

Pulse-Compression SDR Radar with Doppler Processing and Two-Plane Monopulse Localization Using ADALM-PLUTO

Denys I. Zaikin

Whitecell Systems ApS

Aalborg, Denmark

e-mail: denys.zaikin@whitecell-systems.com

ORCID: 0000-0003-4080-5631

Abstract—This paper presents the design, implementation, and experimental evaluation of a low-cost pulse-compression radar based on the ADALM-PLUTO software-defined radio platform. The radar operates near 2.4 GHz and transmits 1 ms linear-frequency-modulated chirps with a bandwidth and sampling rate of 30.72 MHz. A modified dual-receive, dual-transmit ADALM-PLUTO provides coherent full-duplex acquisition, while separate patch-antenna arrays form the transmit and receive apertures. The implemented processing chain combines pulse compression, coherent Doppler processing, clutter-map subtraction, CFAR detection, and two-plane monopulse localization using amplitude comparison in elevation and phase comparison in azimuth.

The work addresses practical SDR-radar limitations, including transmit-to-receive leakage, transmitter-induced spurious components, channel-delay mismatch, acquisition-buffer timing offsets, and host-processing latency. Analytical models are developed for the antenna patterns, monopulse response, detection range, and localization accuracy.

Five outdoor experiments involving pedestrian, vehicle, and maritime targets produced dominant detection sequences consistent with the target routes and demonstrated Doppler-based radial-velocity discrimination and qualitative trajectory formation in urban and coastal environments at ranges up to approximately 600 m. Range estimation was generally more stable than cross-range and height estimation, which remained sensitive to multipath propagation, calibration errors, and angular-estimation uncertainty. The platform and accompanying open-source implementation provide an accessible framework for studying pulse compression, Doppler processing, monopulse localization, and low-cost SDR-based radar systems.

Index Terms—software-defined radar, ADALM-PLUTO, pulse compression, linear frequency modulation, Doppler processing, moving target indication, clutter map, CFAR, monopulse tracking.

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I. INTRODUCTION

Software-defined radio (SDR) platforms provide a flexible and comparatively inexpensive basis for implementing radar waveforms and signal-processing algorithms. Their programmable transmit and receive chains support experimental pulse compression, Doppler processing, adaptive detection, and direction finding without dedicated radar hardware [1]–[3]. However, coherent radar operation on a general-purpose SDR requires careful treatment of practical limitations such as restricted bandwidth, transmit-to-receive leakage, channel mismatch, converter nonlinearities, data-transfer latency, and limited real-time processing capability.

Pulse compression combines the energy of a long waveform with the range resolution associated with a short pulse. In particular, a linear frequency-modulated (LFM) chirp can be compressed using a matched filter, with processing gain determined by its time–bandwidth product [4]–[7]. Coherent processing across successive chirps additionally provides integration gain and Doppler-frequency discrimination, although range–Doppler coupling, sidelobes, blind speeds, and Doppler ambiguity must be considered [4], [6], [8].

Target angle can be estimated using simultaneous observations from multiple receive channels. Amplitude-comparison monopulse estimates the angle from the relative magnitudes of overlapping antenna patterns, whereas phase-comparison monopulse uses the phase difference between spatially separated channels [9]–[12]. When combined with pulse-compression range measurements, these angular estimates provide two-plane monopulse localization and enable the formation of qualitative target trajectories in Cartesian coordinates.

The ADALM-PLUTO integrates a programmable RF transceiver, data converters, programmable logic, and a host interface in a compact platform [13]–[15]. Its programmable carrier frequency, bandwidth, sampling rate, and receiver gain make it suitable for experimental sensing and radar applications. Nevertheless, transmitter scaling and nonlinearities must be considered because transmitter-induced spurious components can raise the interference floor after pulse compression [16], [17].

Previous ADALM-PLUTO-based work has demonstrated phase-sensitive displacement measurement, radar signal processing, digital beamforming, and direction finding [18]–[22]. These implementations establish the platform’s suitability for low-cost coherent sensing but generally emphasize individual radar functions.

In contrast, this work integrates LFM pulse compression, coherent Doppler processing, clutter-map subtraction, CFAR detection, and two-plane monopulse localization in a single architecture using only two receive channels. The clutter-map path complements conventional nonzero-Doppler moving-target indication by detecting newly appearing, displaced, or slowly varying targets whose responses occur in or near the zero-Doppler channel [23], [24]. Particular attention is given to practical full-duplex effects, including transmit leakage, transmitter-induced spurious components, channel-delay mismatch, acquisition-buffer timing offsets, and host-processing latency.

The radar uses a modified ADALM-PLUTO with two coherent receive channels and two combined transmit channels. It operates at 2.425 GHz and transmits 1 ms LFM chirps with a bandwidth of 30.72 MHz. Squinted receive antennas provide amplitude-comparison monopulse measurements in elevation, while their spatial separation enables phase-comparison monopulse measurements in azimuth. Range and angular estimates are combined to calculate Cartesian target coordinates and form qualitative target trajectories. The experimental evaluation is intended to demonstrate two-plane monopulse localization and trajectory formation rather than to provide a complete validation of accurate 3D tracking.

The principal contributions of this work are:

- **An integrated SDR radar architecture** that combines LFM pulse compression, coherent Doppler processing, adaptive clutter-map subtraction, CFAR detection, and two-plane monopulse localization on a low-cost ADALM-PLUTO platform.
- **A practical synchronization and calibration procedure** that compensates for acquisition-buffer timing offsets and

channel-dependent cable delays while preserving coherence among the received chirps.

- **An analysis of transmit leakage and transmitter-induced spurious components**, including their effects on the post-compression interference floor, together with practical mitigation techniques based on antenna isolation, waveform weighting, DAC-level optimization, and reference-signal subtraction.
- **A complementary target-detection architecture** comprising a Doppler filter bank for targets with nonzero radial velocities and a clutter-map processing path for newly appearing, displaced, or slowly varying targets whose responses occur in or near the zero-Doppler channel.
- **A two-plane monopulse localization method using two receive channels** that combines amplitude-comparison monopulse processing in elevation with phase-comparison monopulse processing in azimuth. The resulting angular estimates are combined with the measured range to calculate Cartesian coordinates and form qualitative target trajectories.
- **Analytical models for target-detection range and localization accuracy**, supported by antenna-pattern calculations, electromagnetic simulations, signal-processing simulations, and hardware measurements. Outdoor experiments demonstrate moving-target detection and qualitative trajectory formation while identifying practical limitations associated with antenna-pattern nulls, multipath propagation, phase ambiguity, and channel mismatch.
- **An open implementation framework** comprising host-based signal-processing software and supporting antenna-design and hardware-development materials.

The remainder of this paper is organized as follows. Section II describes the radar hardware and software architecture, and Section III summarizes the implemented signal-processing algorithm. Section IV develops the received-signal model. Section V discusses transmit leakage and antenna isolation, while Section VI presents pulse compression, matched-filter normalization, waveform weighting, and transmitter-induced spurious components. Section VII derives the detection-range and tracking-accuracy relationships. Sections VIII–X describe Doppler moving-target indication, clutter-map processing, and CFAR detection. Section XI presents two-plane monopulse localization and reference-object tracking. Section XII reports the experimental evaluation of target detection and qualitative trajectory formation, Section XIII discusses limitations and future work, and Section XIV concludes the paper.

II. RADAR ARCHITECTURE

A. Radar description

The presented radar is constructed using the ADALM-PLUTO software-defined radio (SDR) platform as a full-duplex receiver (RX) and transmitter (TX) [13]. This SDR is configured as described in Appendix E, enabling additional RX and TX channels, resulting in two RX channels (RX0 and RX1) and two TX channels (TX0 and TX1).

The transmitter generates a linear frequency-modulated (LFM) signal (chirp) characterized by a high time–bandwidth product $\approx \tau B$. At the receiver, after applying a digital matched filter (i.e., complex convolution), the received signal is compressed in time to an equivalent pulse duration of $\approx 1/B$. As a result, the range resolution is significantly improved and can approach the theoretical limit $\Delta R = c/(2B)$, where c is the speed of light in free space and B is the chirp bandwidth [4].

The range to the target is determined by measuring the time delay Δt between the transmitted signal and the received compressed pulse, according to $R = c \Delta t/2$.

The RX channels are directly connected to patch antenna arrays (PAAs), each consisting of a 2×2 array of microstrip patch antennas (MPAs) implemented on a printed circuit board (PCB), as shown in Fig. 1. The two RX channels form a monopulse receiver that provides two-plane target localization. The TX antennas are identical to the RX antennas but are not configured with squint. The signals from the two TX channels are combined, amplified, and subsequently split between the two TX antennas.

The radar boresight is aligned with the positive y -axis. The elevation and azimuth angles are expressed using the standard spherical-coordinate angles θ and φ , measured from the positive z - and x -axes, respectively. Consequently, the boresight direction corresponds to $\theta = \varphi = 90^\circ$.

The amplitude comparison monopulse (ACM) technique is used to track objects in the E-plane (elevation angle), while phase comparison monopulse (PCM) is used to track objects in the H-plane (azimuth angle) [9]–[12]. Thus, the RX antennas are squinted in the E-plane and separated by a distance d in the H-plane, as demonstrated in Fig. 1.

Two prototypes of the presented radar are shown in Fig. 2. Prototype #1 has the TX antenna positioned above the RX antenna, with a larger separation between them, resulting in better TX–RX isolation and lower transmit leakage. Prototype #2 has the RX antenna positioned above the TX antenna, with a smaller separation between them. Owing to the higher transmit leakage in Prototype #2, its TX power is reduced by decreasing the digital amplitude applied to the TX digital-to-analog converter (DAC), as described in Sections VI-E and V.

The main hardware components used to construct the presented radar are listed in Table I.

TABLE I
MAIN COMPONENTS OF THE PRESENTED RADAR.

Component	Quantity	Description
ADALM-PLUTO Rev. C	1	SDR platform
RF2126 module	2	RF power-amplifier [25]
Wilkinson power splitter/combiner	2	[26]
2×2 patch antenna array	4	[27]
DC–DC converter, 15 V $\rightarrow$ 5 V	1	for the SDR
DC–DC converter, 15 V $\rightarrow$ 6 V	1	for the RF2126
Ethernet-to-USB adapter	1	Used with PoE
AC–DC adapter, 15 V, 3 A max.	1	Mains power supply

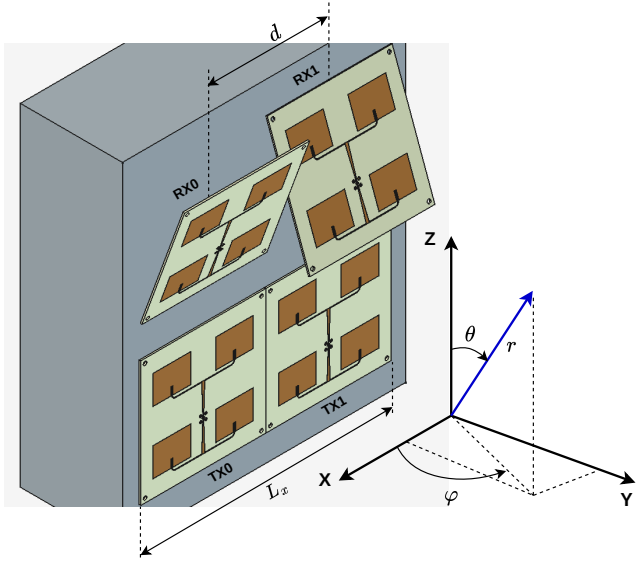


Fig. 1. RX and TX antennas and main coordinate system for the presented radar.

TABLE II
RADAR CHARACTERISTICS

Parameter	Value
SDR platform	ADALM-PLUTO Rev. C
Center frequency, f_0	2.425 GHz
Chirp frequency sweep deviation, B	30.72 MHz
Sample rate, f_s	30.72 MHz
Range resolution, Δ_R	5 m
RF noise bandwidth, B_n	30 MHz
System noise temperature, T_s	425 K
Number of RX channels, N_{RX}	2
Number of TX channels, N_{TX}	2 (combined)
Chirp duration, τ	1 ms
Pulse repetition interval, T_{PRI}	1.1 ms
Loop interval (update rate), T_L	0.1–1.0 s
Number of chirps integrated, n	2–16
TX power, P_t (Prototype #1, $K_{DAC} = 2^{14}$)	26 dBm
TX power, P_t (Prototype #2, $K_{DAC} = 2^{13}$)	20 dBm
TX antenna gain, sum-pattern, G_{TX}	13.2 dBi
TX sum-pattern beam width, E-plane $\theta_{0.5P}^{E,\Sigma,TX}$	56°
TX sum-pattern beam width, H-plane $\theta_{0.5P}^{H,\Sigma,TX}$	26°
RX antenna gain, sum-pattern, G_{RX}	11.9 dBi
RX sum-pattern beam width, E-plane $\theta_{0.5P}^{E,\Sigma,RX}$	66°
RX sum-pattern beam width, H-plane $\theta_{0.5P}^{H,\Sigma,RX}$	26°
Target RCS, m^2	1
Predicted R_{max} , $\sigma = 1 m^2$, $n = 16$, Prototype #1	1000 m

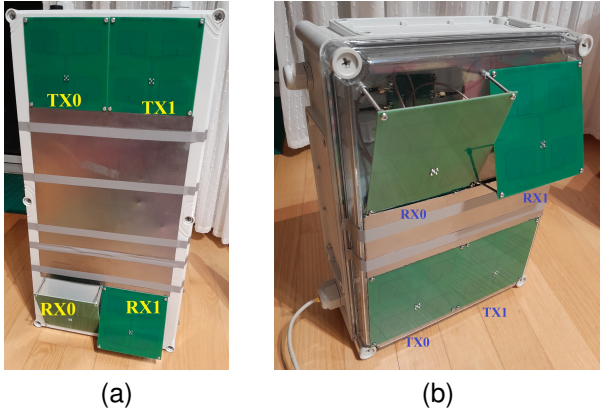


Fig. 2. Prototypes #1 and #2 of the presented radar. (a) Prototype #1. (b) Prototype #2.

B. Main radar characteristics

Main radar parameters and characteristics are summarized in Table II. The development of the antenna characteristics and their monopulse capabilities can be found in Appendix A.

C. Hardware block diagram

A simplified hardware (HW) block diagram of the discussed radar is shown in Fig. 3. The RX input channels of the SDR are directly connected to the RX antennas, while the TX0 and TX1 output channels are first combined using Wilkinson power combiners and then amplified by radio-frequency (RF) power amplifiers. The amplified TX signal is subsequently split between two TX antennas. This approach enables equal power to be applied to the TX antennas independently of gain differences between the two RF power amplifiers.

Based on the AD9363 integrated circuit (IC), the ADALM-PLUTO SDR uses a common local oscillator for both the

transmitter and receiver and operates with a zero intermediate frequency at the receiver.

D. Software block diagram

The user interface of the application is described in Appendix D. The source code and design materials are available online [28], [29]. A simplified software (SW) block diagram of the discussed radar is shown in Fig. 4.

The reference chirp (RC), stored in memory, is used both to generate the TX signal and as a basis for the matched filter in the receiver. During the main loop interval T_L , a fixed number of chirps n is transmitted and received. The received n chirps from both RX channels are stored in the RX buffer as complex data for further signal processing. The n chirps (one burst) constitute a coherent time interval (CTI).

After processing the CTI, a new loop interval begins, and a new series of n chirps is transmitted and received. The time diagram of the SDR operation and signal processing is shown in Fig. 5.

Some selected features are briefly described below, while the remaining aspects are discussed in the following sections.

1) *TDD Synchronization*: To synchronize the transmitter and receiver for coherent signal processing, the ADALM-PLUTO SDR uses Time-Division Duplexing (TDD), which acts as a multi-channel pulse generator and allows control over the timing of the transmit and receive bursts [30].

In the current implementation, the *TDD sync* signal generated by the software initializes the n chirps (burst or CTI) to be transmitted by the transmitter (TDD channel #2) and synchronously received by the receiver (TDD channel #1).

The TX cyclic buffer mode required for TDD operation mandates that the TX buffer contain only a single reference chirp, which is transmitted n times. In contrast, the RX buffer stores all n received chirps.

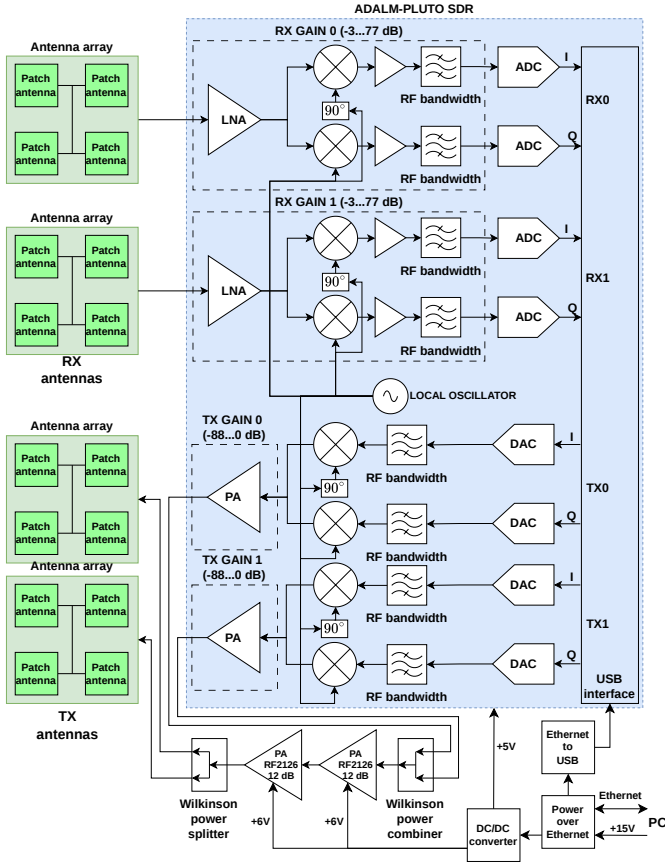


Fig. 3. Block diagram of the HW architecture of the presented radar.

2) *CTI Alignment*: In the ADALM-PLUTO SDR, the TX and RX buffers have been observed to exhibit a random offset of one sample between successive CTIs. This timing jitter may affect range measurements, coherent processing across different CTIs, and the representation of target tracks in the waterfall display. The sample offset is identical for all chirps within a given CTI; therefore, coherence among the chirps within that CTI is preserved.

To compensate for the timing jitter, successive CTIs are aligned using cross-correlation. One CTI is selected as the reference, and a central portion of its first chirp is used as the reference segment. The corresponding segments from subsequent CTIs are cross-correlated with this reference segment to estimate the sample offset. Each CTI is then shifted by the estimated offset before further processing. Using only the central portion of the chirp reduces the computational load while retaining sufficient signal magnitude for reliable offset estimation, even when an amplitude window is applied to the transmitted LFM waveform. The alignment procedure is illustrated in Fig. 6.

3) *Cable-Delay Compensation for Channels RX1 and TX1*: As described in Appendix E, the SDR is modified to enable an additional RX channel (RX1) and TX channel (TX1). However, these additional channels are connected to the SMA connectors using approximately 10 cm coaxial cables, which

introduce an additional delay in the signal paths of the RX and TX channels. These cable delays can be compensated for separately in software (SW).

Let us assume that the complex RX0 and RX1 RF input signals have the form

$$s_{RX0}(t) = a_{RX0}(t)e^{j\omega_0 t}, \quad (1)$$

$$s_{RX1}(t) = a_{RX1}(t - t_d)e^{j\omega_0(t - t_d)}, \quad (2)$$

where a_{RX0} and a_{RX1} are the complex envelopes of the analytic signals s_{RX0} and s_{RX1} , respectively; t_d is the additional delay introduced by the RX1 cable; $\omega_0 = 2\pi f_0$ and $j = \sqrt{-1}$.

After mixers and analog-to-digital converters (ADC), the complex signals for data processing become:

$$s_{RX0}^{ADC}(t) = a_{RX0}(t), \quad (3)$$

$$s_{RX1}^{ADC}(t) = a_{RX1}(t - t_d)e^{-j\omega_0 t_d}, \quad (4)$$

Assuming that the input signals are narrowband [5], i.e., $\Delta B \ll f_0$, and that the cable delay is on the order of $t_d \propto 1/f_0$, the envelopes a_{RX0} and a_{RX1} can be considered slowly varying. Therefore, it can be assumed that $a_{RX1}(t - t_d) \approx a_{RX1}(t)$.

Under this assumption, the RX1 channel cable delay can be compensated by multiplying Eq. (2) or (4) by the term $e^{j\omega_0 \hat{t}_d}$, where $\hat{t}_d$ is a delay compensation parameter that can be adjusted by the operator:

$$s'_{RX0}(t) = a_{RX0}(t)e^{j\omega_0 t}, \quad (5)$$

$$s'_{RX1}(t) \approx a_{RX1}(t)e^{j\omega_0(t - t_d + \hat{t}_d)}, \quad (6)$$

$$s_{RX0}^{ADC'}(t) = a_{RX0}(t), \quad (7)$$

$$s_{RX1}^{ADC'}(t) \approx a_{RX1}(t)e^{j\omega_0(-t_d + \hat{t}_d)}. \quad (8)$$

The same compensation procedure can be applied to the TX1 channel.

