}{|s_k|} \quad (3.7)$$

3.1.1. Model 1: structural model with track mass

For the bridge on rigid supports and added track mass ("Model 1" in Fig. 5), the real parts of the first bending-dominated, first torsion-dominated, and second bending-dominated mode shapes $\Re(\phi_k)$ are depicted in Fig. 6 along with the corresponding natural frequencies and effective modal damping ratios. Since the stand-alone bridge is composed of steel components only, the modal damping ratios follow directly from the *Rayleigh* parameters for steel elements (Table 4) and Eq. (3.2) evaluated at the natural frequency of the corresponding mode. A clear asymmetry in the mode shape displacements can be observed in Fig. 6. This is explained by the higher stiffness of the cross-section at path P1 compared to path P3 illustrated in Fig. 3.

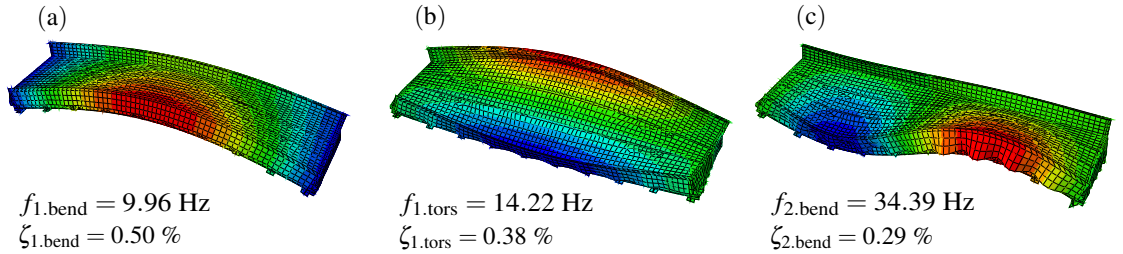


Fig. 6. Rigidly supported bridge structure with track mass. Real part of the mode shape, natural frequency, and effective damping ratio. (a) First bending-dominated mode, (b) first torsion-dominated mode, and (c) second bending-dominated mode.

3.1.2. Model 2: structural model with full track

Figure 7 shows the real parts of the mode shapes of the structural model considering the full track ("Model 2" in Fig. 5) with their corresponding natural frequencies and effective damping ratios. A clear influence of the increased system stiffness due to the full track subsystem can be seen in the natural frequencies of Fig. 7. The relative deviations in natural frequencies of the model with track mass only from the structural model with full track are listed in Table 5. The influence of the ballast stiffness on the natural frequencies of the coupled structure of bridge and track can be seen for all three modes considered and is very pronounced for the first bending- and torsion-dominated modes.

Due to the higher damping of the ballast compared to the damping of the steel components, the effective modal damping ratios of the model with track are significantly larger. The effective damping ratios of the bridge model, where the track is considered by its mass only, deviate from those of model including the full track by up to 50.5 % for the first bending-dominated mode, as shown in Table 5.

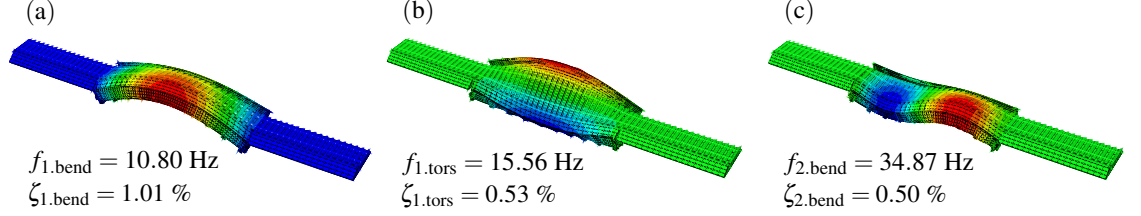


Fig. 7. Rigidly supported bridge structure with full track. Real part of the mode shapes, natural frequencies, and effective damping ratios. (a) First bending-dominated mode, (b) first torsion-dominated mode, and (c) second bending-dominated mode.

Table 5. Relative difference of the natural frequencies and the effective damping ratios $\Delta\zeta$ of the reduced structural model with track mass (Fig. 6) from those of the structural model with full track (Fig. 7).

Natural frequency	Difference (in %)	Effective damping ratio	Difference (in %)
$\Delta f_{1.\text{bend}}$	7.78	$\Delta\zeta_{1.\text{bend}}$	50.5
$\Delta f_{1.\text{tors}}$	8.61	$\Delta\zeta_{1.\text{tors}}$	28.3
$\Delta f_{2.\text{bend}}$	1.38	$\Delta\zeta_{2.\text{bend}}$	42.0

3.2. Models with soil-structure interaction

Considering the significant influence of the track on the overall system stiffness and damping in case of the rigid subsoil, the soil-structure interaction model is at first analyzed with the track mass only ("Model 3" in Fig. 5) and then with the full track (Fig. 3(a) and "Model 4" in Fig. 5). For the coupled soil-structure system, where the subsoil surface is discretized by boundary elements, the eigenvalue problem becomes non-linear due to the frequency-dependent boundary element system matrices (c.f. Eq. (2.2)). Therefore, the eigenvalues are not computed directly but an indirect approach based on frequency-domain evaluations is performed. To this end, a harmonic concentrated force is applied to the bridge deck in the vertical direction and the resulting displacements are computed. Depending on the mode shapes to be estimated, the analyses are performed with the force $F_1(\omega)$ or the force $F_2(\omega)$ with different application points as shown in Fig. 3. The corresponding frequency response functions are then computed for all displacement degrees of freedom of the model, from which the modal properties are estimated. Since the soil-structure interaction system has closely spaced and highly damped modes, the accuracy of the identified modal parameters depends highly on the estimation method used. Therefore, a variety of methods have been tested for the problem at hand: the least-squares complex exponential (LSCE³⁷), the least-squares rational function (LSRF³⁸), the rational fraction polynomial (RFP³⁹), and the least-squares complex frequency-domain (LSCF⁴⁰) methods. The LSCF method proved to be the most efficient and reliable for identifying the stable physical poles and the corresponding mode shapes. Hence, it is used for all evaluations presented in the following subsections. To reliably identify the stable physical poles, a model-order range

$[N_{\min}, N_{\max}]$ is defined, where $N_{\min}$ and $N_{\max}$ are the minimum and maximum orders, respectively. The LSCF algorithm is applied to each model order within this range, and the identified natural frequencies and damping ratios (cf. Eq. (3.7)) are stored. Then, the assessment of stability is carried out for each identified mode based on the stability criteria

$$\frac{|f_k^{(N_{\max})} - f_k^{(n)}|}{f_k^{(N_{\max})}} \leq 0.01 \quad (3.8)$$

$$\frac{|\zeta_k^{(N_{\max})} - \zeta_k^{(n)}|}{\zeta_k^{(N_{\max})}} \leq 0.05 \quad (3.9)$$

where k denotes the k -th mode and $n = N_{\min}, N_{\min} + 1, \dots, N_{\max} - 1$ the model order. A pole is considered as stable only if the criteria of Eqs (3.8) and (3.9) are met for at least 20% of the model order range considered.

3.2.1. Model 3: soil-structure model with track mass

In this section, the influence of soil-structure interaction on the modal parameters of the coupled bridge-foundation-subsoil system is analyzed. As before, the mass of the track is simply added to the deck plate of the steel-girder bridge ("Model 3" in Fig. 5). Considering the soil density and the *Poisson's* ratio from Table 2, the modal parameters are first analyzed for a fixed soil *Young's* modulus of $E_{\text{so}} = 200$ MPa. Taking into account all active degrees of freedom and the time-harmonic force $F_1(\omega)$ applied to the center of the bridge structure, the corresponding real parts of the first 18 complex-valued mode shapes $\Re(\phi_k)$ are estimated and plotted in Fig. 8. For each mode, the natural frequency f and damping ratio ζ are shown in Fig. 8. The identified mode shapes can be classified into three main categories, which are *primary structural modes*, where bending and torsion modes of the steel-girder structure dominate the mode shape (Fig. 8(1a)–(1c)), *non-structural modes*, which are mainly governed by deformations of the surrounding subsoil (Fig. 8(2a)–(2j)), and *secondary structural modes* involving only locally isolated deformations of the steel members (Fig. 8(3a)–(3e)).

To separate these categories of modes, which are differently affected by soil-structure interaction, the modal collinearity factor (MCF) or modal phase collinearity (MPC) is commonly used, e.g. by Zangeneh, et al.⁴¹ Pappa, et al.⁴² define the MPC based on the variances and covariances of the real and imaginary parts of the complex modes given by $S_{xx} = \Re(\phi_k)^T \Re(\phi_k)$, $S_{yy} = \Im(\phi_k)^T \Im(\phi_k)$, and $S_{xy} = \Re(\phi_k)^T \Im(\phi_k)$ reading as

$$\text{MPC}(\phi_k) = \frac{(S_{xx} - S_{yy})^2 + 4S_{xy}^2}{(S_{xx} + S_{yy})^2} \quad (3.10)$$

The terms MCF and MPC are often used interchangeably, with MCF defined as⁴³

$$\text{MCF}(\phi_k) = 1 - \text{MPC}(\phi_k) \quad (3.11)$$

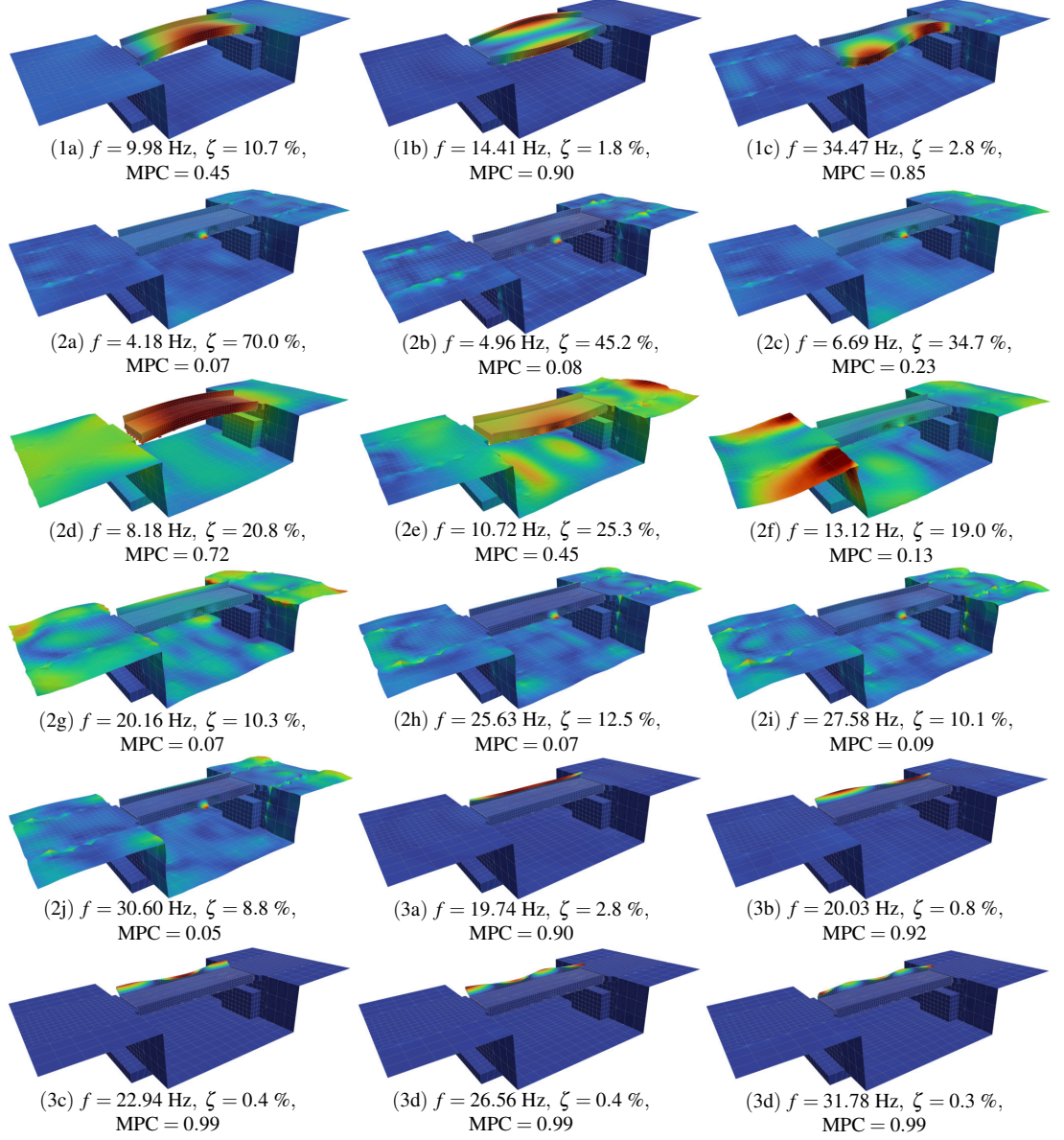


Fig. 8. Modal properties of the soil-structure interaction model with track mass and a soil *Young's* modulus of $E_{so} = 200$ MPa. Real parts of the first 18 mode shapes $\Re(\phi_k)$ with their corresponding natural frequencies f , damping ratios ζ , and modal phase collinearity (MPC). (1a)–(1c) Primary structural modes. (2a)–(2j) Non-structural modes. (3a)–(3e) Secondary structural modes.

The MPC quantifies the correlation of phase angles at the degrees of freedom for a given mode. It ranges from 0 for completely uncorrelated phase angles to 1 for com-

pletely correlated phase angles, and thus also quantifies the complexity of the mode shape, with a maximum value of 1 for real-valued modes. Since the non-classical damping associated with the wave propagation in the subsoil is the main contributor to the complexity of the mode shapes, lower MPC values indicate modes with a proportionally larger contribution of the subsoil to the overall system deformation.

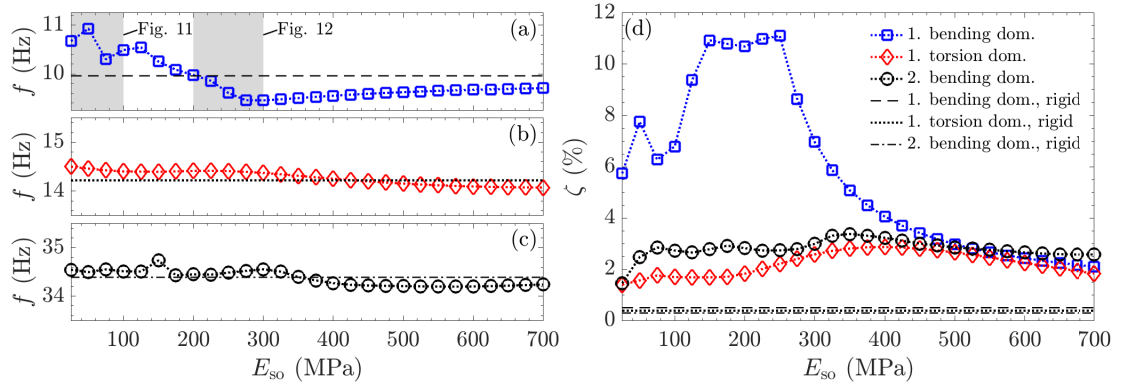


Fig. 9. Primary structural modes of the soil-structure interaction model with track mass as a function of soil modulus E_{so} . Natural frequency of (a) first bending-dominated mode, (b) first torsion-dominated mode, and (c) second bending-dominated mode. (d) Corresponding effective damping ratios.