4) *TX-Leakage Compensation*: The effect of transmit leakage into the RX path is described in more detail in Sections VI-E and V. Here, a software-based approach for its compensation is presented.

The reference chirp, stored in memory and used to generate the TX signal and implement the matched filter, is also employed to subtract the leakage component from the RX signal, as shown in Fig. 4. A common leakage-reference magnitude and delay are applied to both receive channels in the current implementation. The magnitude α_r and delay τ_d of the reference chirp can be adjusted by the operator in the software application to achieve optimal compensation:

$$a_{RX0}^{comp}(t) = a_{RX0}(t) - \alpha_r a_{REF}(t - \tau_d), \quad (9)$$

$$a_{RX1}^{comp}(t) = a_{RX1}(t) - \alpha_r a_{REF}(t - \tau_d). \quad (10)$$

This approach reduces the main lobe of the transmit leakage and, consequently, mitigates sidelobe effects after matched filtering, since the window function applied to the transmitted chirp has limited capability for sidelobe suppression of the compressed pulse (see Section VI-D).

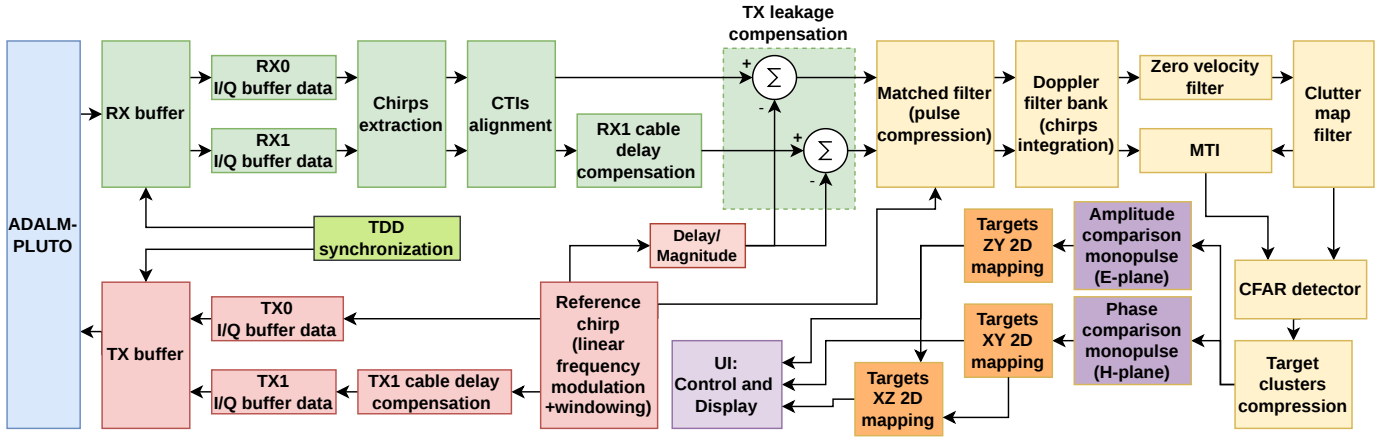


Fig. 4. Block diagram of signal-processing chain of the radar.

It should be noted that the compensation described above is effective only in reducing the non-ideal sidelobes caused by the window function. This limitation arises because the reference chirp does not exactly match the actual transmitted signal, which may contain additional spurs and noise. In the ideal case, an exact replica (“bucked-off”) of the actual TX signal should be subtracted at the receiver input [4, Chapter 3.2].

Spur and noise components present in the real TX signal cannot be compensated for using only the reference chirp stored in memory. Additional mitigation methods based on increased TX–RX isolation and reduced transmitter spurious components are discussed in Section V and Section VI-E, respectively.

5) *Target-Cluster Compression*: Target-cluster compression represents a sequence of adjacent detected range bins by a single detection. Such a sequence may be produced by a spatially extended target, range sidelobes, or multiple unresolved targets occupying nearby range bins. The concept is illustrated in Fig. 7.

In the MTI processing path, target-cluster compression is first applied independently to the output of each enabled nonzero-Doppler filter after CFAR detection. Within each sequence of adjacent detections, only the range bin having the greatest signal magnitude is retained. The retained detection is assigned the corresponding peak magnitude, while the remaining detections in the cluster are discarded. The Doppler filter having the largest positive CFAR threshold exceedance is subsequently selected among the retained cluster peaks at each range bin. The complex RX0 and RX1 samples associated with the retained peak and selected Doppler filter are used for monopulse processing.

Target-cluster compression may also be applied after fusion of the MTI and clutter-map detections. This second compression stage removes adjacent detections that may remain because the peak range bin can differ between Doppler filters or between the MTI and clutter-map processing paths.

The procedure prevents a spatially extended response from being reported as multiple targets. However, it may also merge

two or more unresolved targets occupying adjacent range bins. The CFAR processing and Doppler-filter selection procedure are described in Section X.

6) *RX-Gain Parameter Selection*: The total receiver noise consists of the noise generated by the receiver front-end circuitry (amplifiers, mixers, and filters) and the external noise received by the antenna. The total noise reaching the ADC determines the effective dynamic range of the receiver.

The noise and signal levels at the ADC input can be adjusted using the *rx_gain* parameter of the PlutoSDR. The maximum ADC input level corresponding to 0 dBFS is 0.625 V peak [14].

If the noise level at the ADC input is too high relative to the ADC’s own noise, the dynamic range of the receiver chain is reduced due to the increased probability of clipping and quantization limitations. Conversely, if the noise level at the ADC input is significantly lower than the ADC’s intrinsic noise floor, the receiver sensitivity is degraded because a portion of the available ADC dynamic range is unused. As a result, target detection performance based on the signal-to-noise ratio deteriorates [6, Chapter 6.10].

The *rx_gain* parameter should be selected such that the receiver noise floor remains sufficiently above the ADC intrinsic noise floor while maintaining adequate margin to avoid ADC saturation.

Figure 8 presents the measured receiver noise level at the ADC output (sum-channel response) as a function of the *rx_gain* parameter. The noise is measured at the output of Doppler filter #2 ($k = 2$; see Section VIII-C), with the transmitter disabled and moving target indication (MTI) enabled. Since the measurements are performed using the actual radar hardware, both antenna noise and receiver thermal noise contribute to the measured values, even though the radar is operated indoors in a laboratory at a room temperature of 293 K.

The nonlinear dependence in Fig. 8 arises because the *rx_gain* parameter does not correspond to a single gain stage. Instead, the overall gain is distributed among multiple am-

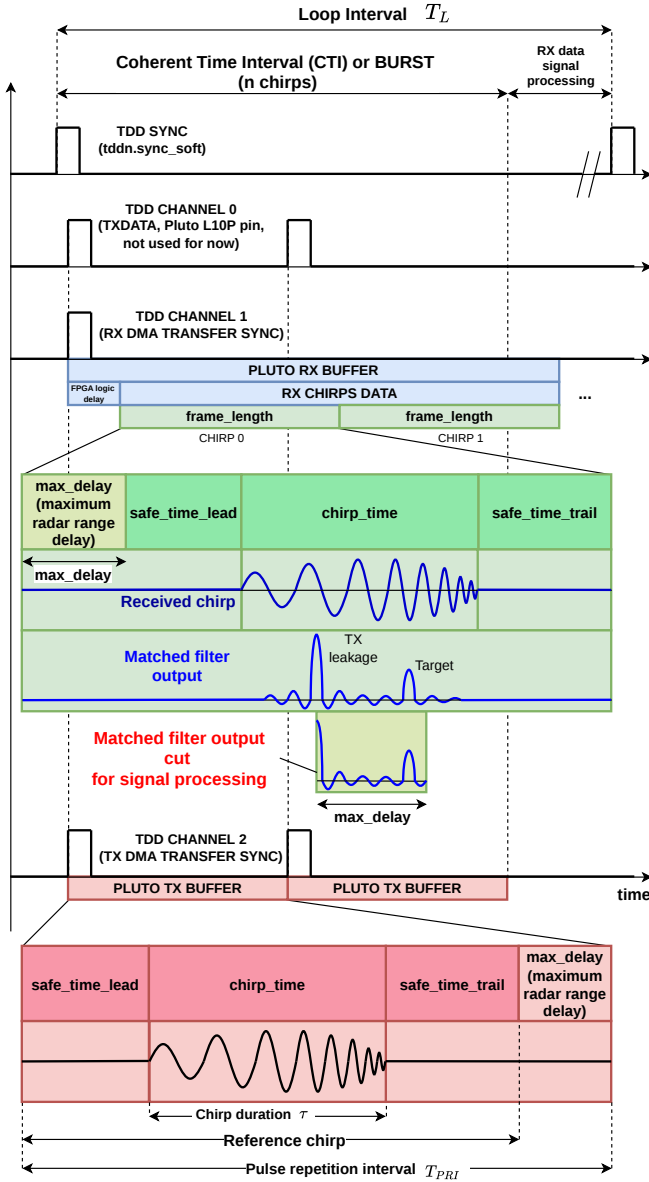


Fig. 5. Time diagram of the coherent time interval and chirp acquisition for the presented radar.

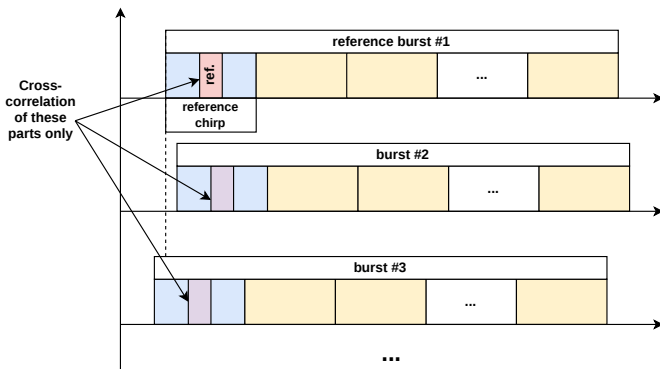


Fig. 6. Time diagram of the CTI-alignment procedure.

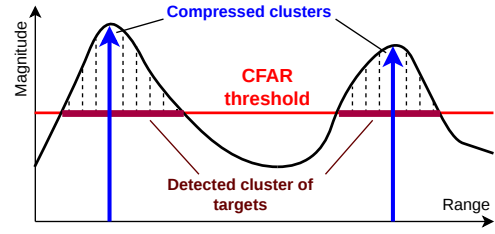


Fig. 7. Target-cluster compression in which adjacent detections are replaced by the detection having the greatest magnitude.

plifiers, mixers, and filters in the receive chain according to internal gain-control tables [14], [15].

It can be observed in Fig. 8 that rx_gain values in the range of 30–40 provide a reasonable compromise between ADC utilization and receiver dynamic range. Based on these results, $rx_gain = 40$ dB is selected for the presented radar.

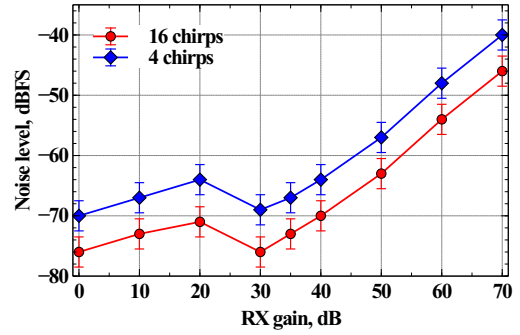


Fig. 8. Measured noise level as a function of the rx_gain parameter. Results for coherent integration of $n = 16$ and $n = 4$ chirps are shown (Section VIII-C).

III. RADAR SW ALGORITHM

The radar software implements the complete signal-processing chain required to transform the complex baseband samples acquired from the receiver into target detections and estimated target coordinates. The processing is performed repeatedly for each coherent time interval (CTI) and operates on data from two receive channels. The main processing stages include signal acquisition, chirp extraction, transmitter-reference compensation, matched filtering, Doppler processing, clutter suppression, CFAR detection, target localization, and reference-object tracking.

The received data are first divided into individual chirps according to the configured radar-frame structure. Transmitter leakage and channel-dependent delays are then compensated before pulse compression is performed. Pulse compression is implemented using a matched filter based on the transmitted reference chirp. This operation improves the range resolution and increases the signal-to-noise ratio of the received echoes.

After pulse compression, a Doppler filter bank is applied across the chirps of the coherent time interval. In the moving-target-indication (MTI) processing path, CFAR processing is

applied independently along the range dimension to every enabled nonzero-Doppler filter. For each Doppler filter and range bin, the software calculates the CFAR detection margin as the difference between the signal magnitude and the corresponding filter-specific CFAR threshold, both expressed in dBFS. If several enabled Doppler filters produce valid detections at the same range bin, the filter having the largest positive CFAR margin is selected. If none of the enabled filters exceeds its threshold, no MTI detection is declared at that range bin. The complex samples from both receive channels are subsequently taken from the same selected Doppler filter, thereby preserving the interchannel magnitude and phase relationships required for monopulse processing. This MTI processing path is used to detect targets with nonzero radial velocity.

A separate clutter-map processing path is used to detect newly appearing, displaced, or slowly varying targets whose responses lie in or near the zero-Doppler channel. In this path, the zero-Doppler response is compared with a clutter map accumulated from previous measurements. Subtracting the clutter map suppresses persistent reflections from static objects and highlights changes in the observed environment, thereby providing MTI functionality for objects with zero or very low radial velocity.

Target detection is performed using a constant false-alarm rate (CFAR) algorithm. The CFAR threshold is calculated independently for the MTI response, the clutter-map residual, and the individual Doppler-filter outputs. The resulting MTI and clutter-map detections can be processed separately or combined. When both processing paths are enabled, the stronger valid detection is selected independently for each range bin.

Adjacent detected range bins may be combined into target clusters to avoid reporting multiple detections originating from the same physical object. The complex samples from the two receive channels are subsequently used to estimate the spatial coordinates of each detected target. The estimated range is obtained from the corresponding range-bin position, while the angular information is derived from the amplitude and phase relationships between the receive channels.

The software also calculates an error signal for optional large-object tracking. When tracking is enabled, this error signal is used to adjust the relative delay of the receive channels and therefore steer or correct the effective antenna response.

Finally, the processed signals, CFAR thresholds, detected targets, clutter-map data, target coordinates, and tracking information are passed to the graphical user interface. If data logging is enabled, each target detection is stored together with its range-bin index, estimated coordinates, and associated Doppler-filter index.

Algorithm 1 summarizes the principal operations performed during one execution of the radar data-generation and target-detection procedure.

The Python implementation of the radar signal-processing chain must be optimized to minimize the loop time T_L shown in Fig. 5. The loop time determines the achievable update rate of the radar system and is influenced by several factors,

Algorithm 1 Radar Data Generation and Target Detection

Require: Number of samples N_s , CFAR parameters, processing mode

Ensure: Display data S , target coordinates (X, Y, Z) , tracking error e , and peak information

1: Measure elapsed time since the previous call

Radar signal processing

2: Acquire radar samples

3: Extract individual chirps (Fig. 5)

4: CTI alignment (Section II-D2)

5: Apply cable-delay compensation to the RX1 and TX1 channels (Section II-D3)

6: Compensate for transmitter leakage in the received signals (Section II-D4)

7: Apply matched filtering to perform pulse compression (Section VI)

8: Apply the Doppler filter bank (Section VIII-C)

MTI processing

9: Apply range CFAR independently to each enabled nonzero-Doppler filter

10: Compress adjacent detections independently within each Doppler-filter output (Section II-D5)

11: At each range bin, select the filter with the largest positive threshold exceedance among the retained cluster peaks

12: Retain the selected detection, Doppler-filter index, and complex RX0 and RX1 data

Clutter-map processing (Section IX)

13: Compute the zero-Doppler response

14: Apply one of three clutter-map subtraction methods

15: Convert the clutter residual to dBFS

16: Apply CFAR to the combined clutter-map response of the two receive channels to obtain detections

Detection fusion

17: **for** each range bin l_s **do**

18: **if** the MTI detection is stronger (in dBFS) **then**

19: Select the MTI detection and its complex channel data

20: **else if** the clutter-map detection is stronger **then**

21: Select the clutter-map detection and its complex channel data

22: **else**

23: Mark the range bin as having no detection

24: **end if**

25: **end for**

26: Optionally compress adjacent fused detections into target clusters (Section II-D5)

Output selection

27: **if** MTI and clutter-map processing are enabled **then**

28: Use the fused detections

29: **else if** only MTI processing is enabled **then**

30: Use the MTI detections

31: **else if** only clutter-map processing is enabled **then**

32: Use the clutter-map detections

33: **else**

34: Use detections from the selected Doppler filter

35: **end if**

36: Update the clutter map for the next processing cycle

Target localization and tracking (Section XI)

37: Estimate the target coordinates (X, Y, Z) using amplitude- and phase-comparison monopulse techniques (Section XI-A)

38: Estimate the tracking error and distance to the large reference object within the specified range gate (Section XI-B)

39: **if** reference-object tracking is enabled **then**

40: Adjust the receive-channel delay using the tracking error

41: **end if**

42: Assemble data for GUI display

43: **if** logging is enabled **then**

44: Log each detected target and its Doppler-filter index

45: **end if**

46: **return** Display data, target coordinates, tracking error, peak value, peak distance, and elapsed time

including hardware communication, signal processing, and data visualization. The principal contributors to the loop time are listed below:

- **PlutoSDR receive-buffer read time.** For the radar configuration used in this work, a coherent time interval (CTI) consists of 16 chirps, each with a duration of 1 ms and a pulse repetition interval of $T_{PRI} = 1.1$ ms. The corresponding CTI duration is approximately 17.6 ms. The corresponding receive buffer contains approximately 5.4×10^5 complex samples per channel. The minimum buffer-transfer time is therefore constrained by the communication interface bandwidth. For a 100 Mb/s Ethernet connection, the transfer time is on the order of 0.4 s, while USB 2.0 reduces this value to approximately 0.1 s under typical operating conditions.
- **Signal-processing time.** The execution time of pulse compression, Doppler filtering, clutter suppression, CFAR detection, target localization, and tracking depends on the computational capabilities of the host computer connected to the Pluto SDR. In general, the processing time increases with the length of the coherent time interval (CTI), since a larger number of chirps and samples must be processed. Among all processing stages, pulse compression, implemented by cross-correlation (matched filtering) between the received and reference chirps, is typically the most computationally intensive operation and therefore represents the dominant contribution to the overall signal-processing time.
- **Data-visualization time.** Rendering the processed radar data on the display also contributes to the loop time. The computational load increases with the number of detected targets, since additional graphical objects must be generated, updated, and stored in memory.

The total loop time for a coherent time interval consisting of 16 chirps was measured to be approximately 0.8 s. The loop time is continuously measured and displayed by the radar application as part of the user interface (Appendix D).

The Python source code of the radar application is publicly available online and can be accessed through the repositories listed in [28], [29].

IV. SIGNAL PATH

The complex signal of the LFM chirp with index m (out of n chirps, $m = 0, 1, 2, \dots, n-1$) is defined as

$$a_{REF}(t) = \begin{cases} 0, & t < -\frac{\tau}{2} + mT_{PRI}, \\ A_0 e^{j\left(\frac{\beta(t-mT_{PRI})^2}{2} + \phi_{ref}\right)}, & -\frac{\tau}{2} \leq (t - mT_{PRI}) \leq \frac{\tau}{2}, \\ 0, & t > \frac{\tau}{2} + mT_{PRI}, \end{cases} \quad (11)$$

where $\beta = \frac{2\pi B}{\tau}$ and ϕ_{ref} is the initial phase of the reference chirp.

The signal in Eq. (11) is mixed with the local oscillator (LO) signal at frequency f_0 (the carrier frequency)

and transmitted after amplification. Assuming that the m -th chirp ($m = 0, 1, 2, \dots, n-1$) occupies the time interval $-\tau/2 + mT_{PRI} \leq t \leq \tau/2 + mT_{PRI}$, the transmitted signal can be expressed as

$$S_{TX}(t) = A_1 e^{j\left(\omega_0 t + \frac{\beta(t-mT_{PRI})^2}{2} + \phi_{ref} + \phi_{LO}\right)}, \quad (12)$$

where ϕ_{LO} is the initial phase of the LO.

After reflection from a moving target located at distance R_0 and moving radially with velocity v , such that $R = R_0 + vt$, the received signal at the radar is given by

$$s_t(t) = A_{RX} e^{j\left(\omega_0 t - 2k(R_0 + vt) + \frac{\beta(t-mT_{PRI})^2}{2}\right)} \times e^{j(\phi_{ref} + \phi_{LO} + \phi_t)}, \quad (13)$$

where ϕ_t is the phase introduced upon reflection from the target.

After mixing with the same LO frequency f_0 , the signal at the ADC of a given channel is

$$s_{ch}(t) = A_{ch} e^{j\left(\frac{\beta(t-mT_{PRI})^2}{2} - 2kvt - 2kR_0 + \phi_{ref} + \phi_t + \phi_{ch}\right)}, \quad (14)$$

which represents the complex envelope of the received RF signal.

We define the Doppler angular frequency as

$$\omega_d = -\frac{2(\omega_0 + \beta t)v}{c} \approx -\frac{4\pi f_0 v}{c}, \quad (15)$$

where the approximation follows from the narrowband condition $\beta t \ll \omega_0$. According to the adopted sign convention, a positive Doppler frequency corresponds to a negative radial velocity, i.e., motion towards the radar, whereas a negative Doppler frequency corresponds to a positive radial velocity, i.e., motion away from the radar.

Using the exact expression for the wavenumber $k(t) = (\omega_0 + \beta t)/c$, the signal can be rewritten as

$$s_{ch}(t) = A_{ch} e^{j\left(\frac{\beta(t-t_d-mT_{PRI})^2}{2} + \omega_d t - \frac{2\omega_0 R_0}{c} + \phi_{ref} + \phi_t + \phi_{ch}\right)}, \quad (16)$$

where $t_d = \frac{2R_0}{c}$ is the round-trip delay, and ϕ_{ch} is the channel-dependent phase related to the spatial position of the target.

Equation (16) is expressed in terms of the absolute time variable t , where each chirp is defined within the interval $-\tau/2 + mT_{PRI} \leq t \leq \tau/2 + mT_{PRI}$, and the time reference corresponds to the beginning of the coherent time interval (CTI). Introducing a local time variable within the m -th chirp, $t' = t - mT_{PRI}$, Eq. (16) can be rewritten as a function of t' and m :

$$s_{ch}(t', m) = A_{ch} e^{j\left(\frac{\beta(t'-t_d)^2}{2} + \omega_d t'\right)} e^{j\omega_d mT_{PRI}} \times e^{j\left(-\frac{2\omega_0 R_0}{c} + \phi_{ref} + \phi_t + \phi_{ch}\right)}. \quad (17)$$

In Eq. (17), the last exponential term is constant for all n chirps within a given CTI and for a given target. The first term can be used to estimate the delay t_d and, consequently, the range

of the target using matched filtering and pulse compression (Section VI). The middle term, $e^{j\omega_d m T_{PRI}}$, can be used to estimate the radial velocity using a Doppler filter bank (Section VIII-C) or, alternatively, via a two-dimensional fast Fourier transform (FFT) applied along the fast-time (within chirps, range bins) and slow-time (across chirps) dimensions [7]. The structure of the received signal is illustrated in Fig. 9.

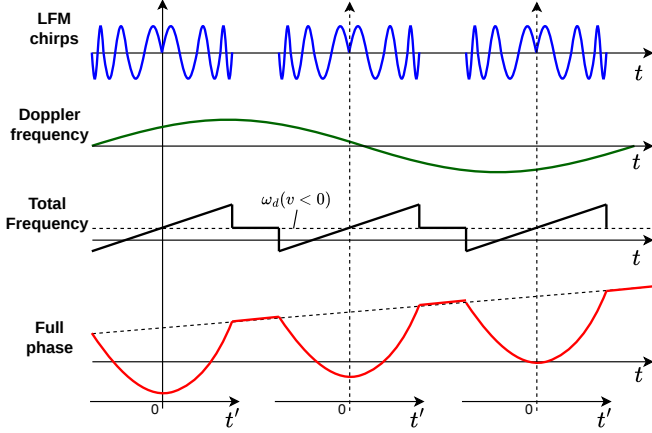


Fig. 9. Structure of the received chirps, showing instantaneous frequency and phase evolution.

V. TRANSMIT LEAKAGE INTO THE RECEIVER AND ANTENNA ISOLATION

The presented radar employs separate antennas for transmission and reception. The radar operates in full-duplex mode, in which the transmitted and received chirps overlap in time. A portion of the transmitted power reaches the receiver not only via reflection from a target but also directly through coupling between the TX and RX antennas, as well as through parasitic coupling in cables and on the printed circuit board (PCB).

This transmit leakage can be significant and may severely degrade receiver performance, potentially leading to damage, saturation, blocking, and loss of sensitivity [4, Chapter 3.2].

To limit transmit leakage, the primary approach is to physically separate the TX and RX antennas [4, Chapter 3.2]. For Prototype #1, the antenna separation is approximately 320 mm, whereas for Prototype #2 it is about 80 mm (Fig. 2). Additionally, RF absorbing material (UVOX CSR-07-25X25) is used to fill the gap between the antennas. Consequently, Prototype #1 achieves approximately 10 dB better TX–RX isolation, as demonstrated in Fig. 10. It can also be observed that the transmit leakage level remains well below ADC saturation, with the signal amplitude reaching approximately ± 200 ADC codes within the full-scale range of $-2048 \dots 2047$.

However, even with a sufficient margin relative to hardware limitations, transmit leakage should be kept as low as possible for the following reasons.

The transmitted signal may contain components that degrade the signal-to-interference ratio. These interference components may arise from several sources, including compressed

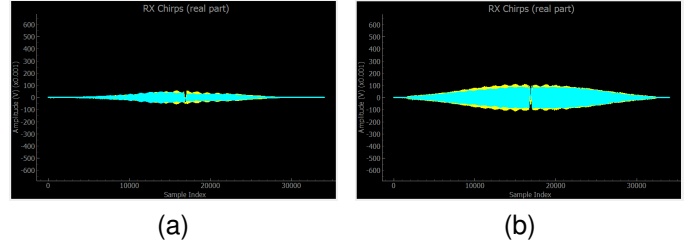


Fig. 10. Measured transmit leakage levels at the PlutoSDR ADC: (a) Prototype #1; (b) Prototype #2. The ADC full-scale range is $-2048 \dots 2047$ (12-bit signed representation). $rx_gain=30$ dB.

pulse sidelobes, local oscillator phase noise, and transmitter spurious components. Such interference can mask weak targets located near strong signals caused by transmit leakage, even at large ranges (long time delays), particularly in the presence of spurious components.

If a spur is modeled as a sinusoidal component superimposed on the chirp, it affects the entire range profile after pulse compression, producing an elevated interference floor that limits the dynamic range and detection capability of weak targets (Section VI-E). The same reasoning applies not only to strong transmit leakage but also to strong target reflections.

The interference floor must be reduced below the thermal noise floor by implementing the following practical solutions:

- The isolation between TX and RX antennas and signal paths must be increased as much as possible, as described above. In general, the interference floor is proportional to the TX power level; therefore, reducing leakage power lowers the interference floor, regardless of the nature of the interfering components (sidelobes or spurious signals).
- Compressed pulse sidelobes should be reduced by applying amplitude weighting (window functions), as described in Section VI-D.
- The level of spurious components in the TX signal must be minimized, which depends on the DAC performance of the PlutoSDR, as discussed in Section VI-E.
- Local oscillator phase noise should be minimized through appropriate selection of phase-locked loop (PLL) parameters, use of a low-noise voltage-controlled oscillator (VCO), and a high-performance reference clock source. The PlutoSDR provides a high-quality local oscillator; therefore, phase noise is not the dominant contributor to the interference floor in this system [18], [31], [32].
- In general, the signal-to-interference ratio of the TX signal should exceed the dynamic range of the receiver and ADC.

VI. PULSE COMPRESSION

A. Matched filter

[It is known that for a received signal containing the reflected chirp $s_{ch}(t)$ and additive white noise, the optimal

filter that maximizes the SNR has the following frequency response [4], [5]:

$$H^{opt}(f) = G_a S_{ch}^*(f), \quad (18)$$

where $*$ denotes complex conjugation and $G_a = 1 \text{ Hz/V}$ is a unity-valued dimensional conversion factor [4, Eq. (10.1)]. Therefore, a replica of the transmitted reference chirp $a_{REF}(t)$ (Eq. (11)) can be used as the basis for the matched filter:

$$H^{opt}(f) = G_a A_{REF}^*(f), \quad (19)$$

$$S_{ch}^{comp}(f) = G_a S_{ch}(f) A_{REF}^*(f), \quad (20)$$

where

$$A_{REF}(f) = \int_{-\infty}^{\infty} a_{REF}(t) e^{-j2\pi ft} dt \quad (21)$$

Thus, the compressed signal at the output of the optimal filter is given by

$$\begin{aligned} s_{ch}^{comp}(t') &= G_a s_{ch}(t) \otimes a_{REF}^*(-t) = \\ G_a \int_{-\infty}^{\infty} s_{ch}(t) a_{REF}^*(t - t') dt &= G_a s_{ch}(t) \star a_{REF}(t), \end{aligned} \quad (22)$$

where $\otimes$ denotes convolution and $\star$ denotes cross-correlation. By definition, cross-correlation of complex signals includes conjugation.

B. Compressed pulse

Considering the relationship between the RX and TX chirps shown in Fig. 11, we substitute the received chirp from Eq. (17) and the reference chirp from Eq. (11) into Eq. (22) similarly as in [5, Chapter 8.6]:

$$s_{ch}^{comp}(t') = \begin{cases} G_a \int_{-\frac{\tau}{2} + t_d}^{\frac{\tau}{2} + t'} s_{ch}(t) a_{REF}^*(t - t') dt, & -\tau + t_d \leq t' < 0, \\ G_a \int_{-\frac{\tau}{2} + t_d}^{\frac{\tau}{2} + t'} s_{ch}(t) a_{REF}^*(t - t') dt, & 0 \leq t' < t_d, \\ G_a \int_{-\frac{\tau}{2} + t'}^{\frac{\tau}{2} + t_d} s_{ch}(t) a_{REF}^*(t - t') dt, & t_d \leq t' < \tau + t_d. \end{cases} \quad (23)$$

Thus, the compressed pulse can be expressed as

$$\begin{aligned} s_{ch}^{comp}(t', m) &= G_a A_{ch} A_0 \\ &\times \frac{\sin\left(\frac{\beta}{2} \left(t' - t_d + \frac{\omega_d}{\beta}\right) (\tau - |t' - t_d|)\right)}{\frac{\beta}{2} \left(t' - t_d + \frac{\omega_d}{\beta}\right)} \\ &\times e^{j\frac{\omega_d(t' + t_d)}{2}} e^{j\omega_d m T_{PRI}} e^{j\left(-\frac{2\omega_0 R_0}{c} + \phi_{ref} + \phi_t + \phi_{ch}\right)}, \end{aligned} \quad (24)$$

for the time interval $-\tau + t_d \leq t' \leq \tau + t_d$, where $m = 0, 1, 2, \dots, n-1$. As before, the local time variable within the m -th chirp is defined as $t' = t - mT_{PRI}$.

Equation (24) demonstrates that the estimated target range is influenced by the Doppler frequency, resulting in range-Doppler coupling [6, Chapter 8].

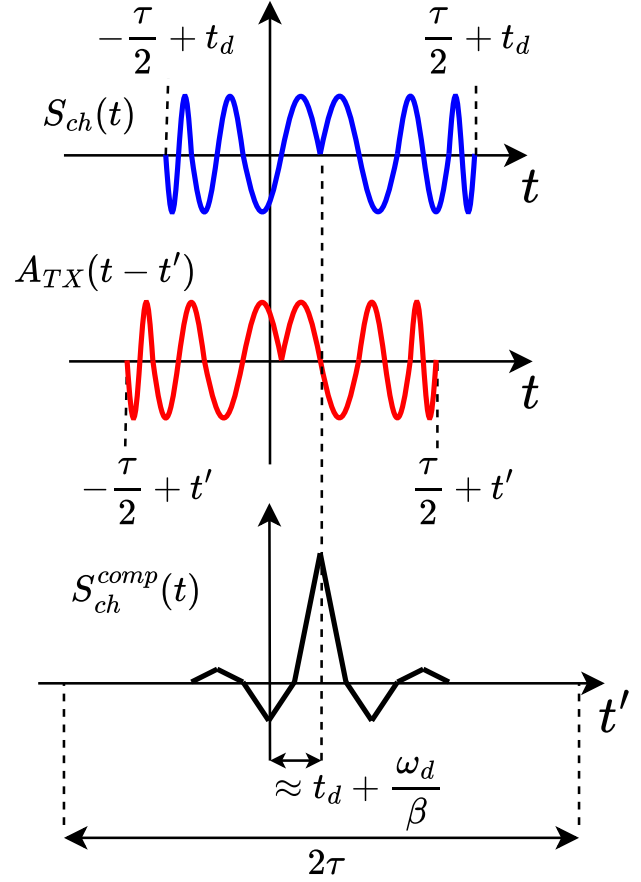


Fig. 11. Pulse compression using cross-correlation.

C. Digital matched filter and normalization

Numerical evaluation of Eq. (22) using the rectangle (Riemann sum) method yields

$$s_{ch}^{comp}(t', m) = G_a \sum_{i=0}^{N_s-1} s_{ch}(i\Delta t_s, m) a_{REF}^*(i\Delta t_s - t') \Delta t_s, \quad (25)$$

where N_s is the number of samples in one chirp and $\Delta t_s = 1/f_s$ is the sampling period.

Equation (25) leads to the digital matched filter implementation:

$$s_{ch}^{comp}(m, l_s) = G_a \Delta t_s \sum_{i=0}^{N_s-1} s_{ch}(m, i) a_{REF}^*(i - l_s), \quad (26)$$

where l_s is the sample index along fast time (within a chirp) and m is the chirp index along slow time (across chirps). The factor Δt_s accounts for the discrete-time implementation of Eqs. (22) and (25). Again, $G_a = 1 \text{ Hz/V}$ is a unity-valued dimensional conversion factor [4, Eq. (10.1)].

For the presented radar, it is desirable that the output signal $s_{ch}^{comp}(m, l_s)$ preserves the correct thermal noise floor relative to the ADC full scale. In this case, the compressed pulse

may exceed the full-scale level, but the noise floor remains properly represented. This allows the receiver noise figure to be compared with theoretical predictions.

This requirement implies that the matched filter should have unity gain for a harmonic signal within its passband, i.e.,

$$|H^{opt}(f)| = |G_a A_{REF}(f)| = 1. \quad (27)$$

To satisfy this condition, the amplitude A_0 of the reference chirp in Eq. (11) must be selected appropriately. Here, $G_a = 1 \text{ Hz/V}$ is a unity-valued dimensional conversion factor introduced to ensure consistency of units [4, Eq. (10.1)].

Using Parseval's theorem, the energy balance between time and frequency domains is given by

$$\int_{-\infty}^{\infty} |A_{REF}(f)|^2 df = \int_{-\infty}^{\infty} |a_{REF}(t)|^2 dt. \quad (28)$$

For the reference chirp in Eq. (11), this becomes

$$\int_{-B/2}^{B/2} |A_{REF}(f)|^2 df = \int_{-\tau/2}^{\tau/2} |a_{REF}(t)|^2 dt. \quad (29)$$

Assuming a large time-bandwidth product ($B\tau \gg 1$), the spectrum of the LFM signal can be approximated as flat over its bandwidth, i.e., $|A_{REF}(f)| = \text{const}$. Therefore,

$$A_{REF}^2(f)B = A_0^2\tau. \quad (30)$$

Enforcing the unity transfer condition $G_a A_{REF}(f) = 1$ within the filter bandwidth yields the required amplitude of the reference chirp:

$$A_0 = \frac{1}{G_a} \sqrt{\frac{B}{\tau}}. \quad (31)$$

Under this normalization, the matched filter response becomes

$$S_{ch}^{comp}(f) = G_a S_{ch}(f) A_{REF}(f) = S_{ch}(f), \quad (32)$$

ensuring that the noise floor is correctly preserved. The same holds for a harmonic signal, whose amplitude is therefore preserved as well.

D. Window functions

The compressed pulse in Eq. (24) is the result of optimal filtering and is based on the reference chirp with uniform amplitude in the time domain (Eq. 11). Such a compressed pulse exhibits a relatively high level of sidelobes [4]–[6]. These sidelobes can mask weaker targets located near the main peak of the compressed pulse.

In the presented radar, identical amplitude weighting is applied to both the transmitted chirp and the matched-filter reference; that is, both signals are multiplied by the same window function in the time domain. The windowed reference chirp is generated once, and identical replicas are subsequently used to form the TX signal and implement the matched filter.

This approach reduces the computational load of real-time signal processing, as no additional processing is required in the receiver path. In particular, it eliminates the need for applying weighting in the frequency domain to the chirp spectrum,

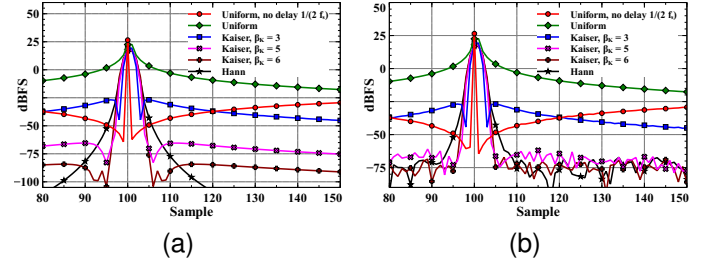


Fig. 12. Simulation of the effect of the implemented window functions on sidelobe reduction and range resolution. (a) RX thermal noise is excluded in the simulation. (b) RX thermal noise is included in the simulation. Parameters are taken from Table II. The Kaiser window is controlled by the shape parameter β_K , which determines the trade-off between sidelobe suppression and main-lobe widening.

which is a more conventional approach in high-power radar systems [4, Section 11.5].

It is well known that amplitude weighting, regardless of whether it is implemented in the time or frequency domain, introduces a trade-off between sidelobe suppression and main lobe widening, thereby affecting the achievable range resolution.

Figure 12 shows a simulation of the effect of different window functions on the compressed pulse.

It should be noted that analysis of Eq. (24) indicates that, under certain conditions such as $f_s = B$, $t_d - \omega_d/\beta = 0$, and $t' \ll \tau$, the zeros of the compressed pulse may coincide exactly with the sampling instants. In this case, sidelobes may not be observed in the discrete-time representation despite being present in the continuous-time signal.

However, this situation is not realistic in practice. In the worst case, when

$$t_d - \frac{\omega_d}{\beta} = \frac{1}{2f_s},$$

the peaks of the sidelobes coincide with the sampling instants, resulting in the maximum observable sidelobe levels.

E. DAC level and spur effects on pulse compression

As shown in Fig. 3, the PlutoSDR uses a digital-to-analog converter (DAC) to generate the TX signal. The DAC has a 12-bit effective resolution, while it is controlled using 16-bit input samples, with the least significant 4 bits truncated [17]. For simplicity, the DAC is here considered as having a 16-bit digital control interface.

Thus, assuming full-scale DAC operation with maximum deviation from the mid-scale value, the DAC scaling factor can be defined as $K_{DAC} = 2^{15}$. In this case, the input control word spans the range $-2^{15} \dots +(2^{15} - 1)$, corresponding to the maximum achievable TX output level.

It was observed that at the maximum level $K_{DAC} = 2^{15}$, the transmitter exhibits a high level of spurious signals, as shown in Fig. 13a. The spur level decreases significantly when the maximum control signal is reduced by factors of 2 and 4 (corresponding to $K_{DAC} = 2^{14}$ and $K_{DAC} = 2^{13}$, respectively), as shown in Figs. 13b and 13c. As can be

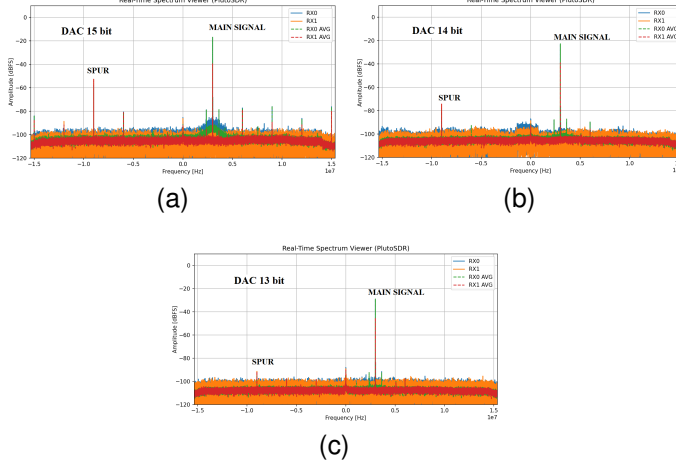


Fig. 13. Measured spur levels as a function of DAC full-scale amplitude: (a) $K_{DAC} = 2^{15}$, (b) $K_{DAC} = 2^{14}$, and (c) $K_{DAC} = 2^{13}$. The transmitted signal is a continuous sinusoid, without chirp modulation or pulse compression.

observed, the main signal scales correctly with the DAC control level, while the relative level of spurious components is reduced significantly.