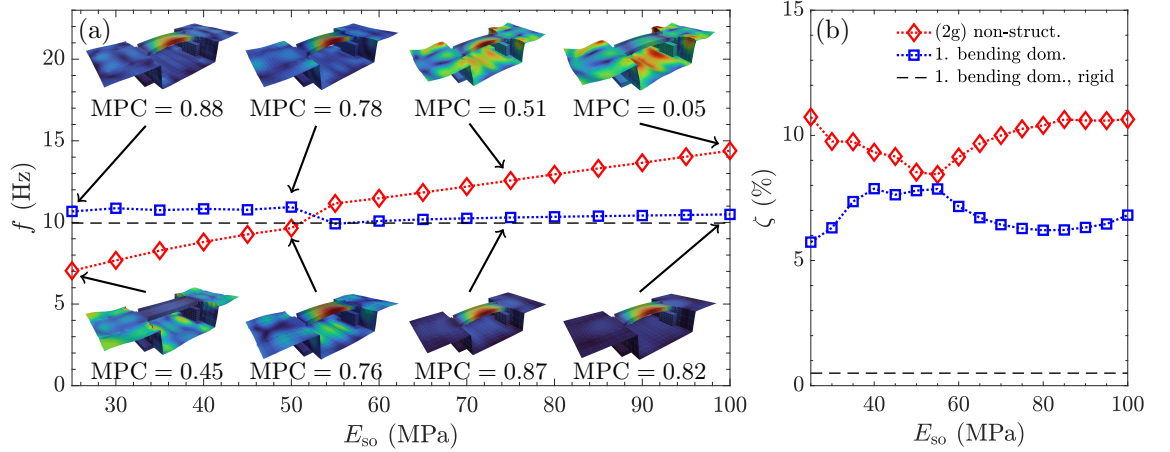


Fig. 10. First bending-dominated mode and closely-spaced non-structural mode for the soil Young's modulus between $E_{so} = 25$ MPa and $E_{so} = 100$ MPa. (a) Natural frequencies and (b) effective damping ratios.

The secondary structural modes are least affected by the subsoil, as indicated

by MPC values larger than 0.9 and the relatively low damping ratio below 3 %, as seen in Fig. 8(3a)–(3e). The primary structural modes, such as the first bending-dominated mode (1a), the first torsion-dominated mode (1b), and second bending-dominated mode (1c), have moderate to high MPC values (0.45 to 0.90) and damping ratios ranging from 1.4 % for the first torsion-dominated mode to 10.7 % for the first bending-dominated mode. The non-structural modes (2a)–(2j) have higher damping ratios ranging from 8.8 % to 70.0 %. In addition, their corresponding MPC values tend to be closer to 0, with the exception of modes (2c) and (2d) with MPC = 0.72 and MPC = 0.45, respectively. Both modes exhibit rigid-body-like displacements of the steel-girder bridge and deformations concentrated at the foundation and its surrounding subsoil, thus showing a considerable amount of damping (20.8 % and 25.3 % in Figs 8(2c) and (2d)). Of the modes described above, the primary structural modes of Figs 8(1a)–(1c) represent the counterparts to the modes of the rigidly supported structure depicted in Fig. 6.

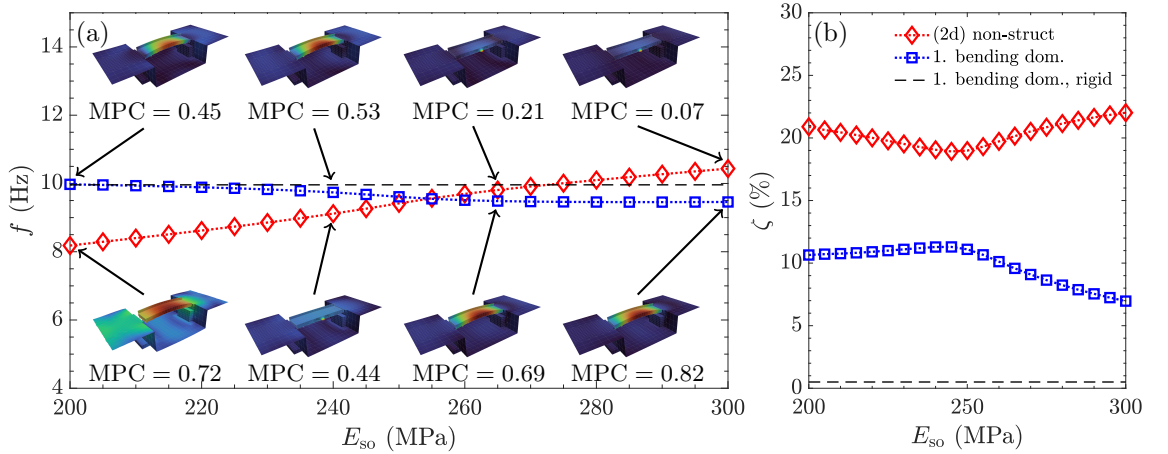


Fig. 11. First bending-dominated mode and closely-spaced non-structural mode for the soil *Young's* modulus between $E_{so} = 200$ MPa and $E_{so} = 300$ MPa. (a) Natural frequencies and (b) effective damping ratios.

Next, the dependence of natural frequencies and damping ratios on *Young's* modulus of the soil is investigated. For this purpose, the *Young's* modulus is varied within the range specified in Table 2. Since the influence of the subsoil on the secondary structural modes is small and the overall contribution of these modes to the bridge deck displacements and accelerations is considered as negligible, they are not analyzed further.

In contrast to the secondary structural modes, the primary structural modes are expected to dominate the response of the bridge deck excited by the passage of a train. Figure 9 depicts the natural frequencies of (a) the first bending-dominated mode, (b) the first torsion-dominated mode, and (c) the second bending-dominated

mode for the considered range of soil *Young's* moduli. In order to ensure sufficient excitation of the first bending-dominated and second bending modes for a reliable modal parameter estimation over the whole range of soil stiffness, the force $F_2(\omega)$ located at the quarter point of the path P3 is applied for the estimation of these two modes. Here, the largest influence of the soil *Young's* modulus is visible for the natural frequency of the first bending-dominated mode (Fig. 9(a)). This strong dependence of the modal parameters of the first bending-dominated mode on the *Young's* modulus is also visible in the corresponding effective modal damping ratio (Fig. 9(d)), which ranges from 2.10 % for the stiffest soil ($E_{so} = 700$ MPa) to 11.10 % for soil of moderate stiffness ($E_{so} = 250$ MPa). The modes with higher natural frequencies (b) and (c) are less influenced by the soil-structure interaction and have modal damping ratios below 4 % over the considered soil stiffness range. Overall, the natural frequencies of the bending- and torsion-dominated modes of the coupled bridge-subsoil system are close to those of the rigidly supported bridge, as shown by the dashed, dotted, and dash-dotted lines. However, the corresponding effective damping ratios are significantly higher for the coupled bridge-subsoil system than for the rigidly-supported bridge due to the additional geometrical damping of the subsoil.

Since a high variability of the natural frequency of the first bending-dominated mode, which is a primary mode, is observed for lower *Young's* moduli, this mode is further analyzed in Figs 10 and 11. Figure 10(a) shows the natural frequency of the identified first bending-dominated mode for *Young's* moduli from $E_{so} = 25$ MPa to $E_{so} = 100$ MPa. In addition, the natural frequency of the closest non-structural mode corresponding to the one in Fig. 8(2g) is depicted in Fig. 10(a). The corresponding mode shapes are shown for four different *Young's* moduli of the soil. While the separation of the two modes into a non-structural mode with a high contribution from the subsoil and the first primary structural bending-dominated mode is straightforward for $E_{so} = 25$ MPa, $E_{so} = 75$ MPa, and $E_{so} = 100$ MPa, it is not easy for the case of $E_{so} = 50$ MPa where the natural frequencies are closest to each other. For this case, the corresponding mode shapes show both a significant bending of the bridge and a moderate contribution from the subsoil. Also, the corresponding MPC values of 0.78 and 0.76 and damping ratios of 8.53 % and 7.80 % suggest that two similar bending-dominated modes can be identified for this particular soil stiffness. As the *Young's* modulus of the soil increases, the separation of these two modes becomes more pronounced and is supported by the MPC value approaching 0 in the case of the non-structural mode and 1 in the case of the primary structural bending-dominated mode.