Spurious components are transmitted along with the main signal, whereas the matched filter in the receiver uses a reference chirp that is a clean, digitally generated replica without any spurs. As a result, at the output of the matched filter, the compressed pulse exhibits a reduced signal-to-spur ratio. The simulated effect of spurs on the compressed pulse is illustrated in Fig. 14. Thus, when the RX signal level is high, for example due to transmit leakage, the spurious components appear as additional noise significantly above the thermal noise floor. This results in masking of weaker targets, even at large range separations from the strong signal.

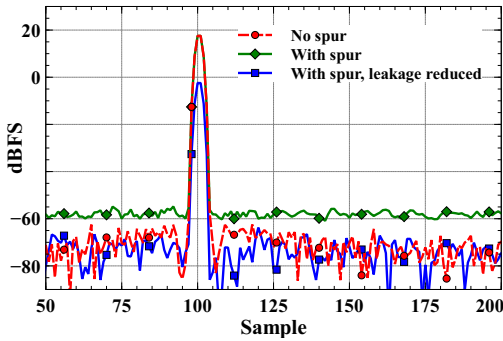


Fig. 14. Simulation of the effect of spur noise in the received signal on pulse compression. The spur is modeled as a sinusoidal component with an amplitude 40 dB lower than the main signal. Receiver thermal noise is included in the simulation. Parameters are taken from Table II. The Kaiser window is applied with a shape parameter $\beta_K = 5$.

As a practical solution for the presented radar, in addition to improving TX–RX antenna isolation, the DAC level is reduced. Specifically, the DAC level is set to $K_{DAC} = 2^{14}$ for Prototype #1 and $K_{DAC} = 2^{13}$ for Prototype #2 (Fig. 2).

A higher DAC level can be used for Prototype #1 due to the better isolation between its TX and RX antennas. The PlutoSDR transmitter attenuation settings are configured for maximum output power, corresponding to $tx_gain0=0$ and $tx_gain1=0$.

To compensate for the reduction in DAC level, and thus in TX power, additional RF power amplifiers are employed.

VII. RADAR RANGE EQUATION

A. Based on SNR Requirements

The radar range equation can be derived from the signal-to-noise ratio (SNR) required for target detection. For specified probabilities of detection p_d and false alarm p_f , the required SNR determines the maximum achievable detection range [4].

The radar range equation for a radar that employs neither pulse compression nor pulse integration can be written as follows [4], [33]:

$$R_{max}^4 = \frac{P_t G_{TX} G_{RX} \lambda_0^2 \sigma}{(4\pi)^3 k T_s B_n (S/N)_1^\Sigma L_t}, \quad (33)$$

where P_t is the total transmitted power; G_{TX} and G_{RX} are the gains of the TX and RX antennas (sum patterns), respectively; λ_0 is the wavelength; σ is the radar cross section (RCS) of the target; $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant; B_n is the noise-equivalent bandwidth of the receiver; $T_s = T_a + T_e$ is the system noise temperature; T_a is the antenna noise temperature; and $T_e = T_0(F_n - 1)$ is the effective receiver noise temperature, where F_n is the receiver noise figure and $T_0 = 290$ K is the reference temperature. The system noise temperature is calculated assuming negligible losses in the transmission line between the antenna and the receiver [4], [33], [34].

The parameter $(S/N)_1^\Sigma$ denotes the signal-to-noise ratio (SNR) in the sum channel (RX0+RX1) required for target detection using a single chirp. In the following analysis, $(S/N)_1^\Sigma$ may also be interpreted as the SNR at the output of the complete signal-processing chain. Such an SNR is required to achieve a specified probability of detection p_d for a given probability of false alarm p_f .

The parameter L_t is the total loss factor, incorporating all relevant loss mechanisms in the radar signal path, including transmitter, receiver, and propagation losses [4], [33], [34].

When pulse compression is employed, Eqs. (31) and (24) show that the peak signal power increases by a factor of $B\tau$ after compression. As discussed in Section VI-B, the matched filter does not change the noise floor due to the adopted normalization. Furthermore, if n chirps are coherently integrated (Section VIII-C), the required SNR can be reduced by the pulse-compression gain and coherent integration gain. Consequently, the radar range equation becomes

$$R_{max}^4 = \frac{P_t G_{TX} G_{RX} \lambda_0^2 \sigma}{(4\pi)^3 k T_s B_n ((S/N)_1^\Sigma \frac{1}{n} \frac{1}{B\tau}) L_t}, \quad (34)$$

or, equivalently,

$$R_{max}^4 = \frac{P_t G_{TX} G_{RX} \lambda_0^2 \sigma B \tau n}{(4\pi)^3 k T_s B_n (S/N)_1^\Sigma L_t}. \quad (35)$$

Assuming that the receiver noise-equivalent bandwidth is matched to the chirp bandwidth, i.e., $B_n = B$, Eq. (35) reduces to

$$R_{max}^4 = \frac{P_t \tau G_{TX} G_{RX} \lambda_0^2 \sigma n}{(4\pi)^3 k T_s (S/N)_1^\Sigma L_t}. \quad (36)$$

Equation (36) shows that, for a fixed target RCS and required detection performance, the maximum detection range increases with transmitted power, antenna gain, chirp duration, and the number of coherently integrated chirps.

As can be seen from Eq. (36), under the assumption that the receiver noise-equivalent bandwidth is matched to the chirp bandwidth, $B_n = B$, the maximum radar range does not depend directly on the chirp bandwidth (frequency sweep range) B . Therefore, the chirp bandwidth can be reduced to permit a lower sampling rate, since $f_s = B$ in the current implementation (Table II). Reducing the sampling rate decreases the amount of data to be processed and may therefore shorten the processing time and loop interval T_L . However, reducing the chirp and receiver bandwidths degrades the range resolution, which is given approximately by

$$\Delta R = \frac{c}{2B}. \quad (37)$$

Consequently, targets with a range separation smaller than ΔR cannot be resolved.

The product τn in Eq. (36) represents the total coherent illumination time of the n integrated chirps and is proportional to the coherent time interval T_{CTI} shown in Fig. 5. More precisely, $T_{CTI} \approx n T_{PRI}$, while τn is the portion of this interval occupied by the transmitted chirps. Thus, the maximum radar range depends on the total duration over which the received signal is coherently integrated. In principle, the same coherent integration gain, and therefore the same theoretical maximum range, can be obtained using fewer chirps of longer duration, provided that the product τn remains constant and coherence is preserved. However, the number of chirps n also determines the number of Doppler-filter channels and affects the Doppler-frequency resolution, as described in Section VIII-C. Therefore, reducing n while increasing τ may preserve the range-processing gain but degrade Doppler resolution and radial-velocity discrimination.

The parameters of the presented radar used in the radar range equations are listed in Table III. The total system loss factor L_t is estimated from the individual loss mechanisms summarized in Table IV. The dominant contribution arises from transmitter spurs and phase noise, which elevate the interference floor and consequently reduce the achievable detection range. The antenna gains and beamwidths were determined from analytical directivity calculations combined with antenna efficiency simulations performed in COMSOL Multiphysics®, as described in Appendix A. The mismatch losses of the RX and TX antennas are evaluated in Appendix B.

Figure 15 illustrates the estimated maximum detection range of the presented radar prototypes as a function of the target radar cross section (RCS) σ . The required SNR, $(S/N)_1^\Sigma$, is assumed to be constant.

TABLE III
PARAMETERS USED IN THE RADAR RANGE EQUATIONS

Parameter	Value
Wavelength, λ_0	12.4 cm
Antenna noise temperature, T_a [4]	200 K
Effective receiver noise temperature, T_e [16]	225 K
System noise temperature, T_s	425 K
Chirp duration, τ	1 ms
Number of coherently integrated chirps, n	16
TX power, P_t (Prototype #1)	26 dBm
TX power, P_t (Prototype #2)	20 dBm
TX antenna gain (sum pattern), G_{TX}	13.2 dBi
TX sum-pattern beamwidth, E-plane, $\theta_{0.5P}^{E,\Sigma,TX}$	56°
TX sum-pattern beamwidth, H-plane, $\theta_{0.5P}^{H,\Sigma,TX}$	26°
RX antenna gain (sum pattern), G_{RX}	11.9 dBi
RX sum-pattern beamwidth, E-plane, $\theta_{0.5P}^{E,\Sigma,RX}$	66°
RX sum-pattern beamwidth, H-plane, $\theta_{0.5P}^{H,\Sigma,RX}$	26°
Target RCS, σ	1 m ²
Required $(S/N)_1^\Sigma$ ($p_d = 0.5$, $p_f = 1.9 \times 10^{-10}$) [4, Fig. 2.7]	13.4 dB
Total system loss factor, L_t	20 dB

TABLE IV
SYSTEM LOSS FACTOR BUDGET

Component	Loss factor
Transmitter spurs and phase noise	10 dB
RX cable and mismatch losses	2 dB
TX cable and mismatch losses	4 dB
Beam-shaping loss	4 dB
Total loss factor, L_t	20 dB

B. Based on Cross-Range Accuracy

The down-range (radial) tracking accuracy (RMS error) caused by thermal noise for a radar employing pulse compression with an LFM chirp can be estimated as follows [4, Eq. 11.26] [34], [35]:

$$\sigma_{dr} = \frac{c}{2B} \cdot \frac{\sqrt{3}}{\sqrt{2\pi} \sqrt{(S/N)_1^\Sigma}}. \quad (38)$$

The angular tracking accuracy (RMS error) caused by thermal noise for a monopulse radar can be estimated as follows [4] [35, Eq. 8.29] [34]:

$$\sigma_\theta = \frac{\theta_{0.5P}^{E,\Sigma,RX}}{2\sqrt{(S/N)_1^\Sigma}}, \quad (39)$$

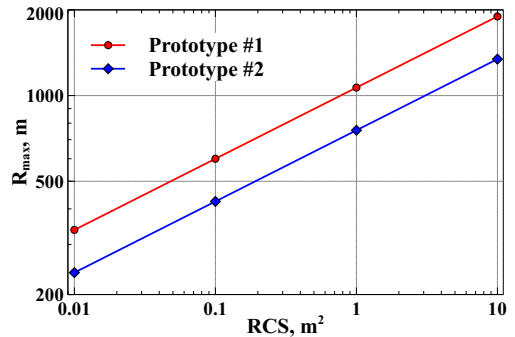


Fig. 15. Estimated maximum radar range R_{max} as a function of the target radar cross section σ .

$$\sigma_\varphi = \frac{\theta_{0.5P}^{H,\Sigma,RX}}{2\sqrt{(S/N)_1^\Sigma}}, \quad (40)$$

where $\theta_{0.5P}^{E,\Sigma,RX}$ is the RX antenna sum-pattern beamwidth in the E-plane, and $\theta_{0.5P}^{H,\Sigma,RX}$ is the RX antenna sum-pattern beamwidth in the H-plane (Appendix A).

The corresponding cross-range (transverse) tracking accuracies in the vertical and horizontal directions at a radial distance R are given by

$$\sigma_{crv} = R \frac{\theta_{0.5P}^{E,\Sigma,RX}}{2\sqrt{(S/N)_1^\Sigma}}, \quad (41)$$

$$\sigma_{crh} = R \frac{\theta_{0.5P}^{H,\Sigma,RX}}{2\sqrt{(S/N)_1^\Sigma}}. \quad (42)$$

Comparing Eq. (38) with Eqs. (41)–(42), it can be seen that the cross-range tracking error is typically much larger than the down-range tracking error. Therefore, the cross-range error can be used as the worst-case criterion when determining SNR requirements or estimating the maximum radar range for a given cross-range accuracy requirement.

Furthermore, for the presented radar, the angular accuracy in the E-plane is worse than in the H-plane due to the wider beamwidth in the E-plane, as can be seen from Eqs. (39) and (40) and Appendix A. Therefore, the worst-case cross-range tracking accuracy is determined by the E-plane beamwidth of the RX antenna and can be expressed as

$$\sigma_{cr} = R \frac{\theta_{0.5P}^{E,\Sigma,RX}}{2\sqrt{(S/N)_1^\Sigma}}. \quad (43)$$

The antenna gains can be estimated from their beamwidths (in degrees) as follows [36]:

$$G_{RX} \approx \frac{26000}{\theta_{0.5P}^{E,\Sigma,RX} \theta_{0.5P}^{H,\Sigma,RX}}, \quad (44)$$

$$G_{TX} \approx \frac{26000}{\theta_{0.5P}^{E,\Sigma,TX} \theta_{0.5P}^{H,\Sigma,TX}}. \quad (45)$$

Substituting G_{RX} and G_{TX} from Eqs. (44)–(45) and $(S/N)_1^\Sigma$ from Eq. (43) into Eq. (36), the following radar range equation is obtained:

$$R_{max}^6 = \frac{4.5 \cdot 10^9 P_t \tau \lambda_0^2 \sigma n \sigma_{cr}^2}{\left(\theta_{0.5P}^{E,\Sigma,RX}\right)^3 \theta_{0.5P}^{H,\Sigma,RX} \theta_{0.5P}^{E,\Sigma,TX} \theta_{0.5P}^{H,\Sigma,TX} k T_s L_t}. \quad (46)$$

Figure 16 illustrates the estimated maximum detection range of the presented radar prototypes as a function of the required cross-range tracking accuracy for a reference target with an RCS of $\sigma = 1 \text{ m}^2$.

It should be noted that Eqs. (36) and (46) do not account for propagation-factor effects arising from ground and sea-surface reflections. Consequently, the predicted detection ranges should be interpreted as free-space estimates. The influence of radar height, ground reflections, and Fresnel-zone clearance on propagation conditions is discussed in Section XII-F.

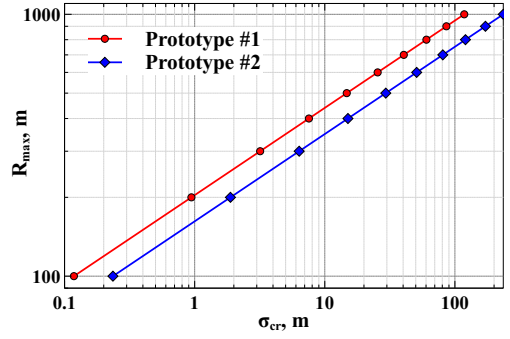


Fig. 16. Estimated maximum detection range R_{max} of the presented radar prototypes as a function of the required cross-range tracking accuracy σ_{cr} for a target with an RCS of $\sigma = 1 \text{ m}^2$.

VIII. MOVING TARGET INDICATION AND CHIRP INTEGRATION

A. Delay-line canceler

The first method used to implement moving target indication (MTI) is the single delay-line canceler (DLC) [4], [6]. As shown in Fig. 17, the DLC operates on two chirps within a CTI and subtracts the first chirp ($m = 0$) from the second chirp ($m = 1$). The subtraction is performed on a sample-by-sample basis for all samples of the chirp data. This operation eliminates the DC component (zero Doppler frequency), which is responsible for the response from stationary clutter. The

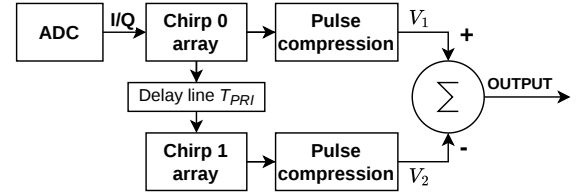


Fig. 17. MTI using delay line canceler.

voltages V_1 and V_2 corresponding to the chirps with indices $m = 0$ and $m = 1$, respectively, can be expressed using Eq. (24) as

$$V_1(t') = K(t'), \quad (47)$$

$$V_2(t') = K(t')e^{j\omega_d T_{PRI}}, \quad (48)$$

where $K(t')$ is the same for both chirps:

$$K(t') = G_a A_{ch} A_0 \times \frac{\sin\left(\frac{\beta}{2} \left(t' - t_d + \frac{\omega_d}{\beta}\right) (\tau - |t' - t_d|)\right)}{\frac{\beta}{2} \left(t' - t_d + \frac{\omega_d}{\beta}\right)} \times e^{j\frac{\omega_d(t'+t_d)}{2}} e^{j\left(-\frac{2\omega_0 R_0}{c} + \phi_{ref} + \phi_t + \phi_{ch}\right)}. \quad (49)$$

As before, the local time variable within the m -th chirp is defined as $t' = t - mT_{PRI}$ (Fig. 9).

The output of the DLC is therefore

$$\begin{aligned} V_2(t') - V_1(t') &= K(t')(e^{j\omega_d T_{PRI}} - 1) \\ &= K(t')2j \sin \frac{\omega_d T_{PRI}}{2} e^{j\frac{\omega_d T_{PRI}}{2}}. \end{aligned} \quad (50)$$

Thus, the magnitude response of the DLC is

$$|H_{DLC}(\omega_d)| = 2 \left| \sin \left(\frac{\omega_d T_{PRI}}{2} \right) \right|. \quad (51)$$

The frequency response of the single DLC is shown in Fig. 18. As can be seen, the zero-Doppler component is effectively attenuated, allowing stationary clutter to be suppressed before further signal processing.

The z -transform representation of a single DLC, treated as a digital filter, is given by

$$H_{DLC}(z) = 1 - z^{-1}. \quad (52)$$

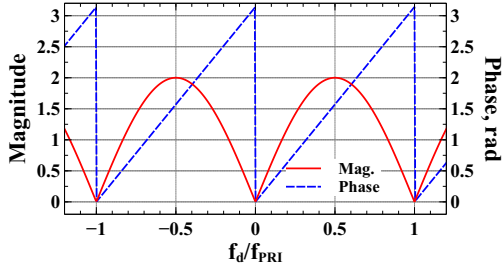


Fig. 18. Response of the delay-line canceler. Here, $f_d = \omega_d/(2\pi)$ is the Doppler frequency and $f_{PRI} = 1/T_{PRI}$ is the pulse repetition frequency.

As can be seen from Fig. 18, the effects of sampling and aliasing are clearly visible. The frequency response is periodic with a period equal to f_{PRI} . Each sample within a chirp (fast time) is processed across successive chirps (slow time) with an effective sampling frequency of $1/T_{PRI} = f_{PRI}$. Consequently, Doppler frequency estimation is subject to aliasing.

For example, the magnitude response and phase response at a Doppler frequency of $f_d = 0.7f_{PRI}$ are identical to those at $f_d = -0.3f_{PRI}$. As a result, target motion cannot be unambiguously determined when the Doppler frequency exceeds the Nyquist limit

$$|f_d| = \frac{f_{PRI}}{2}.$$

Therefore, both the radial velocity and the direction of motion become ambiguous for targets whose Doppler frequencies satisfy $|f_d| > f_{PRI}/2$.

B. FIR filter

The DLC MTI filter is an example of a finite impulse response (FIR) filter [4], [6]. The DLC described in Section VIII-A is a two-tap FIR filter, or equivalently a first-order FIR filter, since it uses one delay line. The following properties of FIR filters are relevant to MTI processing:

- An N th-order FIR filter uses $N + 1$ taps (chirps) and N delay lines.

- FIR filters are non-recursive, i.e., they do not contain feedback paths and use only feed-forward connections. As a result, they exhibit good transient response and unconditional stability [4], [6].
- The impulse response (i.e., the output produced by a Kronecker delta input) of an N th-order discrete-time FIR filter contains at most $N + 1$ nonzero samples.
- Processing $N + 1$ chirps, an N th-order FIR filter produces one filtered chirp at its output.
- FIR MTI filters can effectively attenuate clutter at or near zero Doppler frequency. However, due to their relatively broad passband, they provide only limited suppression of thermal noise. Consequently, the integration efficiency of DLC-based filters is relatively low and does not increase significantly with filter order.
- In general, FIR filters of fourth or fifth order provide sufficient clutter suppression. Increasing the filter order beyond this point offers only marginal improvement, as the noise bandwidth changes little while the filter response time and processing delay increase [4], [6], [8].
- Blind speeds occur at Doppler frequencies

$$f_d = \frac{i}{T_{PRI}}, \quad (53)$$

where $i = \pm 1, \pm 2, \dots$

The block diagram of a general FIR filter is shown in Fig. 19.

The transfer function of a general N th-order FIR filter ($N + 1$ taps) in the z -domain is given by

$$\begin{aligned} H_{FIR}(z) &= \\ a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N} &= \sum_{i=0}^N a_i z^{-i}, \end{aligned} \quad (54)$$

where N is the filter order and a_i are the FIR tap coefficients. It can be seen that a single DLC described above can be represented as a first-order FIR filter with coefficients

$$a_0 = 1, \quad a_1 = -1. \quad (55)$$

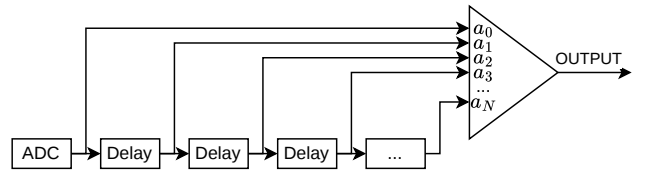


Fig. 19. Block diagram of an N th-order FIR filter implemented using delay lines.

An example of a higher-order FIR filter frequency response is shown in Fig. 20. Compared to a single DLC, the wider rejection bandwidth around zero Doppler frequency provides improved clutter attenuation. In general, there is a trade-off between clutter suppression and the usable Doppler passband. Increasing the filter order broadens the clutter-rejection bandwidth and improves attenuation of low-Doppler clutter, but

reduces sensitivity to slowly moving targets and increases the filter response time.

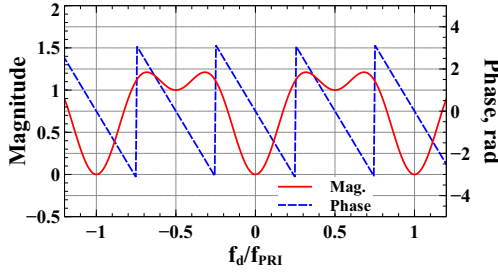


Fig. 20. Magnitude response of a 4th-order FIR filter (5 taps). Compared to a single delay-line canceler, the wider rejection bandwidth around zero Doppler frequency provides improved clutter attenuation. The filter coefficients are $a_0 = -0.1523$, $a_1 = -0.25$, $a_2 = 0.8046$, $a_3 = -0.25$, and $a_4 = -0.1523$. Here, $f_d = \omega_d/(2\pi)$ denotes the Doppler frequency and $f_{PRI} = 1/T_{PRI}$ denotes the pulse repetition frequency.

C. Doppler filter bank

Although DLC and higher-order FIR MTI filters effectively suppress stationary clutter, their relatively wide passbands provide limited reduction of thermal noise. A more efficient approach is to divide the Doppler spectrum into multiple narrow frequency bands using a bank of filters. In this case, each filter processes only a small portion of the Doppler spectrum, resulting in a reduced noise bandwidth and improved signal-to-noise ratio.

When n chirps are available within a coherent time interval (CTI), a bank of n FIR band-pass filters (BPF) can be implemented to cover the entire Doppler frequency range. This structure is commonly referred to as a Doppler filter bank and provides coherent integration of the received signal over n chirps [4, Chapter 4.5], [8, Chapter 3.2].

Since each band-pass filter has a bandwidth approximately n times narrower than that of the DLC filter, effective coherent integration of n chirps is achieved. This coherent integration can significantly improve the signal-to-noise ratio $(S/N)_1^\Sigma$ and increase the maximum radar detection range R_{max} , as discussed in Section VII. The effect of the number of coherently integrated chirps on the resulting noise floor is illustrated in Fig. 8.

Each band-pass filter in the bank is implemented as an FIR filter, and its output is produced after processing all n chirps. Consequently, n filter outputs are generated for each CTI. To obtain the response of the k th band-pass filter, the following procedure is performed [4, Chapter 4.5], [8, Chapter 3.2]:

- 1) Start with the FIR filter structure shown in Fig. 19 and described by Eq. (54).
- 2) Replace the real filter coefficients with complex coefficients according to

$$a_{i,k} = e^{-j\frac{2\pi(i-1)k}{n}}, \quad (56)$$

where $i = 1, 2, \dots, n$ is the tap index and $k = 0, 1, \dots, n-1$ is the index of one of the n band-pass filters.

Using Eqs. (56) and (54), the frequency response of the filter with index k out of a bank of n filters can be obtained as

$$H_k(f_d) = \left| \frac{\sin[\pi n (f_d T_{PRI} - \frac{k}{n})]}{\sin[\pi (f_d T_{PRI} - \frac{k}{n})]} \right|. \quad (57)$$

The resulting filter bank partitions the Doppler spectrum into n frequency bins, each centered at

$$f_k = \left(p + \frac{k}{n}\right) \frac{1}{T_{PRI}}, \quad (58)$$

where p is an integer. The bandwidth of each band-pass filter is approximately

$$\Delta f \approx \frac{0.9}{n T_{PRI}}. \quad (59)$$

The magnitude responses of the band-pass filters are shown in Fig. 21. It should be noted that the responses are not normalized; therefore, the peak magnitude of each filter response is equal to the number of integrated chirps (taps), n .

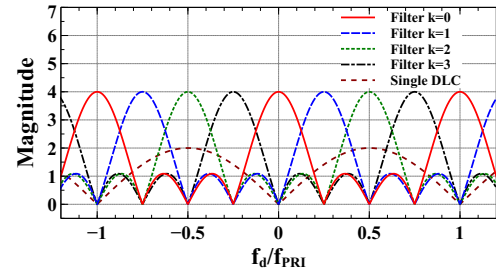


Fig. 21. Magnitude responses of a 4-chirp filter bank ($n = 4$) (Eq. (57)). Compared to a single delay-line canceler (Eqs. (50)–(52)), the filter bank provides multiple Doppler channels and coherent integration gain. The responses are not normalized, and the peak magnitude of each channel is equal to $n = 4$. Here, $f_d = \omega_d/(2\pi)$ denotes the Doppler frequency and $f_{PRI} = 1/T_{PRI}$ denotes the pulse repetition frequency.

As can be seen from Fig. 21, the effects of sampling and aliasing are clearly visible. Similar to the delay-line canceler, the magnitude response of each filter in the bank is periodic with a period of $1/T_{PRI} = f_{PRI}$. This periodicity introduces ambiguity in radial velocity measurements.

Another ambiguity arises when the target Doppler frequency exceeds the unambiguous Doppler interval. In such cases, the measured Doppler frequency is aliased into the response of another filter channel and may even fall within the clutter-response region. In addition, blind speeds occur at Doppler frequencies

$$f_d = \frac{i}{T_{PRI}}, \quad (60)$$

where $i = \pm 1, \pm 2, \dots$, if the $k = 0$ filter is disabled for clutter suppression.

These ambiguities can be resolved using techniques such as periodic variation of the pulse repetition interval T_{PRI} or changing the RF carrier frequency f_0 , allowing the Doppler frequency to be determined unambiguously [8, Chapter 3.6].

Thresholding and CFAR processing can be applied independently to each Doppler filter channel, depending on the

application. However, in the presented radar implementation, the same CFAR parameters are used for all filters with $k \neq 0$. For the clutter-map filter ($k = 0$), the CFAR bias can be configured separately, as described in Section X and Appendix D. The filter with index $k = 0$ is used to generate and display the clutter map (Section IX), while the remaining filters with indices $k = 1, 2, \dots, n - 1$ are used for MTI processing.

As can be seen from Fig. 21, each Doppler filter exhibits relatively high sidelobe levels, which may cause a target to be detected in neighboring filter channels or produce responses from clutter. These sidelobes can be reduced by applying amplitude weighting (a window function) across the chirps in the CTI [4, Chapter 4.5]. However, such weighting widens the main lobe of each filter response. Consequently, stationary or low-Doppler clutter may appear not only in the zero-Doppler filter ($k = 0$) but also in the adjacent filters ($k = 1$ and $k = n - 1$, corresponding to $k = 15$ for $n = 16$), potentially producing false MTI detections. Therefore, Doppler-domain weighting is disabled by default in the current radar application but can be enabled as a hard-coded option.

The radar software also implements an individual masking mechanism that allows each Doppler filter channel to be enabled or disabled independently. This feature provides additional flexibility for suppressing specific clutter patterns or restricting detection to selected ranges of radial velocity.

In the digital implementation, the output of the k th Doppler filter, whose coefficients are given by Eq. (56), can be expressed as

$$s^{BPF}(k, l_s) = \sum_{i=1}^n s_{ch}^{comp}(i-1, l_s) e^{-j\left(\frac{2\pi(i-1)k}{n}\right)}, \quad (61)$$

where $s_{ch}^{comp}(i-1, l_s)$ is the complex sample obtained after pulse compression (Eq. (26)) at the fast-time index (range bin) $l_s = 0, \dots, N_s - 1$ from the $(i-1)$ th received chirp of a given RX channel. The index $i = 1, 2, \dots, n$ corresponds to the chirps within the coherent time interval (CTI).

The index $(i-1)$ corresponds directly to the chirp index m in Eq. (24), i.e., $(i-1) = m$. As a result, the factor $e^{j\omega_d m T_{PRI}}$ becomes the dominant component processed by the bank of band-pass filters. This component represents a complex analytic signal sampled at the rate $1/T_{PRI}$ and contains the Doppler information of the target. Consequently, the filter bank is capable of distinguishing both positive and negative Doppler frequencies, depending on the direction of the target's radial motion.

Equation (61) can be compared with the general expression for the discrete Fourier transform (DFT) of a sampled signal x [5]:

$$X_k = \sum_{i=0}^{n-1} x_i e^{-j\left(\frac{2\pi i k}{n}\right)}. \quad (62)$$

It can be seen that Eqs. (61) and (62) are equivalent. Therefore, the Doppler filter bank can be implemented directly using a

DFT. Furthermore, when the number of integrated chirps is selected as

$$n = 2^p, \quad p \in \mathbb{N}, \quad (63)$$

the filter-bank outputs can be calculated efficiently using the fast Fourier transform (FFT).

It should also be noted that, owing to the signal definition in Section IV and the features of the PlutoSDR receive path shown in Fig. 3, a negative Doppler frequency corresponds to an object moving away from the radar along the radial direction, whereas a positive Doppler frequency corresponds to an object moving toward the radar.

As can be seen from Eqs. (61) and (62) and Fig. 21, the Doppler-filter response is periodic with a period of $1/T_{PRI}$. Similar to DFT-based spectral analysis, provided that the Doppler frequency satisfies the Nyquist criterion, negative Doppler frequencies are represented by filter indices $k = n/2, \dots, n-1$ rather than by negative indices $k = -n/2, \dots, -1$, since the filter index k is defined only for nonnegative integer values. Optionally, the frequency bins can be rearranged using an FFT-shift operation to represent the Doppler interval from $-1/(2T_{PRI})$ to $1/(2T_{PRI})$ in the conventional order.

For the presented radar, with $T_{PRI} = 1.1$ ms and $n = 16$ chirps per CTI (Table II), each Doppler-filter index k corresponds to the center Doppler frequency and radial velocity listed in Table V.

TABLE V
CENTER DOPPLER FREQUENCIES AND CORRESPONDING RADIAL VELOCITIES FOR THE $n = 16$ DOPPLER-FILTER BANK.

Filter index, k	f_d (Hz)	v_d (m/s)	v_d (km/h)
0	0.00	0.00	0.00
1	56.82	-3.51	-12.65
2	113.64	-7.03	-25.30
3	170.45	-10.54	-37.96
4	227.27	-14.06	-50.61
5	284.09	-17.57	-63.26
6	340.91	-21.09	-75.91
7	397.73	-24.60	-88.57
8	454.55	-28.12	-101.22
9	-397.73	24.60	88.57
10	-340.91	21.09	75.91
11	-284.09	17.57	63.26
12	-227.27	14.06	50.61
13	-170.45	10.54	37.96
14	-113.64	7.03	25.30
15	-56.82	3.51	12.65

The Doppler-frequency spacing between adjacent filters is

$$\Delta f_d = \frac{1}{nT_{PRI}} = 56.82 \text{ Hz}. \quad (64)$$

The corresponding radial-velocity spacing is

$$|\Delta v_d| = \frac{\lambda_0 \Delta f_d}{2} = 3.51 \text{ m/s} = 12.65 \text{ km/h}. \quad (65)$$

Because the filter indices are defined over the interval $k = 0, \dots, n-1$, negative Doppler frequencies are represented

by the upper half of the filter-bank indices. For example, $k = 15$ represents the negative-frequency channel equivalent to $k = -1$. The channel with index $k = 8$ corresponds to the Nyquist-frequency bin. Depending on the adopted DFT convention, this bin can equivalently be represented by either $f_d = +454.55$ Hz or $f_d = -454.55$ Hz. These frequencies represent the same discrete-time frequency bin.

As can also be seen, the bandwidth of each filter in the bank, given by Eq. (59), and the Doppler-frequency spacing, given by Eq. (64), are inversely proportional to the coherent time interval $T_{CTI} = nT_{PRI}$. Consequently, increasing the coherent time interval reduces the filter bandwidth and improves the Doppler-frequency resolution.

IX. CLUTTER MAP FILTER (BACKGROUND SUBTRACTION)

A clutter map can be used to complement conventional Doppler moving-target indication by detecting newly appearing, displaced, or slowly varying targets whose responses occur in or near the zero-Doppler channel and may therefore not be detected by the nonzero-Doppler filter channels [4], [23]. In this approach, the clutter map is formed from the output of the zero-Doppler channel ($k = 0$). A low-pass filter is then applied across successive CTIs to estimate the persistent background response, which can subsequently be compared with the current radar response [23, Eq. (1)]:

$$s_i^{CM}(l_s) = \alpha_{cm} s_i^{BPF}(0, l_s) + (1 - \alpha_{cm}) s_{i-1}^{CM}(l_s), \quad (66)$$

where $l_s = 0, \dots, N_s - 1$ is the fast-time index (range bin, sample number within a chirp); $s_i^{CM}(l_s)$ is the updated clutter-map array after processing the CTI with index i ; $s_{i-1}^{CM}(l_s)$ is the clutter-map array obtained after processing the previous CTI with index $(i - 1)$; $s^{BPF,i}(0, l_s)$ is the output of the zero-Doppler filter ($k = 0$) for the CTI with index i , obtained from the sum channel formed by combining RX0 and RX1; and α_{cm} is the clutter-map update coefficient. The parameter α_{cm} determines the adaptation rate of the clutter map. Small values of α_{cm} result in slow adaptation and better suppression of temporary moving objects, whereas larger values enable faster tracking of changes in the clutter environment.

Knowing the clutter map $s_{CM}(l_s)$, the following methods are implemented to detect newly appearing, displaced, or slowly varying targets whose responses occur in or near the zero-Doppler channel. The current zero-Doppler response and clutter map are represented as complex-valued signals. Depending on the selected method, subtraction or comparison is first performed in the complex or magnitude domain. The resulting detection statistic is then converted to dBFS before CFAR processing. Consequently, the results are represented in the dBFS scale, which improves target visualization and dynamic-range utilization when processing signals with a wide range of amplitudes.

a) Method 1: This method is similar to that described in [23, Eq. (2)]. The condition for detecting a target at the fast-time sample (range bin) l_s during the current CTI with index i can be expressed as

$$20 \lg |s^{BPF,i}(0, l_s) - s^{CM,i}(l_s)| > L_{CFAR}, \quad (67)$$

where L_{CFAR} is the detection threshold calculated for each CTI by the constant false alarm rate (CFAR) processor described in Section X.

It should be noted that the left-hand side of Eq. (67) serves as the input array for the CFAR processor. Consequently, both the CFAR threshold calculation and target detection are performed using the residual signal obtained after clutter-map subtraction. The same approach is used in the methods described below.

b) Method 2: For this method, the target detection condition can be expressed as

$$20 \lg |s^{BPF,i}(0, l_s) - s^{CM,i}(l_s)| - 20 \lg |s^{CM,i}(l_s)| + K > L_{CFAR}, \quad (68)$$

where K is a scaling coefficient used to match the input data presented to the CFAR processor to the corresponding noise-floor level of the RX0+RX1 sum channel.

This method normalizes the clutter-map-subtracted signal by the local clutter-map level, thereby making the detection threshold less sensitive to variations in the stationary background level. This is particularly beneficial at short ranges, where strong clutter reflections can greatly exceed the target signal level. By effectively measuring the relative deviation from the local clutter background, weak targets become easier to detect.

c) Method 3: For this method, the target detection condition can be expressed as

$$20 \lg |s^{BPF,i}(0, l_s)| - 20 \lg |s^{CM,i}(l_s)| + K > L_{CFAR}, \quad (69)$$

where K is a scaling coefficient used to match the input level to the CFAR processor. Unlike Method 1, which uses the difference between the current signal and the clutter map, this approach compares the instantaneous signal level directly with the estimated clutter level. The method is similar to the conventional approach described in [24] and is the default clutter-map-based MTI mode implemented in the presented radar.

X. CFAR DETECTION

The constant false alarm rate (CFAR) technique is used to adaptively adjust the detection threshold so that the probability of false alarm remains approximately constant despite variations in noise and clutter levels [4, Chapter 10.8]. The CFAR implementation used in the presented radar is shown in Fig. 22. It is based on the reference implementation from [19] and has been further optimized in Python for improved execution speed [28], [29].

Because the implemented detector operates on magnitude expressed in dBFS and calculates the reference-cell statistic in the logarithmic domain, its bias does not follow the conventional analytical CA-CFAR scaling law derived for linear power samples. The term CFAR is used here to describe adaptive range-dependent thresholding. The bias values are selected empirically, and the resulting false-alarm probability

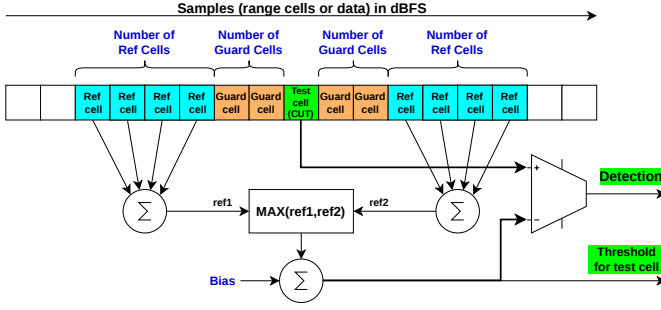


Fig. 22. Conventional range-CFAR implementation. In the MTI processing path, the illustrated CFAR window is applied independently to the range profile of every enabled nonzero-Doppler filter.

must be measured experimentally for a specified operating environment and number of enabled Doppler filters.

The main CFAR parameters are:

- Number of reference cells on each side of the cell under test (CUT), typically 5–30;
- Number of guard cells on each side of the CUT, typically 1–10;
- Bias, which determines the detection threshold offset and is typically chosen to be comparable to the required post-processing SNR, $(S/N)_I^\Sigma$, expressed in dB.

The guard cells prevent the target response and its adjacent range sidelobes from directly contributing to the local background estimate. The reference cells are divided into two groups located on opposite sides of the CUT. Depending on the selected CFAR method, the threshold is derived from the average of all reference cells, the greater of the two side-window averages, or the smaller of the two side-window averages. The selected bias is then added to the estimated background level.

In the presented radar software, the CFAR input is the signal magnitude expressed in dBFS. For the Doppler-filter-bank processing path, the data are obtained from the coherent sum of the RX0 and RX1 complex outputs within each Doppler filter. CFAR is applied independently along the range dimension to every enabled nonzero-Doppler filter. The same number of reference cells, number of guard cells, CFAR method, and bias are used for these filters. The clutter-map path is processed separately and has an independently configurable bias parameter (Appendix D).

A target may be detected simultaneously in several Doppler filters because the filter responses overlap, as shown in Fig. 21. Therefore, Doppler-filter selection is performed after the individual CFAR operations. For each enabled filter and range bin, the software calculates the difference between the filter-output magnitude and its corresponding filter-specific threshold. A valid detection requires this difference to be positive. If several enabled filters produce valid detections at the same range bin, the filter with the largest positive threshold exceedance is selected. If none of the enabled filters exceeds its corresponding threshold, no MTI detection is declared at that range bin.

The complex RX0 and RX1 samples are subsequently taken from the same selected Doppler filter. This preserves the interchannel magnitude and phase relationships required for amplitude-comparison and phase-comparison monopulse processing. The selected Doppler-filter index is also retained for display and target logging. The magnitude and phase measured in the selected filter are used for target tracking (Section XI-B) and for estimating the down-range position and cross-range coordinates using angle-of-arrival measurements (Section XI-A).

It should be noted that two different selection criteria are used. Within the Doppler filter bank, the filter having the largest positive CFAR threshold exceedance is selected at each range bin. During the subsequent fusion of the MTI and clutter-map processing paths, the valid detection having the greater signal level in dBFS is selected.

Applying CFAR before selecting the Doppler filter prevents the local background estimate from being calculated from the maximum noise magnitude across the complete Doppler-filter bank. Selecting the maximum filter output before CFAR would produce an extreme-value noise statistic and would generally increase the estimated background level. The implemented approach instead estimates the local background independently within each Doppler channel and then selects among valid detections. Since every Doppler-filter output is obtained by coherent processing of all n chirps in the CTI, each channel retains the corresponding coherent-integration gain and reduced noise bandwidth. The corrected CFAR ordering therefore preserves the sensitivity improvement provided by n -chirp integration without contaminating the range-dependent background estimate with the maximum noise value across the Doppler-filter bank.

The search over multiple Doppler filters nevertheless creates multiple opportunities for a false detection. If N_F enabled Doppler-filter outputs are assumed to be statistically independent and each filter has a false-alarm probability $P_{FA,F}$ at a particular range bin, the probability that at least one filter produces a false detection is

$$P_{FA,MTI} = 1 - (1 - P_{FA,F})^{N_F}. \quad (70)$$

For $P_{FA,F} \ll 1$, this can be approximated as

$$P_{FA,MTI} \approx N_F P_{FA,F}. \quad (71)$$

The Doppler-filter outputs may not be completely independent in practice because of spectral leakage, slow-time weighting, clutter, and target spreading across adjacent Doppler channels. Consequently, the CFAR bias required to obtain a specified overall MTI false-alarm rate should be verified experimentally for the selected number of enabled filters.