Figure 11 shows another case for a non-structural mode closely spaced to the first primary structural bending-dominated mode for the range $E_{so} = 200$ MPa to $E_{so} = 300$ MPa. Here, the natural frequency of the first rigid-body-like non-structural mode, also shown in Fig. 8(2d) for $E_{so} = 200$ MPa, increases with E_{so} . The visual distinction between the first primary structural bending-dominated mode and the non-structural rigid-body-like mode is simple here, and the separation of modal

damping ratios is pronounced, as demonstrated in Fig. 11(b). However, comparing MPC values for low *Young's* moduli in the range of this figure fails to identify the two types of modes, as for $E_{\text{so}} = 200$ MPa, the non-structural rigid-body-like mode has an MPC value of 0.72, which is significantly larger than for the bending-dominated mode, which has a value of $\text{MPC} = 0.45$.

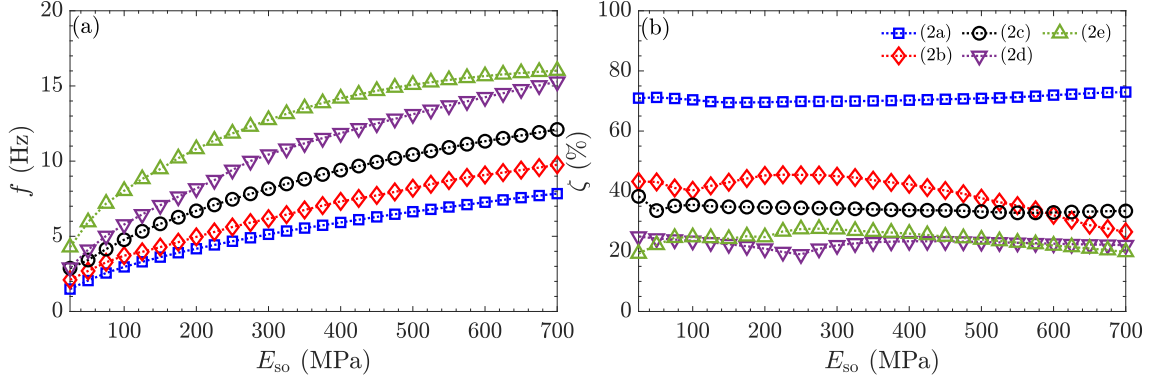


Fig. 12. Non-structural modes of the soil-structure model with track mass as a function of soil modulus E_{so} . (a) Natural frequencies and (b) effective damping ratios.

In addition, the natural frequencies and effective damping ratios of the first five non-structural modes (cf. Fig. 8(2a)–(2e)) are depicted in Fig. 12. A large influence of the soil *Young's* modulus on the natural frequency of these modes can be observed in Fig. 12, as it increases continuously with increasing E_{so} . The damping ratios associated with these modes tend to be higher and less dependent on the *Young's* modulus of the subsoil.

3.2.2. Model 4: soil-structure model with full track

The complete interaction system of Fig. 3(a) ("Model 1" in Fig. 5), consisting of the track, the bridge, the foundation, and the subsoil, is analyzed in the same manner as the reduced model with added track mass of section 3.2.1. The mode shapes are determined for a *Young's* modulus of the soil of $E_{\text{so}} = 200$ MPa and the other soil parameters listed in Table 2, based on the frequency-domain computations applying the single force $F_1(\omega)$. Figure 13 shows the real parts of the first 18 mode shapes. As can be observed, the bending and torsion-dominated primary structural modes (see Fig. 13(1a)–(1c)) have the highest MPC values. The identified non-structural modes are shown in Figs 13(2a) to (2h). Similar to the model with added track mass, two rigid-body-like modes are identified in Fig. 13(2c) and (2e). They are associated with larger deformations of the subsoil and high effective modal damping ratios of 26.6 % and 18.6 %. The secondary structural modes are shown in Figs 13(3a) to (3e). Due to the importance of the primary structural bending-dominated and

torsion-dominated modes for the response of the bridge deck, Fig. 14 shows their corresponding natural frequencies and effective modal damping ratios as a function of the soil *Young's* modulus.

In addition, the influence of material damping of the subsoil can be considered in the present frequency-domain analysis. This is achieved by applying the correspondence principle, defining the complex *Lamé* constants $\bar{G}_{so} = G_{so}(1 + i2\zeta_{so})$, with ζ_{so} as the viscous damping ratio, $G_{so} = \frac{E_{so}}{2(1+\nu_{so})}$ as the elastic shear modulus of the soil, and $\bar{\lambda} = \lambda(1 + i2\zeta_{so})$, where $\lambda = \frac{\nu_{so}E_{so}}{(1+\nu_{so})(1-2\nu_{so})}$.²⁰ The resulting shear and compression wave velocities are complex quantities. In a further analysis, material damping is assigned to the substrate via a viscous damping ratio of $\zeta_{so} = 2\%$. The corresponding results are shown in Fig. 14 with orange markers. As can be seen, the influence of this additional damping mechanism on the natural frequencies and damping ratios of the primary structural bending- and torsion-dominated modes is negligible. Therefore, material damping of the soil is omitted in the subsequent analyses.

According to the EN1991-2,¹² the design damping ratio for the given steel-girder bridge with $L_b = 16.8$ m is 0.9 %. Thus, the estimated modal damping ratio for the first bending-dominated mode is more than 1.7 times the one of the recommended design value for the stiffest considered soil with $E_{so} = 700$ MPa and up to 9.6 times for the soil of lower stiffness ($E_{so} = 125$ MPa).

Overall, the influence of the track on the modal parameters of the bending-dominated and torsion-dominated structural modes is significant. This fact is visualized in Fig. 15, where the corresponding natural frequencies and damping ratios of the model with full track (results of Fig. 14) are compared with those of the model with added track mass (results of Fig. 6). As can be seen in Figs 15(a), (b), and (c), the increase in overall stiffness due to the track results in an increase in the natural frequencies of all three primary structural modes and for all soil stiffnesses.

Furthermore, for the first bending- and torsion-dominated modes, a strong dependence of the effective modal damping ratio on the soil *Young's* modulus, together with the full track, is visible in Figs 15(d) and (e). While for low to medium stiff soil ($E_{so} = 125$ to $E_{so} = 300$ MPa), the estimated modal damping ratio of the model with track mass overestimates the modal damping ratio of the first bending-dominated mode by up to 53 %, the modal damping ratio for the stiffest soil is underestimated by up to 13 %. The opposite trend is observed for the first torsion mode, where for the soils of lower stiffness ($E_{so} < 350$ MPa) the modal damping ratio is underestimated for the model with added track mass, and the modal damping ratios are slightly overestimated for the higher stiffnesses ($E_{so} \geq 350$ MPa). These discrepancies in the estimated modal damping ratios as a function of the soil stiffness are not as pronounced for the second bending-dominated mode, where a larger modal damping ratio is computed for the model with full track over the entire range of soil stiffness considered.