The CFAR thresholds are recalculated once per loop interval T_L after processing the CTI, using the corresponding Doppler-filter and clutter-map data, as illustrated in Fig. 5.

For the presented radar, a simplified CFAR algorithm is also proposed, as shown in Fig. 23. Compared with the conventional implementation, this approach can be executed more efficiently in Python. Since the nonzero-Doppler MTI

processing suppresses stationary clutter, the remaining background is approximately flat. The adaptive threshold for the CUT can therefore be calculated from the average level of a set of neighboring cells that includes the CUT itself. This simplified function is not directly selectable in the application and is currently implemented as a hard-coded alternative to the conventional CFAR function.

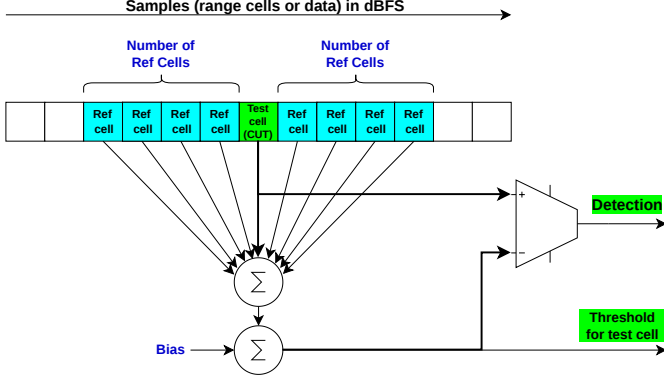


Fig. 23. Simplified adaptive range-threshold detector.

If a target occupies several adjacent range cells with magnitudes above their corresponding detection thresholds, target-cluster compression is applied, as illustrated in Fig. 7 and described in Section II-D. The detected cluster is represented by the range bin having the largest magnitude. This prevents a single spatially extended response from being reported as multiple targets during positioning and visualization, although it may also merge unresolved targets occupying adjacent range bins.

XI. MONOPULSE TRACKING

The RX antennas of the presented radar are squinted in the E-plane and separated by a distance d in the H-plane, as illustrated in Fig. 1. Therefore, amplitude-comparison monopulse (ACM) is used to track targets in the E-plane (elevation angle, $x = 0$ plane), whereas phase-comparison monopulse (PCM) is used to track targets in the H-plane (azimuth angle, $z = 0$ plane) [9]–[12].

The antenna characteristics and monopulse methods used to estimate the angle of arrival of a detected target are derived in Appendix A. In the presented radar, monopulse techniques are first used to estimate the three-dimensional coordinates of targets while the antenna beam remains fixed, i.e., under steady-state conditions. In another operating mode, the radar slowly tracks a large or reference target by electronically steering the receive beam through adjustment of the phase (delay) in the RX1 channel.

A. Angle-of-Arrival Estimation and 3D Target Localization

Equations (A.23) and (A.25) from Appendix A can be rewritten in terms of the measured complex channel voltages after pulse compression and Doppler band-pass filtering:

$$V_{RX0} = A_0 e^{j\phi_0}, \quad V_{RX1} = A_1 e^{j\phi_1}, \quad (72)$$

where A_0 and A_1 are the measured channel magnitudes, and ϕ_0 and ϕ_1 are the corresponding phases obtained from the complex I/Q data.

First, the target elevation angle θ_{AOA} is estimated using the noncoherent form of amplitude-comparison monopulse (ACM) [11, Section 8.10]:

$$\theta_{AOA} \approx \frac{1}{(k_m/\theta_{0.5P}^{E,\Sigma,RX})} \left(\frac{|V_{RX0}| - |V_{RX1}|}{|V_{RX0}| + |V_{RX1}|} \right) + \frac{\pi}{2} = \frac{1}{(k_m/\theta_{0.5P}^{E,\Sigma,RX})} \left(\frac{A_0 - A_1}{A_0 + A_1} \right) + \frac{\pi}{2}, \quad (73)$$

where $\theta_{0.5P}^{E,\Sigma}$ is the half-power beamwidth of the RX antenna sum pattern in the E-plane ($x = 0$), and k_m is the *normalized monopulse slope* in the E-plane [11, Section 5.1.3] [35, Eq. 8.5]. The value of k_m is estimated from the results shown in Fig. A.7a in Appendix A.

Once the elevation angle θ_{AOA} has been estimated, it can be used to compensate (Appendix A, Eq. (A.22)) for the elevation dependence when estimating the azimuth angle φ_{AOA} using phase-comparison monopulse (PCM) in the H-plane:

$$\varphi_{AOA} \approx \arccos \left(\frac{\phi_0 - \phi_1}{k_0 d \sin \theta_{AOA}} \right). \quad (74)$$

Ambiguity outside the angular range $60^\circ \lesssim \varphi \lesssim 120^\circ$ occurs because the magnitude of the phase difference between V_{RX0} and V_{RX1} ,

$$|\phi_0 - \phi_1| = |k_0 d \sin \theta \cos \varphi|, \quad (75)$$

exceeds π for the incident wave, resulting in phase wrapping.

Given the estimated angles of arrival θ_{AOA} and φ_{AOA} , the target position can be expressed in Cartesian coordinates as

$$x_t = R_t \sin(\theta_{AOA}) \cos(\varphi_{AOA}), \quad (76)$$

$$y_t = R_t \sin(\theta_{AOA}) \sin(\varphi_{AOA}), \quad (77)$$

$$z_t = R_t \cos(\theta_{AOA}), \quad (78)$$

where R_t is the radial distance to the target measured by the radar.

B. Reference object tracking

In the H-plane, the receive beam can be steered by adjusting the delay, and consequently the phase, of the RX1 channel. This delay-adjustment mechanism is also used to compensate for the difference in cable lengths between the RX channels. The delay adjustment is implemented entirely in software as part of the signal-processing chain, as described in Section II-D3 and expressed in Eqs. (6)–(8). As discussed in Section II-D3, applying a phase correction at a single carrier frequency is equivalent to compensating for a time delay only under the narrowband assumption. For a waveform with a bandwidth of 30.72 MHz and a carrier frequency of 2.425 GHz, this approximation is considered acceptable.

Such digital receive-beam steering in the H-plane can be used to slowly track a large reference object that serves as a calibration target for maintaining a known RX antenna

boresight direction. PCM is used to measure the angular error signal, from which a phase correction is calculated and applied to the RX1 channel. A zero error signal indicates that the RX antenna boresight is directed toward the reference object. This tracking mode can be enabled or disabled through the radar application's user interface, as described in Appendix D. In the application, the user can also specify the range interval within which the software searches for a large reference object by identifying the maximum magnitude of the reflected signal.

The measured H-plane error characteristic is derived in Appendix A, as given by Eq. (A.22). Assuming operation close to the principal H-plane, i.e., $65^\circ \lesssim \theta \lesssim 115^\circ$, the error signal can be approximated as

$$\varepsilon = \frac{\phi_0 - \phi_1}{k_0 d}, \quad (79)$$

where ϕ_0 and ϕ_1 are the measured phases of the RX0 and RX1 channels, respectively, obtained from the complex I/Q data.

Similar phase wrapping may occur when the unambiguous phase-difference range is exceeded, resulting in ambiguity in the measured error signal and the estimated azimuth angle.

Algorithm 2 summarizes the principal operations performed during reference-object tracking. The parameter K_T is the tracking-loop gain coefficient and must be selected to provide a stable beam-steering response. During each radar processing cycle, the RX1 channel delay $\hat{t}_d$ is updated according to the estimated tracking error. This feedback mechanism continuously adjusts the receive-beam direction toward the reference object at every CTI, as illustrated in Fig. 5.

The RX1 channel delay is updated according to

$$\hat{t}_d^i = \hat{t}_d^{i-1} + K_T \varepsilon^{i-1}. \quad (80)$$

The resulting feedback loop is equivalent to a discrete-time integrator controller. In control-system terminology, the delay-adjustment block acts as an integrating feedback amplifier whose output is the accumulated delay correction. The loop continuously integrates the monopulse error signal until the angular error approaches zero, thereby steering the receive beam toward the reference object.

It should be noted that the error signal is set to zero when the selected range gate contains no valid CFAR detections. Therefore, the CFAR parameters must be configured appropriately to ensure reliable detection of the reference object. Otherwise, the feedback loop receives a zero error signal and no longer updates the beam-steering correction.

C. Effect of RX and TX Antenna Positioning

As described above, the presented radar employs phase-comparison monopulse (PCM) in the H-plane, where the phase difference between the RX0 and RX1 channels is used to estimate the target angle or, equivalently, the H-plane error signal (Eqs. (74) and (79)). Let us now consider a target moving in the radial direction when the transmitter and receiver phase centers are separated by a distance l in the H-plane, as illustrated in Fig. 24. The RX0 and RX1 phase centers

Algorithm 2 Reference-Object Tracking

Require: Zero-Doppler filter outputs from two receive channels

Ensure: Tracking error ε , tracking error threshold ε_{th} , reference-object distance R_t , updated RX1 delay

Target Detection

- 1: Form the sum-pattern signal from the two receive channels
- 2: Convert the sum-pattern magnitude to dBFS
- 3: Apply CFAR detection to the complete range profile
- 4: Restrict detections to the predefined tracking range gate
- 5: **if** no valid detections exist within the range gate **then**
- 6: Return $\varepsilon = 0$
- 7: **end if**
- 8: Select the strongest detected target within the range gate
- 9: Estimate the target distance R_t

Phase-Comparison Monopulse Processing

- 10: Measure the phase of both receive channels
- 11: Compute the inter-channel phase difference
- 12: Estimate the angular error signal ε (Eq. (79))

Tracking Loop

- 13: **if** reference-object tracking is enabled **then**
 - 14: **if** $|\varepsilon| > \varepsilon_{th}$ **then**
 - 15: Update the accumulated RX1 delay correction
 $\hat{t}_d^i = \hat{t}_d^{i-1} + K_T \cdot \varepsilon$
 - 16: Apply the updated delay $\hat{t}_d^i$ compensation to RX1
(Eq. (8))
 $s_{RX1}^{ADC'}(t) = a_{RX1}(t) e^{j\omega_0(-t_d + \hat{t}_d^i)}$
 - 17: **end if**
 - 18: **end if**
 - 19: Return RX1 delay $\hat{t}_d^i$, ε and R_t
-

remain separated by a distance d , as in the radar configuration shown in Fig. 1. Under these conditions, the target's radial motion introduces an additional propagation delay between the transmit and receive paths. This delay manifests itself as a phase error in the PCM measurement and may bias the estimated H-plane angle, as demonstrated below.

Assuming specular reflection from a locally planar object, the reflection point associated with each receive channel lies midway between the corresponding TX and RX phase centers in the H-plane. Consequently, the TX-to-object and object-to-RX propagation segments have equal lengths for each channel.

The propagation phases of the RX0 and RX1 channel signals can be expressed as

$$\phi_0 = 2k_0 \sqrt{R^2 + \left(\frac{l + \frac{d}{2}}{2}\right)^2}, \quad (81)$$

and

$$\phi_1 = 2k_0 \sqrt{R^2 + \left(\frac{l - \frac{d}{2}}{2}\right)^2}. \quad (82)$$

For $R \gg l, d$, the phase difference between the channels,

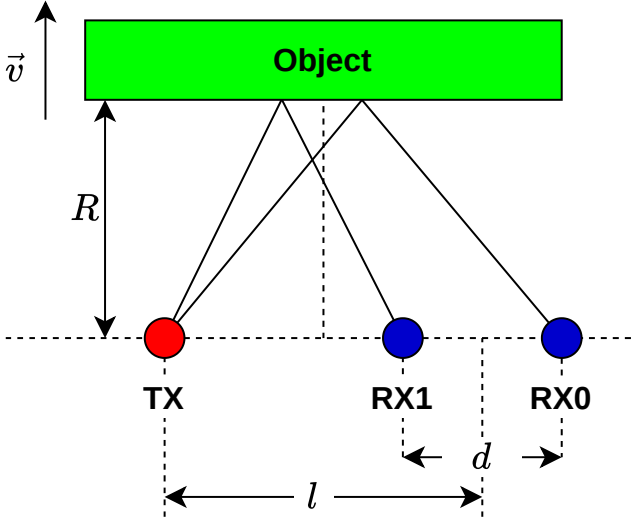


Fig. 24. H-plane propagation geometry for specular reflection from a locally planar object. The distance l is measured between the TX phase center and the midpoint of the RX pair, while d is the separation between RX0 and RX1.

$\Delta\phi = \phi_0 - \phi_1$, can be approximated as

$$\Delta\phi \approx 2k_0 \frac{\left(\frac{l+\frac{d}{2}}{2}\right)^2 - \left(\frac{l-\frac{d}{2}}{2}\right)^2}{2R} = k_0 \frac{ld}{2R}. \quad (83)$$

If the target moves with a relatively low radial velocity v , its range becomes time dependent:

$$R(t) = R_0 + vt. \quad (84)$$

Consequently, the phase difference becomes

$$\Delta\phi(t) = k_0 \frac{ld}{2(R_0 + vt)} \approx k_0 \frac{ld}{2R_0} - k_0 \frac{ldv}{2R_0^2} t, \quad (85)$$

where the first-order approximation

$$\frac{1}{R_0 + vt} \approx \frac{1}{R_0} - \frac{vt}{R_0^2} \quad (86)$$

is valid for $|vt| \ll R_0$. Using $k_0 = \omega_0/c$ and Doppler angular frequency $\omega_d = 2\omega_0 v/c$, the phase difference can be written as

$$\Delta\phi(t) \approx \frac{\omega_0}{c} \frac{ld}{2R_0} - \omega_d t \frac{ld}{4R_0^2}. \quad (87)$$

As shown by Eq. (87), the phase difference caused by the separation of the TX and RX antenna phase centers consists of a static term and a time-varying term. At short ranges, the static term may introduce an additional bias in the monopulse angle estimate, whereas the time-varying term produces a differential phase rate between the receive channels, causing the apparent PCM angle to vary as the target range changes. If this effect is not included in the calibration or angle-estimation model, the radar may interpret purely radial motion as apparent cross-range motion, thereby degrading angular tracking and potentially causing loss of track.

The preceding analysis establishes a requirement for TX and RX antenna positioning in the H-plane, where PCM is

employed: the TX and RX phase centers should coincide or, at least, be positioned as close to each other as possible, corresponding to $l = 0$ in Eq. (87).

In the E-plane, ACM is employed, and the phase centers of the RX0 and RX1 antennas are coincident, as illustrated in Fig. 1 and described in Appendix A. Considering only the principal E-plane ($x = 0$), this might suggest that alignment of the TX and RX phase centers is not required. However, in the more general case of a target moving in a plane for which $x \neq 0$ and $z \neq 0$, the RX phase centers have a nonzero projected separation. If the TX and RX phase centers are also separated, an additional phase difference similar to that in Eq. (87) may arise.

Therefore, for general 3D target tracking, the TX and RX antenna phase centers should ideally coincide in all coordinate-plane projections or, at least, be positioned as close to each other as practicable. This requirement minimizes static and dynamic phase errors that could otherwise bias the estimated target angle and radial velocity.

However, this phase-center alignment requirement conflicts with the conventional practice of increasing the physical separation between the TX and RX antennas to improve isolation [4, Chapter 3.2]. Consequently, achieving accurate 3D tracking with closely spaced phase centers requires additional TX-leakage mitigation techniques, as discussed in Section V.

XII. EXPERIMENTAL EVALUATION

This section describes experiments performed to evaluate radar operation, target detection, Doppler discrimination, and qualitative localization performance. A description of the application interface and operating procedure is provided in Appendix D.

The experiments described in Sections XII-A–XII-C were performed using Prototype #1 placed on a table at a height of approximately 0.6 m above ground level. The experiments described in Sections XII-D–XII-E were performed using Prototype #1 installed on the roof of a building at a height of approximately 5 m above ground level.

A. Person Walking in an Urban Area (Experiment #1)

The experimental environment is shown in Fig. 25. Figure 25a presents a photograph of the measurement area, while Fig. 25b shows the corresponding walking route. The environment contains numerous sources of clutter, including buildings, trees, bushes, a construction site with a metal fence, and a playground. Wind speeds of approximately 2–4 m/s caused movement of the trees and bushes, producing additional MTI detections. Furthermore, significant multipath propagation and reflections were expected because of the surrounding structures and vegetation.

The experiment was performed using Prototype #1 at a center frequency of 2.425 GHz and a sampling rate of 30.72 MHz. The radar was placed on a table approximately 0.6 m above ground level. Each CTI contained $n = 16$ chirps, each with a duration of 1 ms. A Kaiser window with a shape parameter of $\beta_K = 5$ was applied to the reference chirp for pulse



(a) Experimental environment.



(b) Walking route in the experimental area.

Fig. 25. Urban measurement environment and walking route used in the experiment.

compression. The DAC scaling factor was set to $K_{DAC} = 2^{14}$, and the gains of both RX channels were set to 38 dB.

Doppler MTI, clutter-map processing, CFAR detection, and target-cluster compression were enabled. The CFAR bias was set to 22 dB, with two guard cells and six reference cells. The clutter-map bias was set to 9 dB, and clutter-map filter variant 3 was used, as described in Section IX. A hard-coded software parameter limited the processed range to 300 m. The complete configuration is included with the recorded experimental data and supplementary materials [28], [29].

A screenshot of the radar application during the experiment is shown in Fig. 26.

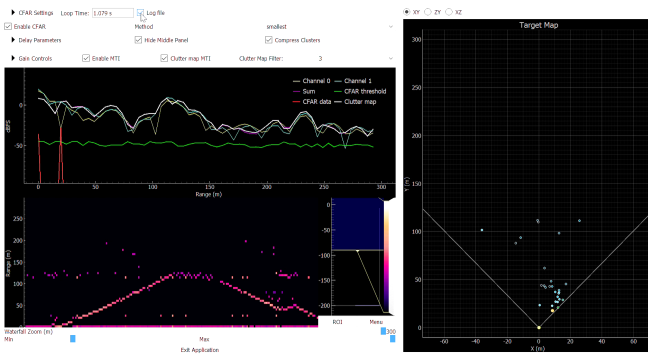


Fig. 26. Radar application display during the urban walking experiment.

The target-tracking results are presented in Figs. 27–32. The marker color indicates the selected Doppler-filter index, while the marker size represents the detected signal magnitude. The estimated radial distance and Cartesian coordinates are

shown as functions of time. These results are followed by the estimated trajectory in the horizontal plane and its overlay on the experimental area.

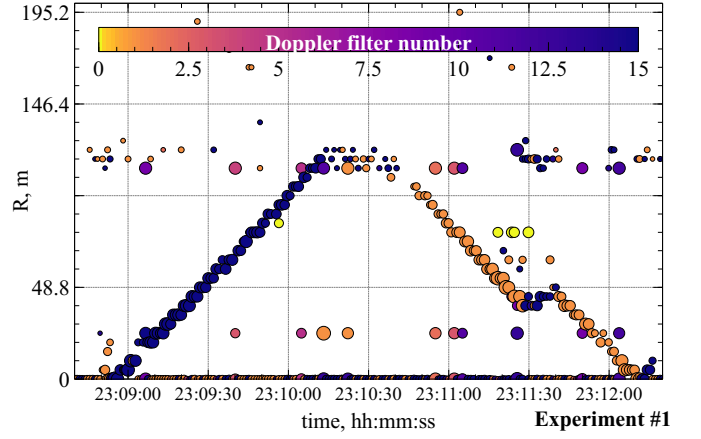


Fig. 27. Estimated target radial distance R as a function of time. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude.

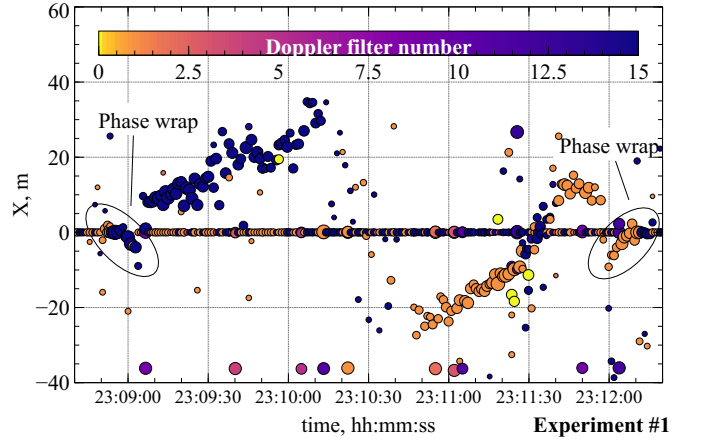


Fig. 28. Estimated target x coordinate as a function of time. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude.