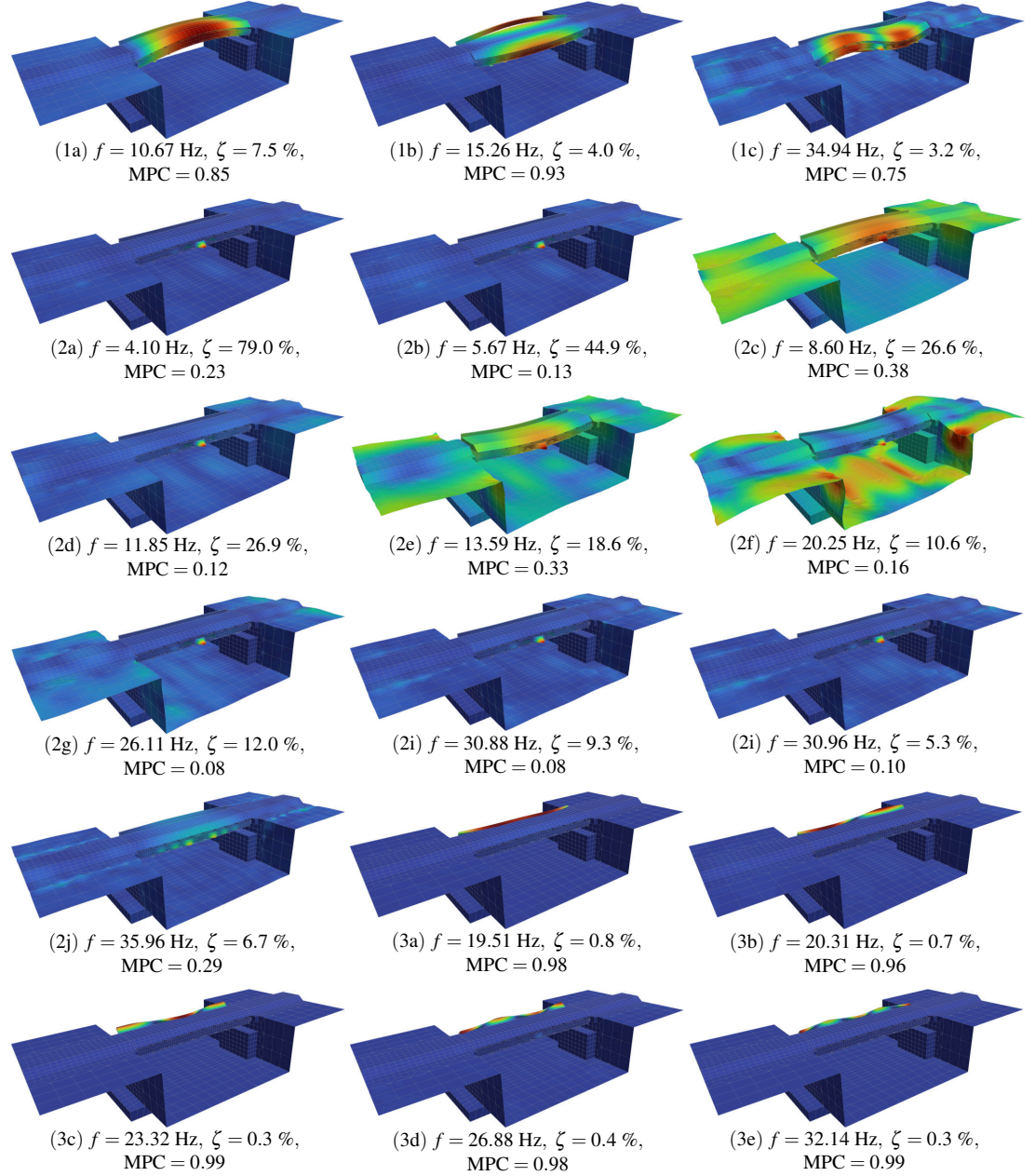


Fig. 13. Modal properties of the soil-structure model with full track and a soil *Young's* modulus of $E_{\text{so}} = 200$ MPa. Real parts of the first 18 mode shapes $\Re(\phi_k)$ with their corresponding natural frequencies f , damping ratios ζ , and modal phase collinearity (MPC). (1a)–(1c) Primary structural modes. (2a)–(2j) Non-structural modes. (3a)–(3e) Secondary structural modes.

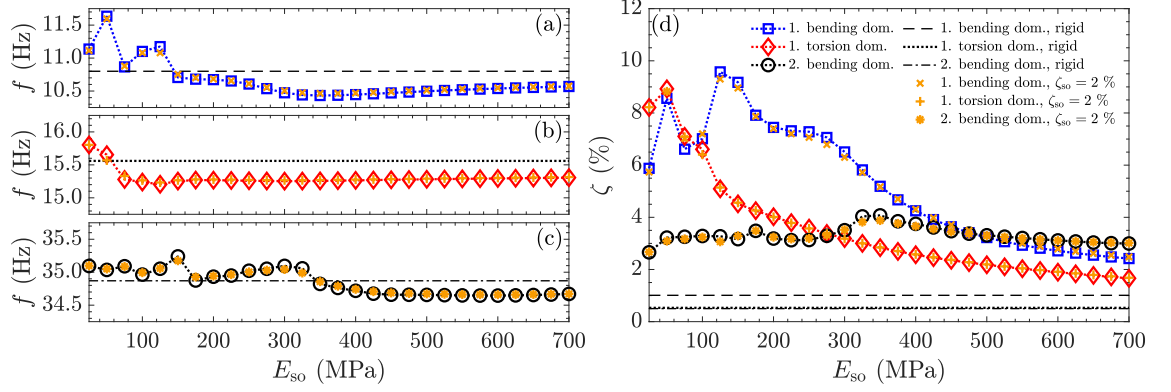


Fig. 14. Modal properties of the boundary element - finite element model with full track as a function of soil modulus E_{so} . Natural frequency of (a) first bending-dominated mode, (b) first torsion-dominated mode, and (c) red second bending-dominated mode. (d) Corresponding effective damping ratios.

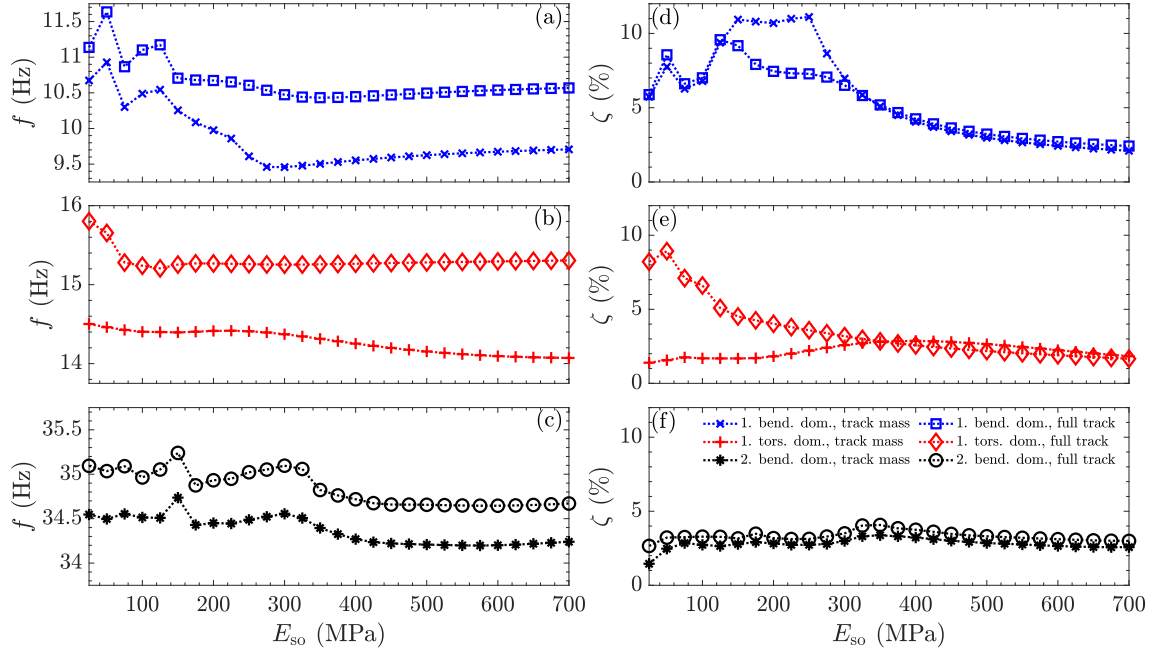


Fig. 15. Modal properties of the boundary element - finite element model as a function of soil modulus E_{so} . Model with full track vs. model with track mass. Natural frequency and effective modal damping ratio of (a),(d) first bending-dominated mode, (b),(e) first torsion-dominated mode, and (c),(f) second bending-dominated mode.

The presented results clearly emphasize the importance of the modeling of the track when considering soil-structure interaction, since ignoring the track could lead to an overestimation of the modal damping ratios and, consequently, to a

non-conservative design, or to an underestimation of the modal damping and to an overly conservative design of the structure. In addition, the influence of the track stiffness on the natural frequencies leads to a change in the response at resonance, where the vibrations are amplified.