The estimated horizontal target trajectory is shown in Fig. 31. Figure 32 overlays the estimated trajectory on a map of the experimental area, allowing the radar-derived positions to be compared with the actual walking route.

The walking target can be distinguished from the background detections by the continuous variation of its estimated position and its relatively large signal magnitude. Different sections of the trajectory are associated with different Doppler-filter indices because the magnitude and direction of the target radial velocity vary along the walking route.

The two sections of the walking route differ in direction by approximately 30° , which is comparable to the sum-channel antenna beamwidth. Consequently, during parts of the route, the target is located in a reduced-gain region of the antenna pattern, resulting in a lower received signal-to-interference-

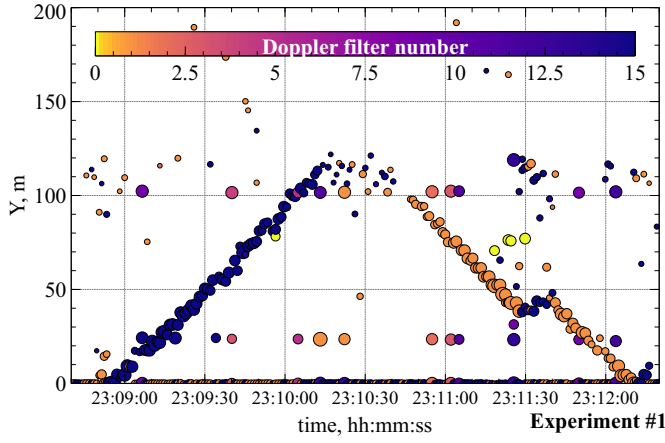


Fig. 29. Estimated target y coordinate as a function of time. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude.

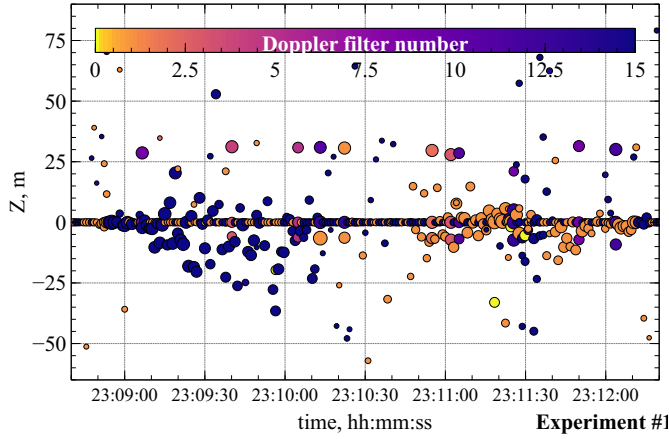


Fig. 30. Estimated target z coordinate as a function of time. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude.

plus-noise ratio. Despite the challenging propagation and clutter conditions, a dominant sequence of detections consistent with the walking route is visible over most of the route. Stationary clutter is substantially suppressed by the nonzero-Doppler processing channels, while isolated detections may be attributed to residual clutter, multipath propagation, moving vegetation, or false alarms.

Phase-wrapping effects are visible in Figs. 31 and 32. When the unwrapped physical phase difference between the RX0 and RX1 channels exceeds the unambiguous interval represented by the principal phase $[-\pi, \pi)$, as described by Eq. (75), the PCM algorithm maps the phase to an incorrect azimuth angle and therefore produces an erroneous cross-range position.

The walking route passes close to the radar on its right-hand side, where the target direction deviates substantially from the antenna boresight. At these large off-boresight angles, the physical inter-channel phase difference exceeds the unambiguous range of the two-channel phase-comparison estimator, resulting in phase wrapping. The affected detections do not

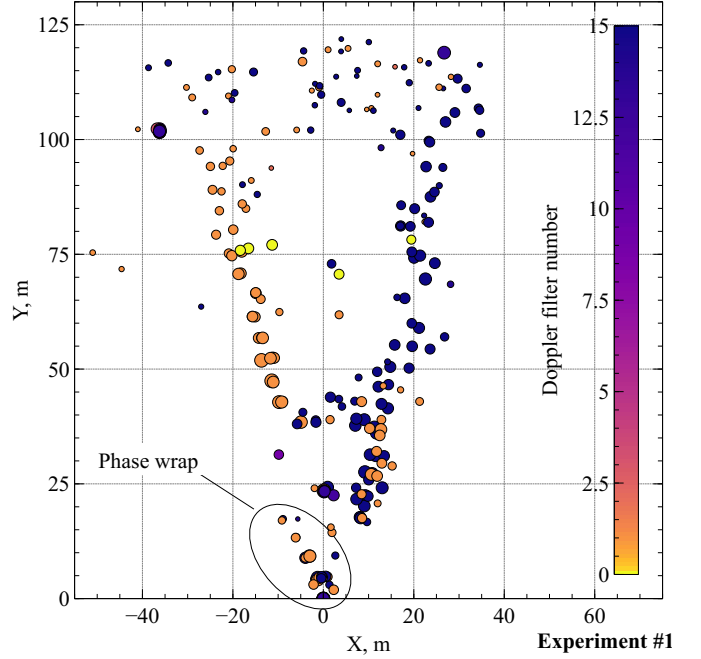


Fig. 31. Estimated target trajectory in the horizontal xy plane. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude. The marked region identifies erroneous position estimates caused by phase wrapping in the PCM azimuth estimate.

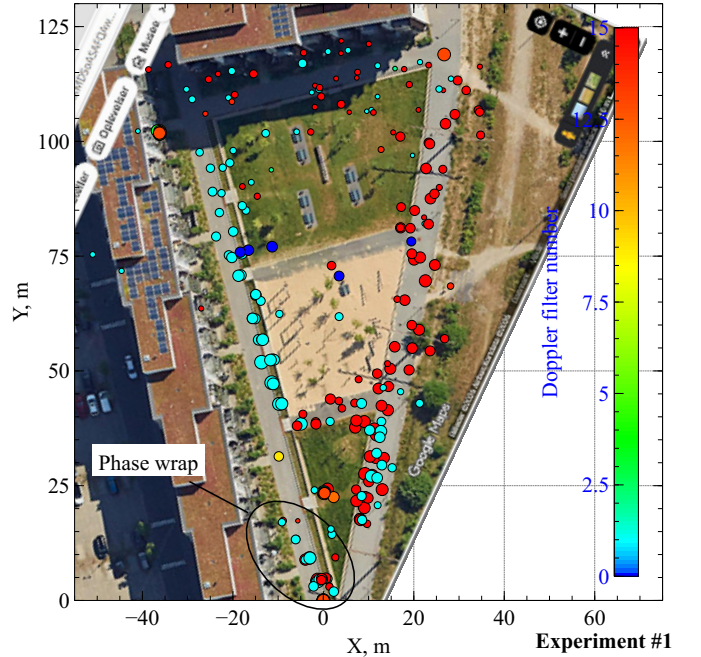


Fig. 32. Estimated target trajectory in the horizontal xy plane overlaid on the experimental area. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude. The marked region identifies erroneous position estimates caused by phase wrapping in the PCM azimuth estimate.



(a) Experimental environment. (b) Walking and running route.

Fig. 33. Urban environment and target route used in Experiment #2.

represent the actual target trajectory and would require rejection or correction using phase-ambiguity resolution, trajectory-continuity constraints, or temporal tracking.

B. Person Walking and Running on a Straight Street (Experiment #2)

Experiment #2 evaluated the radar response to a person walking and running along the predominantly straight urban route shown in Fig. 33. The radar was positioned near one end of the route, and the target moved primarily toward and away from it. This geometry produced a substantial radial-velocity component and enabled evaluation of the Doppler filter bank for different target velocities and directions of motion.

The radar and processing parameters were the same as in Experiment #1, described in Section XII-A. Doppler MTI, clutter-map processing, CFAR detection, and target-cluster compression were enabled. The CFAR bias was 20 dB, with one guard cell and 20 reference cells. The clutter-map detection bias was 6 dB, and filter variant 3 was used (Section IX). A hard-coded software parameter limited the processed range to 300 m. The complete configuration is included with the recorded data and supplementary materials [28], [29].

Figure 34 shows the radar application during the experiment. In the subsequent plots, marker color indicates the selected Doppler-filter index, and marker size indicates the detected post-processing signal magnitude. Index $k = 0$ denotes the zero-Doppler channel.

The estimated radial distance is shown in Fig. 35. The two dominant sloped tracks represent motion away from and toward the radar. Their Doppler-filter indices identify the reversal in radial velocity between the outgoing ($k = 15$) and returning ($k = 1$) portions of the experiment.

A large number of detections also occur in Doppler-filter index $k = 0$. Many of these detections form approximately time-invariant range bands and are therefore distinguishable from the continuously changing range of the moving target. These responses are attributed primarily to the clutter-map processing path and to residual stationary or slowly varying reflections in the urban environment. The clutter-map bias

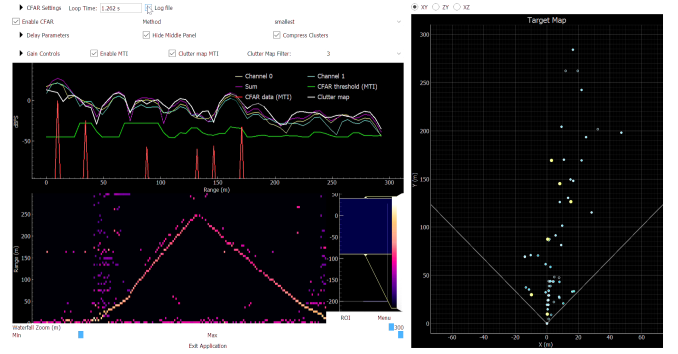


Fig. 34. Radar application display during Experiment #2.

was reduced from 12 dB in Experiment #1 to 6 dB in Experiment #2. This reduction increased the sensitivity to changes in the estimated background but also contributed to a greater number of zero-Doppler detections. The further increase observed when the target was close to the radar may have resulted from strong target reflections and associated multipath components disturbing the clutter-map estimate. Consequently, not every point shown in Fig. 35 represents the walking or running target.

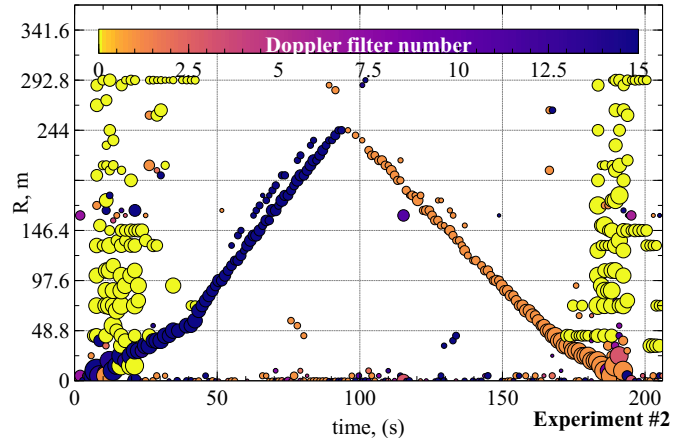


Fig. 35. Estimated radial distance R as a function of time.

Figures 36 and 37 show the Cartesian-coordinate estimates. Because the route was approximately aligned with the y axis, the dominant y -coordinate tracks follow the outgoing and returning range trends. The x -coordinate estimates remain near the street centerline but exhibit greater relative scatter because the cross-range coordinate depends strongly on the phase-comparison monopulse azimuth estimate. Inter-channel phase uncertainty, residual mismatch, multipath, target-aspect variation, and weak detections may all contribute to the observed spread, which should not be interpreted as physical lateral motion alone.

The estimated z coordinate is presented in Fig. 38. Although many detections are concentrated near the nominal ground-reference level, considerable vertical dispersion is present. Because the target remained on the street surface, this disper-

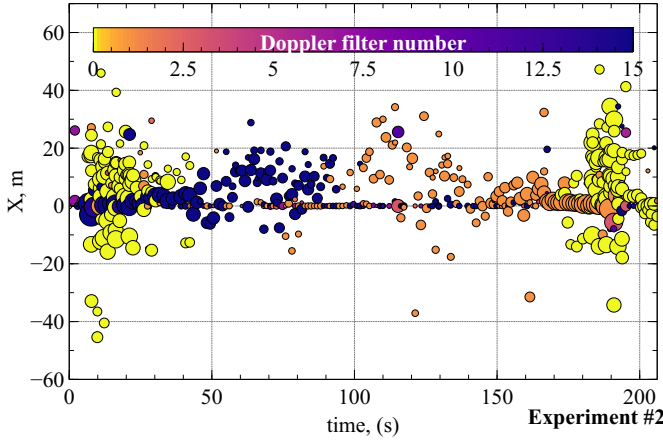


Fig. 36. Estimated target x coordinate as a function of time.

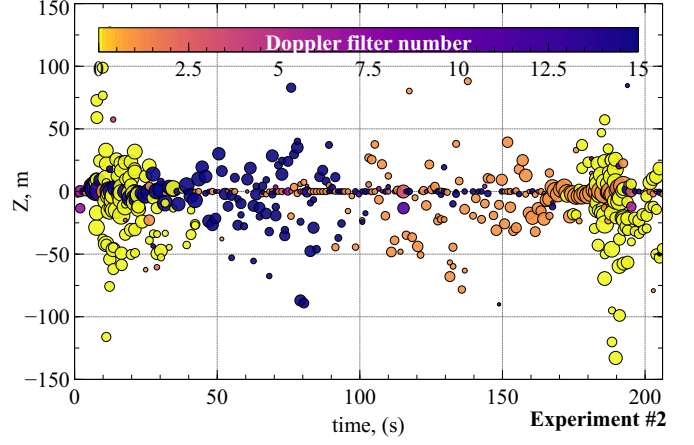


Fig. 38. Estimated target z coordinate as a function of time.

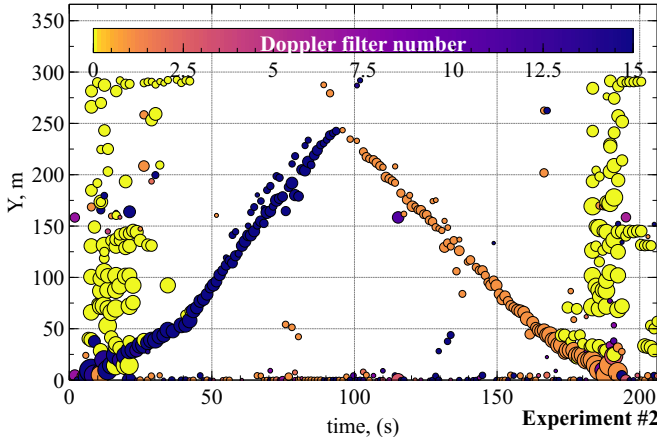


Fig. 37. Estimated target y coordinate as a function of time.

sion primarily reflects uncertainty in the amplitude-comparison monopulse elevation estimate rather than changes in target height.

The antenna alignment and monopulse boresight were not calibrated using the reference-object procedure described in Section XI-B. The measurements may therefore contain residual alignment errors and systematic offsets in both the E- and H-planes. In particular, an elevation-plane offset produces a range-dependent error in the estimated z coordinate. Additional errors may arise from target-aspect variation, ground multipath, antenna-pattern mismatch, receiver-gain mismatch, and weak detections. Accurate height estimation will require reference-object calibration and temporal filtering.

Figure 39 presents the estimated trajectory in the horizontal xy plane. The principal moving-target detections form an approximately straight route, in agreement with Fig. 33b. Their Doppler-filter indices distinguish the outgoing and returning motion. No phase wrapping is observed in the phase-comparison monopulse azimuth measurement.

The trajectory becomes less compact in several regions, and isolated detections occur away from the principal path. These deviations may be caused by angular-estimation un-

certainly, multipath, residual channel mismatch, target-aspect variation, and detections from clutter or other objects. Because no independent position reference was recorded, the results demonstrate qualitative trajectory formation but do not provide a quantitative estimate of cross-range or three-dimensional tracking accuracy.

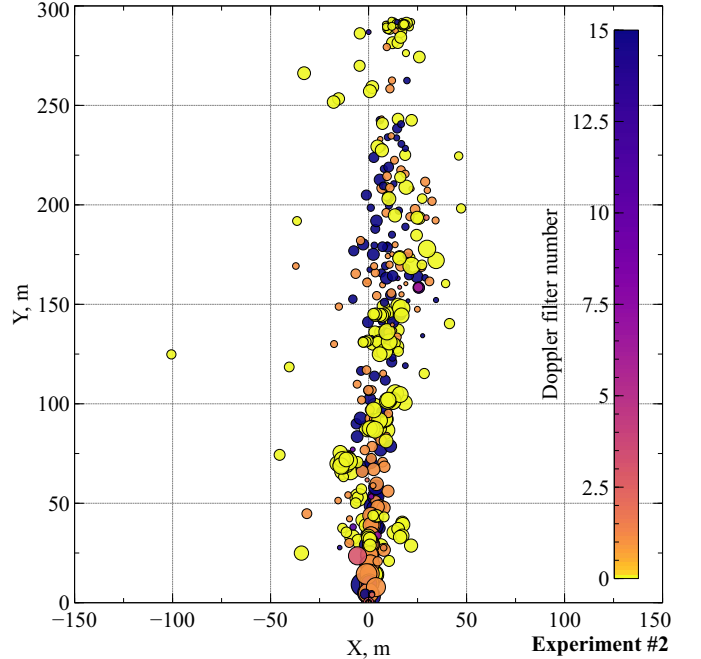


Fig. 39. Estimated target trajectory in the horizontal xy plane.

C. Car Driving on a Straight Street (Experiment #3)

Experiment #3 evaluated the radar response to a Toyota Aygo driven at approximately 20–30 km/h along the predominantly straight urban route shown in Fig. 33. The experimental setup and target route were the same as in Experiment #2.

The radar and processing parameters were also the same as those used in Experiment #2, described in Section XII-B. The

complete configuration is included with the recorded data and supplementary materials [28], [29].

Figure 40 shows the radar application during the experiment. In the subsequent plots, marker color indicates the selected Doppler-filter index, and marker size indicates the detected post-processing signal magnitude. Index $k = 0$ denotes the zero-Doppler channel.

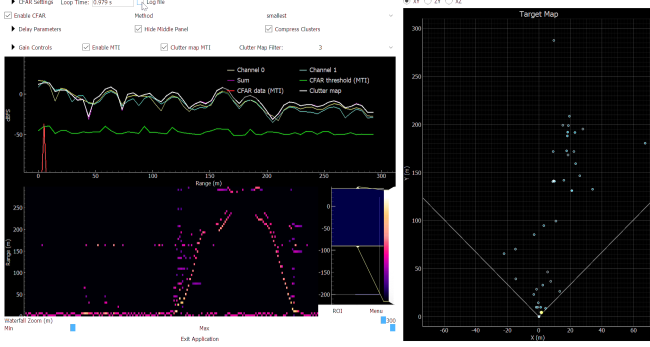


Fig. 40. Radar application display during Experiment #3.

The estimated radial distance is shown in Fig. 41. The dominant tracks represent the outgoing and returning portions of the route. Their Doppler-filter indices identify the reversal in radial velocity, with the outgoing motion primarily detected in filters $k = 13$ – 15 and the returning motion primarily detected in filters $k = 1$ – 3 .

The car responses occupy a broader range of nonzero Doppler-filter indices than the walking-person responses from Experiment #2. This is consistent with the higher radial velocity of the car. The distributions of the selected Doppler-filter indices for the two experiments are compared in Fig. 42.

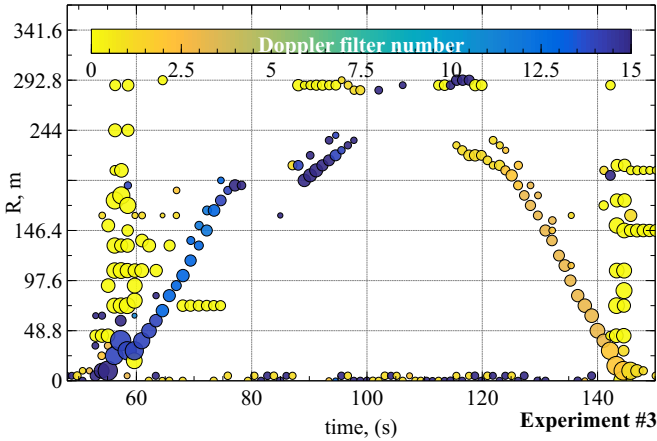
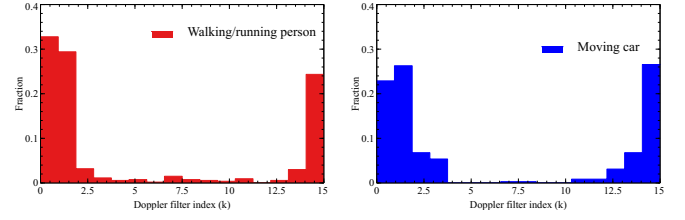


Fig. 41. Estimated radial distance R as a function of time during Experiment #3.