4. Bridge response prediction due to moving train

As shown in the previous section, the consideration of the full track is of great importance for an accurate estimation of natural frequencies and modal damping ratios. Moreover, the material damping of the subsoil has a negligible influence on the properties of the bending-dominated and torsion-dominated structural modes. Therefore, the complete model considering track, bridge, foundations, and elastic subsoil, as illustrated in Fig. 3(a) ("Model 4" in Fig. 5), is considered for the response estimation due to the passage of an ICE3 train model. The conventional train consisting of eight vehicles has the same vehicle length and axle spacing for all vehicles, as shown in Fig. 16. Due to the negligible variation in the weight of each vehicle,⁴⁴ all vertical axle loads, 32 in total, are considered uniformly with 166.03 kN, as shown in Fig. 16.

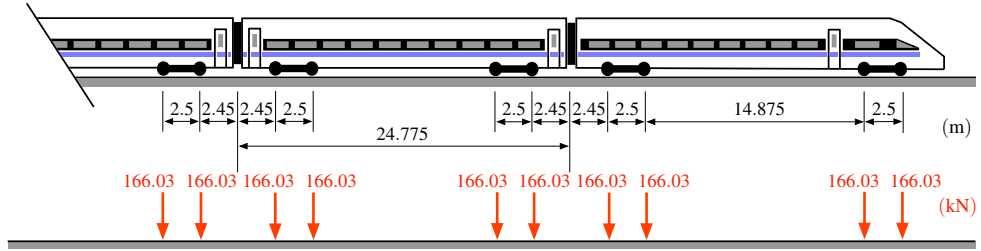


Fig. 16. Single force model of the ICE3 train.

In the numerical model, these moving concentrated loads are applied as time-dependent nodal loads at each node of the two rails. Since quadratic beam elements (B32) are used for the rails, the distribution of the moving load belonging to the a -th wheel $F^a = 166.03/2$ kN depends on the quadratic shape functions of an element (e) evaluated at the load position $\zeta_a^{(e)}$ in the intrinsic element coordinate system at time t . The corresponding shape functions of node "1" located at the intrinsic coordinate position $\zeta_a^{(e)} = -1$, denoted by N_1 , node "2", located at $\zeta_a^{(e)} = 1$, and the center node "3", located at $\zeta_a^{(e)} = 0$, read as¹⁷

$$\begin{aligned}
 N_1(\zeta_a^{(e)}(t)) &= 0.5 \left(-\zeta_a^{(e)}(t) + \left(\zeta_a^{(e)}(t) \right)^2 \right), \quad N_2(\zeta_a^{(e)}(t)) = 0.5 \left(\zeta_a^{(e)}(t) + \left(\zeta_a^{(e)}(t) \right)^2 \right) \\
 N_3(\zeta_a^{(e)}(t)) &= 1 - \left(\zeta_a^{(e)}(t) \right)^2, \quad -1 \leq \zeta_a^{(e)} \leq 1
 \end{aligned}
 \tag{4.1}$$

Figure 17 shows an example of applying a nodal force at node i , which connects two elements $(e - 1)$ and (e) , as well as the center node $i + 1$ of element (e) . Subfigure 17(a) illustrates the time instance $t = t_i$, when the concentrated load $F^{(a)}$ is positioned at node i . Subfigure 17(b) shows the time-instance $t = t_{i+1}$, when the load is located at node $i + 1$. Subfigure 17(c) shows the time-dependent loading $F_i^{(a)}(t)$, which is applied to node i and resulting from the evaluation of N_1 and N_2 . Subfigure 17(d) shows the load $F_{i+1}^{(a)}(t)$, which is applied to node $i + 1$.

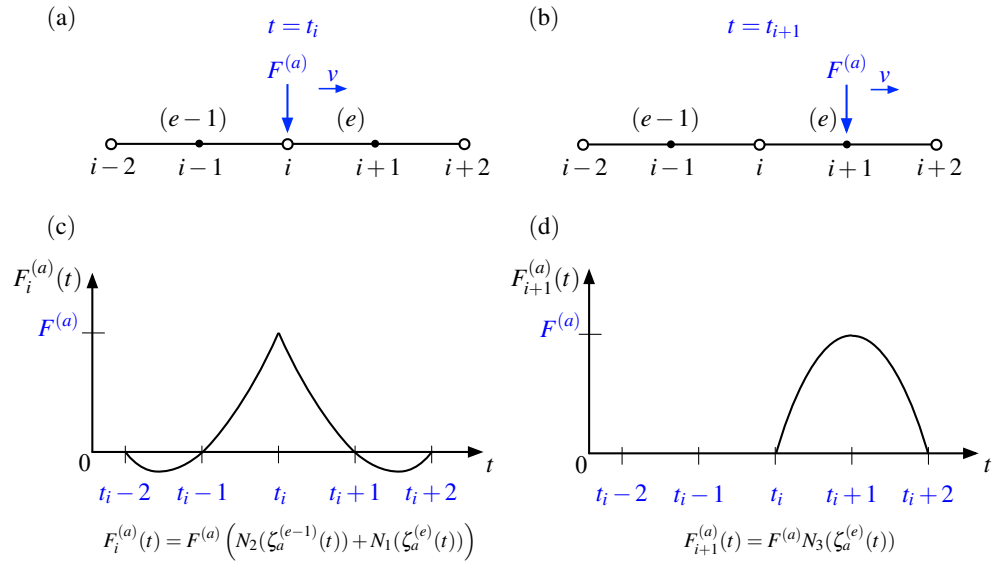


Fig. 17. Nodal loading due to a -th axle load $F^{(a)}$ moving over elements $(e-1)$ and (e) at speed v . (a) Axle load at time t_i located at node i . (b) Axle load at time t_{i+1} located at node $i + 1$. (c) Time-dependent load $F_i^{(a)}$ applied to the i -th node connecting elements $e - 1$ and e and (d) time-dependent load $F_{i+1}^{(a)}$ applied to the center node $i + 1$.

The response computation is performed using the transient time-domain formulations of Eqs (2.4) and (2.10), where a time step ratio of $\Delta t_{\text{FE}}/\Delta t_{\text{BE}} = 0.25$ is used between the boundary element and the finite element domains. Moreover, the time step of the boundary element domain is set to $\Delta t_{\text{BE}} = \beta L_e/c_p$, where the characteristic boundary element length is taken as $L_e = 0.6$ m, corresponding to the shortest element dimension at the coupling interface, and the stability parameter is set to $\beta = 0.5$. The coupling parameters of Table 1 are used for the iterative coupling of the finite and boundary element domains. Element integration is performed using the element subdivision scheme shown in Figs 1 and 2 with a number of $N_{\text{sub}} = 12$ subdivisions per intrinsic coordinate and 2×2 *Gaussian* integration per subdivision. The density ρ_{so} and the *Poisson's* ratio ν_{so} are again taken from Table 2 and five different *Young's* moduli in the range from $E_{\text{so}} = 50$ MPa

Table 6. Resonance speeds corresponding to the first bending-dominated mode $k = 1$ and $i = 3$ evaluated for the analyzed soil stiffnesses.

E_{so} (MPa)	50	100	200	400	600	∞
$v_1^{(3)}$ (m/s)	96.09	91.68	88.14	86.26	87.03	89.19

to $E_{\text{so}} = 600$ MPa as well as one rigidly supported case ($E_{\text{so}} = \infty$) are considered. The spatial and time discretization used allows an accurate representation of the structural response for frequencies below 100Hz in all cases considered. Therefore, the results are filtered using a low-pass *Butterworth* filter with a cut-off frequency of 100 Hz .

The computational results using these parameter configurations are summarized in Fig. 19 for train speeds in the range of $v = 30$ to $v = 120$ m/s. The results are shown separately for the paths P1, P2, and P3, which are located at the bridge deck, as shown in Fig. 3. Figures 19(a) to (c) depict the maximum absolute vertical bridge displacement $\max|w_b|$ at the nodes of (a) path P1, (b) P2, (c) P3 and Figs 19(d) to (f) show the maximum absolute vertical bridge acceleration $\max|\ddot{w}_b|$ at (d) P1, (e) P2, (f) P3. For the rigidly supported bridge ($E_{\text{so}} = \infty$) and soils of higher stiffness ($E_{\text{so}} = 400$ MPa and $E_{\text{so}} = 600$ MPa), pronounced response peaks can be seen at the so-called resonance speeds for repetitive loading defined by⁴⁵

$$v_k^{(i)} = \frac{f_k d_c}{i}$$

where d_c is the characteristic length of the train vehicles given by $d_c = 24.775$ m (see Fig. 16) and $i = 1, 2, \dots$. The peak responses in the considered train speed range are observed for the first bending-dominated mode $k = 1$ and $i = 3$ with the values for the resonance speeds listed in Table 6.

It is important to note that the resonance speeds listed in Table 6 contradict the notion, often found in literature,^{4,6} that resonance speeds (and natural frequencies) must decrease with decreasing soil stiffness. While this trend may be valid for moderate to high soil stiffness, it is not valid for low soil stiffness, as can be seen in the natural frequency of the first bending-dominated mode depicted in Figs 9(a) and 14(a). Here, the natural frequency increases with very low soil stiffness. However, this observation is explained by the fact that the natural frequencies of a completely unsupported structure (i.e., $E_{\text{so}} = 0$) are not necessarily lower than those of the same structure with supports (i.e., $E_{\text{so}} = \infty$). Figure 18 shows this for a simple *Euler-Bernoulli* beam with length L , mass per unit length ρA , and bending stiffness EI . As shown in Fig. 18(b), the bending frequency of the unsupported beam is more than twice that of the simply-supported beam (Fig. 18(a)). Based on these considerations, the increased natural frequencies and resonance speeds for soils with lower stiffness, as listed in Table 6, are justified.

As the *Young's* modulus decreases, the response peaks at resonance become significantly smaller for both $\max|w_b|$ and $\max|\ddot{w}_b|$. In addition, the maximum displacements for cases with lower soil stiffness increase over the entire train speed

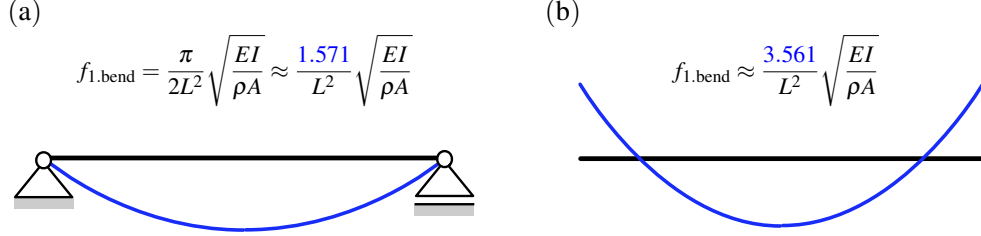


Fig. 18. First bending mode and natural frequency of an *Euler-Bernoulli* beam with (a) fixed supports and (b) free ends.

range and for non-resonant speeds. This increase in total vertical displacements is most pronounced for the case with $E_{so} = 50$ MPa, where the displacements at the foundation and consequently the whole bridge are the largest due to the low support stiffness in the vertical direction.

As can be seen by comparing the results for each path, the overall lowest response amplitudes are computed at the path P1, located at the main outer steel girder, and the highest at the opposite side of the cross-section at path P3. However, the results at P2 more closely reflect the mean deck response, with the peak acceleration $\max|\ddot{w}_b| = 8.90$ m/s² evaluated for the rigidly supported case at the resonance speed $v = 89.19$ m/s². The corresponding peak acceleration at resonance evaluated at P1 is $\max|\ddot{w}_b| = 6.97$ m/s² and $\max|\ddot{w}_b| = 10.66$ m/s² at P3. Therefore, it can be concluded that the maximum acceleration of the bridge deck varies by +23.3 % and -21.5 % with reference to that evaluated at the center path P2. This variance in the bridge response is a direct result of the asymmetry of the cross-section and, consequently, of the first bending-dominated mode. Since the location of the response peak is exactly at the nodes located at the midspan of the bridge, the relative variation of the response amplitudes at resonance by +23.3 % and -21.5 % matches that of the amplitudes of the corresponding mode shape of the first bending-dominated mode at the same node positions.

Figure 20 shows the envelope over all nodal results of the vertical bridge deck accelerations at P1, P2, and P3 transformed into the frequency domain and evaluated for train passages at the resonance speeds corresponding to the different soil stiffnesses. Here, Fig. 20(a) shows the frequency content over the total frequency range considered. It is clearly visible that the largest acceleration peaks occur at the frequencies corresponding to the natural frequencies of the first bending-dominated mode close to 10.80 Hz, and significantly smaller peaks can be observed close to the natural frequencies of the second bending-dominated mode around 34.87 Hz. No significant acceleration response is observed at frequencies higher than that of the second bending-dominated mode. In Fig. 20(b) only the frequency content close to the natural frequencies of the first bending-dominated mode is shown. As can be seen, the response peaks computed for soil *Young's* of $E_{so} = 200$ MPa, $E_{so} = 400$ MPa, and $E_{so} = 600$ MPa occurs at lower frequencies than for the rigidly supported

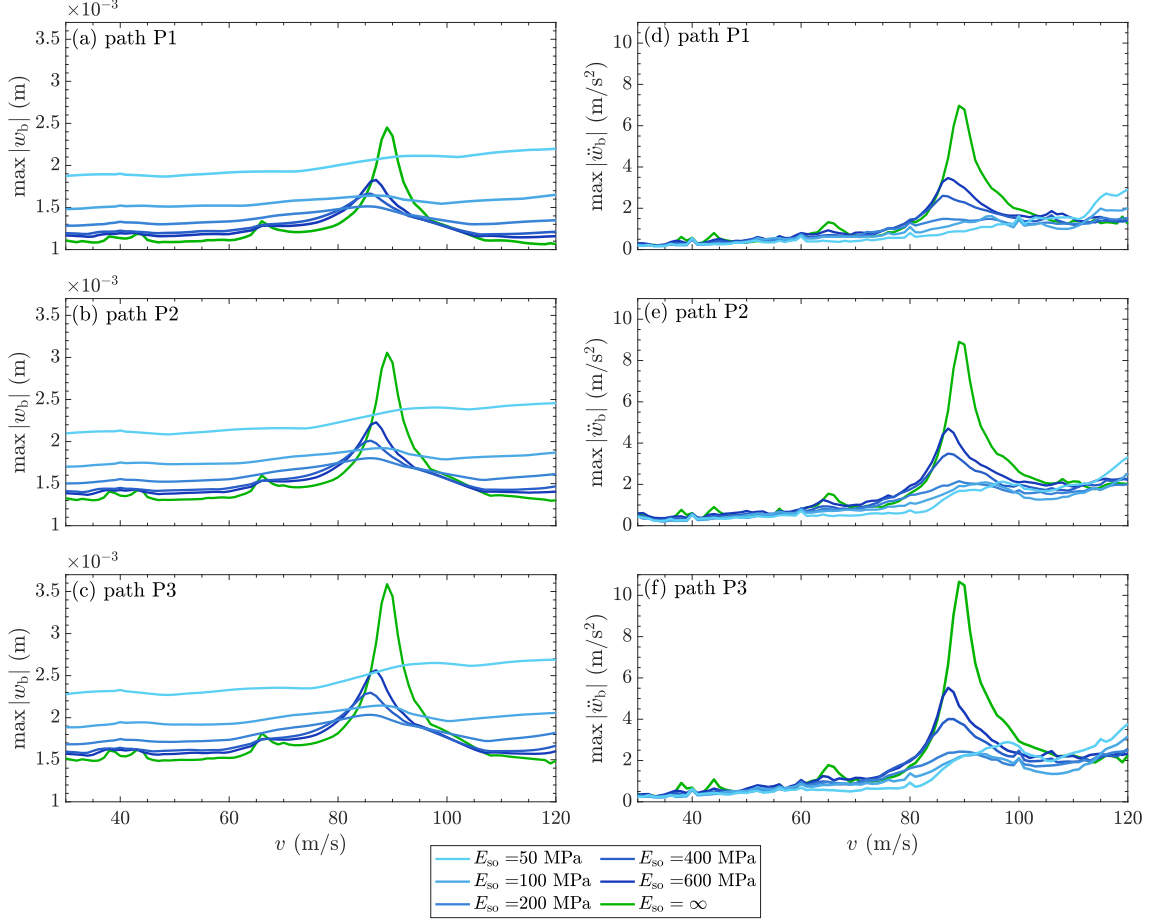


Fig. 19. Peak response of the maximum absolute vertical bridge deck displacement and the maximum absolute vertical bridge deck acceleration, evaluated at the path P1 ((a) and (d)), the path P2 ((b) and (e)), and the path P3 ((c) and (f)).

bridge and at higher frequencies for $E_{so} = 50$ MPa and $E_{so} = 100$ MPa. This observation further confirms the soil stiffness dependent shift of the natural frequency to higher or lower frequencies. The significant reduction of the acceleration response at resonance due to the increased damping from soil-structure interaction is also visible in Fig. 20.

5. Conclusions

This paper presents a comprehensive three-dimensional numerical study examining the dynamic response of a steel-girder railway bridge, accounting for soil-structure interaction. A coupled finite element-boundary element modeling approach was employed to model the interaction between the bridge, track, foundations, and

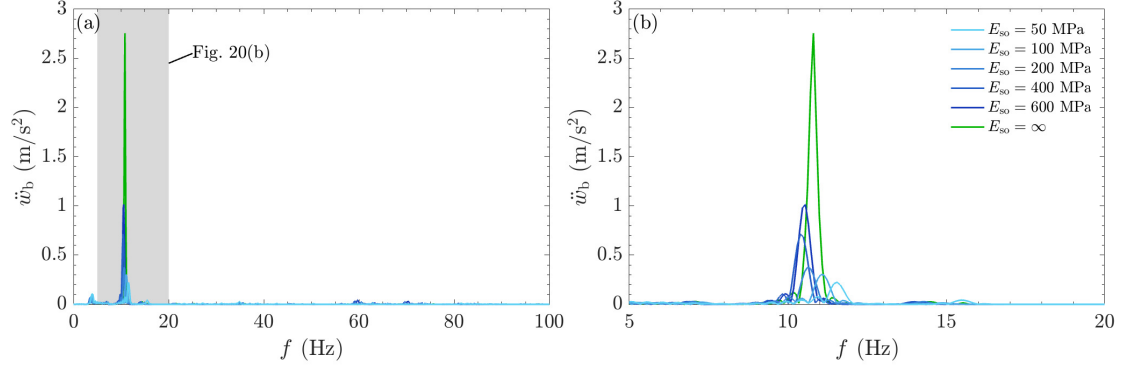


Fig. 20. Envelope of the bridge deck accelerations in frequency domain evaluated at the resonance speeds for different soil *Young's* moduli for (a) $0 \leq f \leq 100$ Hz and (b) $5 \leq f \leq 20$ Hz.

supporting subsoil. This formulation was applied in the frequency and time domains to analyze how subsoil stiffness and track modeling impact modal properties and the transient response caused by moving train loads. Based on the results, the following conclusions can be drawn:

- The three-dimensional coupled finite element-boundary element model allows for a combined frequency- and time-domain assessment of soil-structure interaction effects in steel railway bridges. This model provides consistent predictions of modal properties and transient response under train loading.
- Explicit modeling of the track and supporting subsoil can significantly alter the dynamic behavior of steel railway bridges. This affects both stiffness and damping, showing that simplified modeling approaches may lead to inaccurate response predictions.
- Soil-structure interaction causes shifts in the natural frequencies of the primary structural modes. The direction and magnitude of these shifts depend on subsoil stiffness and directly influence resonance conditions.
- In addition to the primary and secondary structural modes governed by bridge deformations, non-structural modes dominated by subsoil deformation were identified. While modal phase collinearity allows for a clear distinction between these modes when natural frequencies are well separated, it becomes less reliable for closely spaced frequencies, where similar damping ratios and mode shapes may occur.
- Soil-structure interaction primarily increases effective modal damping through geometric damping associated with wave propagation in the supporting subsoil. For the first bending-dominated mode, damping ratios were observed that were nearly an order of magnitude higher than current design recommendations, primarily due to geometric damping associated with

wave propagation in the subsoil.

- Time-domain analyses demonstrate that the increased effective damping significantly reduces bridge deck accelerations at resonance speeds. This highlights the importance of accounting for soil-structure interaction when assessing the dynamics of steel railway bridges to avoid overly conservative results.

The present study is based on a homogeneous elastic representation of the subsoil and idealized bearing conditions. Future studies should investigate the influence of stratified soil profiles, nonlinear track behavior, and bearings with reduced stiffness, which could lead to a partial dynamic decoupling between the bridge and the foundation. Although more computationally efficient frequency- and time-domain formulations may be adopted for future applications, the physical trends and conclusions reported in this study are valid regardless of the specific numerical implementation. Additionally, extending the time-domain formulation to include fully coupled train-bridge interaction models would further enhance the applicability of the approach for realistic serviceability assessments.

The coupled finite element-boundary element approach presented here provides a suitable reference model for analyzing soil-structure interaction in railway bridges. The results can support the calibration of simplified, design-oriented beam models and inform more realistic serviceability evaluations under train loading. Additionally, the time-domain formulation can be extended to include fully coupled train-bridge interaction analyses.

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Appendix A. Model Parameters

Table 7. Parameters of the steel-girder bridge.³³

Description	Variable	Unit	Value
Main girder web	h_1, t_{web1}	mm	2090, 20
Main girder flange	$h_{f1}^u, t_{f1}^u, h_{f1}^l, t_{f1}^l$	mm	200, 60, 310, 60
Secondary T-girder	$h_2, t_{web2}, h_{f2}, t_{f2}$	mm	1140, 12, 350, 60
Through girder	$h_3, t_{web3}, h_{f3}, t_{f3}$	mm	1100, 20, 747, 80
Transversal girder	$h_4, t_{web4}, h_{f4}, t_{f4}$	mm	1100, 20, 350, 30
Deck plate	t_{deck}	mm	35
Steel material	$E_{st}, \rho_{st}, \nu_{st}$	GPa, kg/m ³ , -	210, 7850, 0.3

Table 8. Parameters of the ballast in the finite element model.^{11,34}

Description	Variable	Unit	Value
Ballast dimensions	$h_b, b_{ballast}^l, b_{ballast}^u$	mm	700, 2335, 1500
Ballast material	E_b, ρ_b, ν_b	MPa, kg/m ³ , -	120, 1500, 0.2

Table 9. Parameters of the foundation.

Description	Variable	Unit	Value
Mass	$\tilde{m}_1$	kg	230338
Base area	A_0	m ²	34.4
Reinf. concrete material	$E_{cr}, \rho_{cr}, \nu_{cr}$	GPa, kg/m ³ , -	32.837, 2500, 0.2

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Table 10. Parameters of the rail and sleeper in the finite element model.^{46,47}

Description	Variable	Unit	Value
Half gauge distance	b	mm	720
Rail profile (UIC 60)	A_r, I_y, I_z	cm ² , cm ⁴ , cm ⁴	76.70, 3038.30, 512.30
Sleeper spacing	e	mm	600
Sleeper dimensions	b_{sl}, h_{sl}, l_{sl}	mm	230, 200, 2600
Sleeper material	$E_{sl}, \rho_{sl}, \nu_{sl}$	GPa, kg/m ³ , -	32.837, 2400, 0.2
Rail pad stiffness	k_{rx}, k_{ry}, k_{rz}	MN/m	49, 100, 150

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