Figures 43 and 44 show the Cartesian-coordinate estimates. Because the street was approximately aligned with the y axis, the dominant y -coordinate tracks follow the outgoing and returning range trends. The x -coordinate estimates represent the



(a) Experiment #2: person.

(b) Experiment #3: car.

Fig. 42. Distributions of the selected Doppler-filter indices for Experiment #2 and Experiment #3.

estimated cross-range position and therefore depend strongly on the phase-comparison monopulse azimuth measurement.

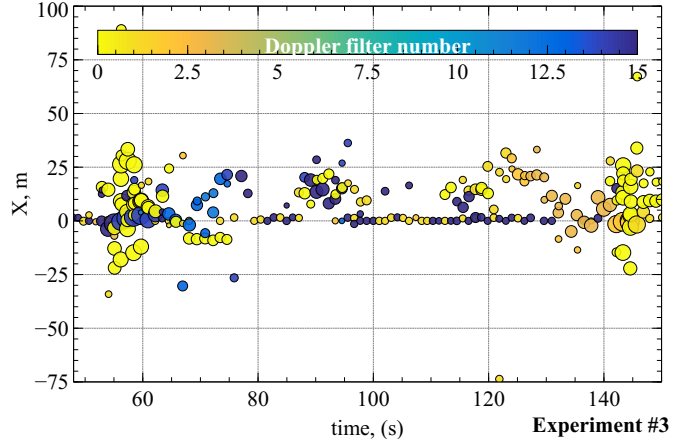


Fig. 43. Estimated target x coordinate as a function of time during Experiment #3.

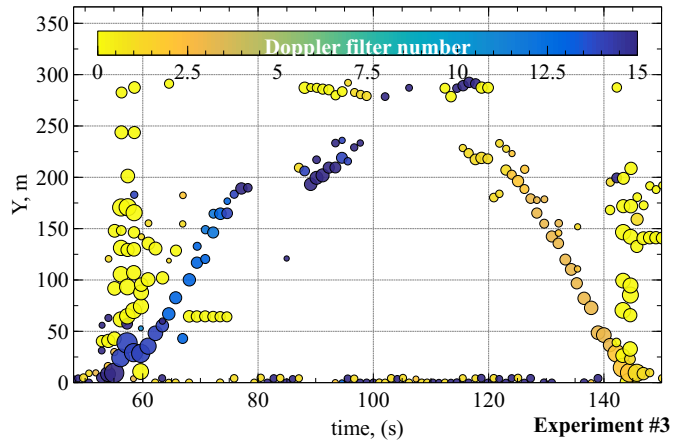


Fig. 44. Estimated target y coordinate as a function of time during Experiment #3.

The estimated z coordinate is presented in Fig. 45. As in Experiment #2, the vertical coordinate depends on the amplitude-comparison monopulse elevation estimate and is sensitive to antenna alignment, amplitude mismatch, multipath propagation, and target-aspect variation. The antenna alignment and monopulse boresight were not calibrated using the

reference-object procedure described in Section XI-B. The z -coordinate results should therefore be interpreted as qualitative elevation estimates rather than validated measurements of vehicle height.

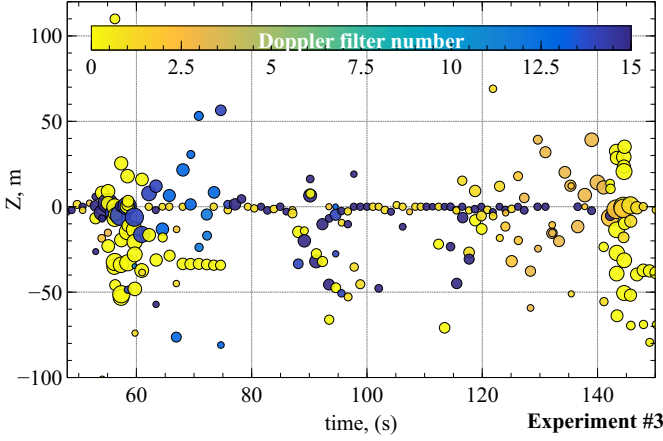


Fig. 45. Estimated target z coordinate as a function of time during Experiment #3.

Figure 46 presents the estimated trajectory in the horizontal xy plane. The principal moving-target detections form an approximately straight route, in agreement with Fig. 33b. The Doppler-filter indices distinguish the outgoing and returning motion. No phase wrapping is observed in the phase-comparison monopulse azimuth measurement during this experiment.

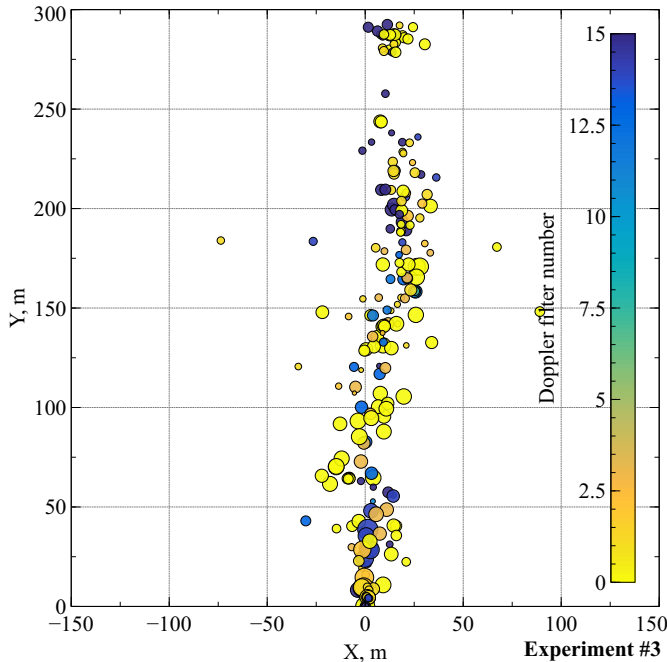


Fig. 46. Estimated car trajectory in the horizontal xy plane during Experiment #3.

Compared with the walking and running results from Experiment #2, the car measurements in Figs. 43–46 generally ex-

hibit lower angular scatter and more compact Cartesian target tracks than those in Figs. 36–39. The car typically produces a stronger radar return because of its larger radar cross section, resulting in a higher signal-to-noise ratio and, consequently, lower uncertainty in the monopulse angle estimates [see Section VII-B]. However, because no independent reference trajectory was recorded, the more compact car track indicates reduced angular-estimation scatter but does not confirm that the estimated angles are closer to their true values.

D. Ferry Crossing the Strait (Experiment #4)

Experiment #4 evaluated the radar response to the ferry vessel *Egholm II* (MMSI: 219015127), a large maritime target with a substantially higher radar cross section than the pedestrian and vehicle targets considered in Experiments #2 and #3.

The experimental environment is shown in Fig. 47. Figure 47a presents a photograph of the measurement area, while Fig. 47b shows the corresponding shipping route.

The radar and processing parameters were the same as in Experiment #1, described in Section XII-A. The radar was placed on an elevated observation point approximately 5 m above sea level. Doppler MTI, clutter-map processing, CFAR detection, and target-cluster compression were enabled. The CFAR bias was 20 dB, with one guard cell and 20 reference cells. The clutter-map detection bias was 12 dB, and filter variant 3 was used (Section IX). A hard-coded software parameter limited the processed range to 1000 m. The complete configuration is included with the recorded data and supplementary materials [28], [29].

The H-plane antenna alignment and PCM boresight were calibrated using a stationary ferry as the reference object, following the procedure described in Section XI-B.

A screenshot of the radar application during the experiment is shown in Fig. 48.

The target-tracking results are presented in Figs. 49–54. The marker color indicates the selected Doppler-filter index, while the marker size represents the detected signal magnitude. The estimated radial distance and Cartesian coordinates are shown as functions of time. These results are followed by the estimated trajectory in the horizontal plane and its overlay on the experimental area.

The estimated radial distance is shown in Fig. 49. Two dominant tracks are visible and correspond to the outbound and inbound portions of the ferry route. Their Doppler-filter indices identify the direction of motion, with outbound detections concentrated primarily in filters $k = 13$ –15 and inbound detections concentrated primarily in filters $k = 1$ –3. Compared with Experiments #2 and #3, the selected Doppler filters exhibit a narrower distribution, indicating a more uniform radial velocity during the observed ferry transit.

Although detections associated with the clutter-map processing path remain visible in the zero-Doppler channel, the ferry responses form a continuous and easily identifiable track throughout the measurement interval. The stronger target



(a) Experimental environment.



(b) Ferry shipping route.

Fig. 47. Harbor measurement environment and ferry ship route used in the experiment.

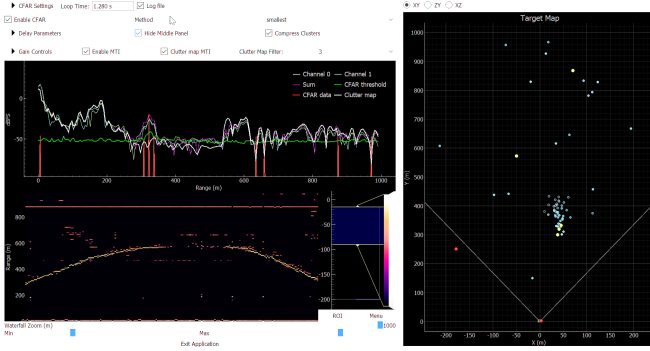


Fig. 48. Radar application display during the ferry shipping experiment.

return produced by the ferry results in a higher signal-to-interference ratio and therefore more robust detection than observed for the pedestrian experiment.

Figures 50 and 51 show the corresponding Cartesian-coordinate estimates. The dominant trajectory forms a smooth outbound and inbound path with substantially less scatter than observed for the pedestrian experiment. This behavior is consistent with the larger radar cross section of the ferry and the resulting improvement in monopulse measurement quality.

The compactness of the coordinate tracks suggests that the phase-comparison monopulse azimuth estimate remained stable throughout the measurement.

The estimated horizontal target trajectory is shown in Fig. 53. Figure 54 overlays the estimated trajectory on a map of the experimental area, allowing the radar-derived positions to be compared with the actual ship travel environment.

The estimated z -coordinate remains comparatively stable

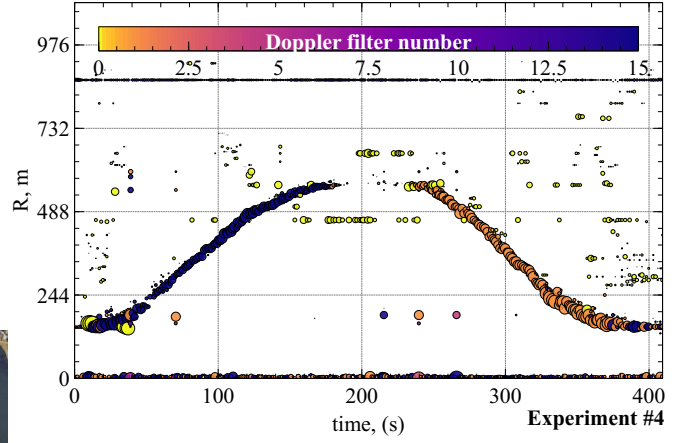


Fig. 49. Estimated target radial distance R as a function of time. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude.

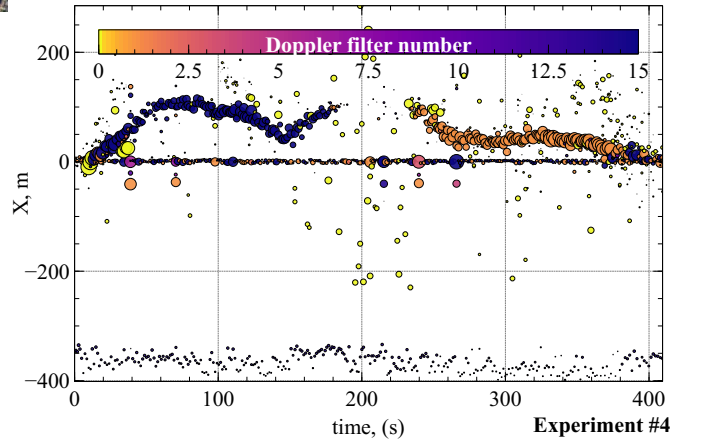


Fig. 50. Estimated target x coordinate as a function of time. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude.

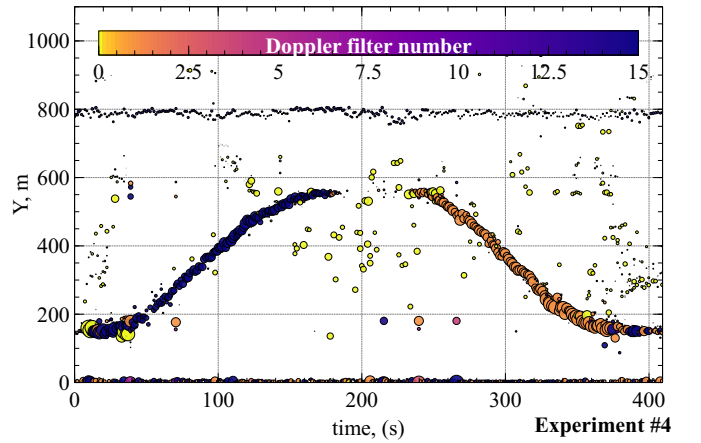


Fig. 51. Estimated target y coordinate as a function of time. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude.

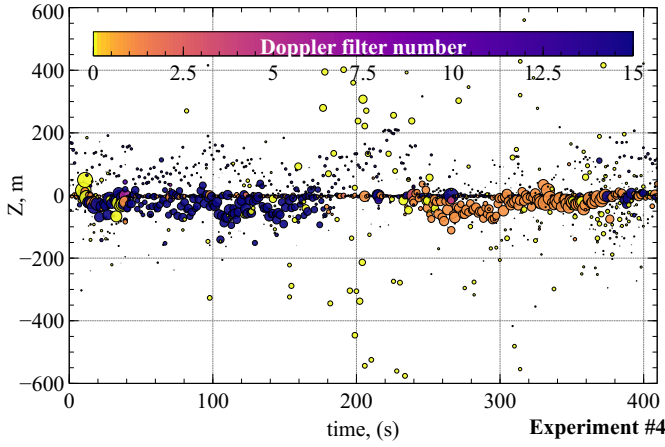


Fig. 52. Estimated target z coordinate as a function of time. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude.

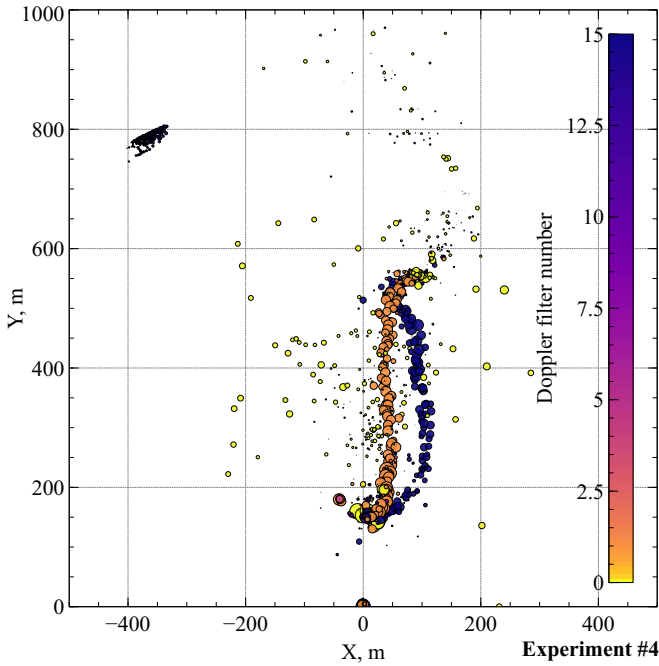


Fig. 53. Estimated target trajectory in the horizontal xy plane. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude.

throughout the experiment. Although some scatter is present, the variation is significantly smaller than that observed for the pedestrian target. The remaining fluctuations are attributed primarily to antenna alignment uncertainty, multipath propagation from the water surface, and residual amplitude mismatch between the receive channels.

The measurements suggest that the mean water level was located below the radar position. This observation is consistent with the known geometry of the measurement site and provides qualitative validation of the amplitude-comparison monopulse elevation estimate. Nevertheless, the absolute height estimates remain sensitive to antenna alignment and calibration errors,

and should therefore be interpreted with appropriate caution.

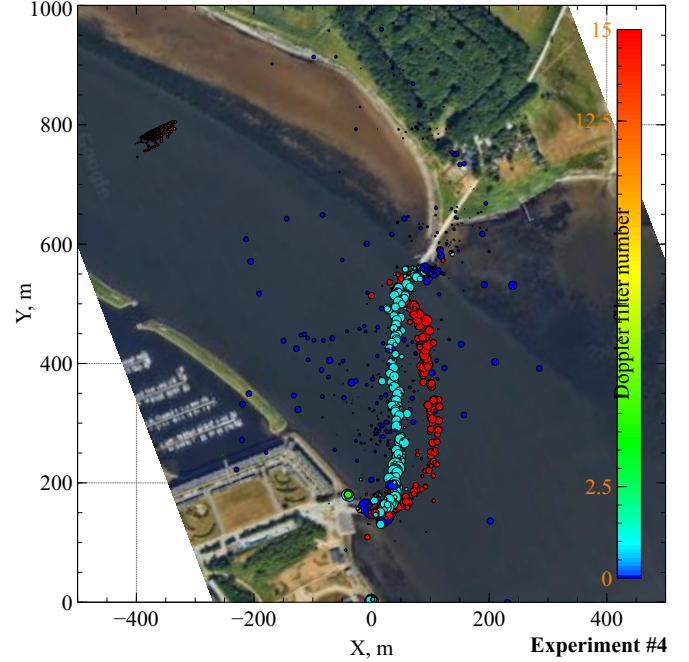


Fig. 54. Estimated target trajectory in the horizontal xy plane overlaid on the experimental area. Marker color indicates the selected Doppler-filter index, while marker size represents the detected signal magnitude.

Figure 54 overlays the estimated trajectory on a map of the experimental area. The reconstructed path follows the expected ferry route and correctly distinguishes the outbound and inbound portions of the transit. Although some positional offset and scatter remain, particularly near the harbor area where reflections from shoreline structures and vessels may contribute additional multipath components, the reconstructed trajectory is generally consistent with the expected ferry route.

The distribution of selected Doppler-filter indices is shown in Fig. 55. Most detections occur near the extreme Doppler bins associated with the outbound and inbound motion of the ferry. The limited occupancy of intermediate Doppler filters is consistent with nearly constant vessel speed during the crossing.

Overall, Experiment #4 demonstrates continuous detection, Doppler discrimination, trajectory formation, and qualitative localization of a large maritime target. Among the presented outdoor experiments, the ferry produced the most stable range, azimuth, and elevation estimates owing to its larger radar cross section and stronger reflected signal. The map-overlay comparison further provides qualitative validation of the estimated trajectory and demonstrates the applicability of the proposed radar architecture to maritime surveillance and tracking scenarios.

E. People Walking on a Straight Coastal Route (Experiment #5)

Experiment #5 evaluated the radar response to two persons walking along the predominantly straight coastal route shown

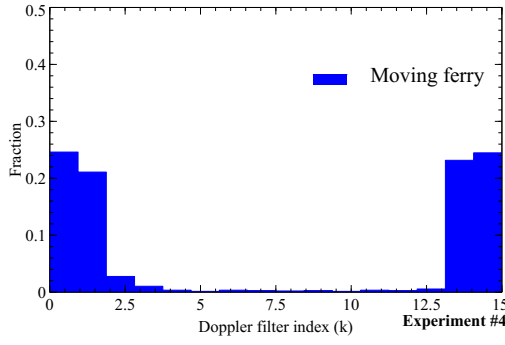


Fig. 55. Distributions of the selected Doppler-filter indices for Experiment #4.



Fig. 56. Walking route used in Experiment #5.

in Fig. 56. The radar was positioned near one end of the route, and the targets moved primarily toward and away from the radar. Unlike Experiment #2, the radar boresight was intentionally offset such that the route was located close to the zero crossing of the horizontal antenna pattern. This configuration increases the sensitivity of the phase-comparison monopulse (PCM) azimuth estimate to small angular deviations and therefore provides a more challenging localization scenario.

Another important difference from Experiment #2 in Section XII-B was the radar installation height. In Experiment #5, the radar was located approximately 5 m above sea level, whereas in Experiment #2 it was positioned approximately 0.6 m above ground level.

The H-plane antenna alignment and PCM boresight were calibrated using a stationary ferry as a reference object, following the procedure described in Section XI-B.

The radar and processing parameters were the same as in Experiment #1, described in Section XII-A. Doppler MTI, clutter-map processing, CFAR detection, and target-cluster compression were enabled. The CFAR bias was 19 dB, with one guard cell and 20 reference cells. The clutter-map detection bias was 12 dB, and filter variant 3 was used (Section IX). A hard-coded software parameter limited the processed range to 300 m. The complete configuration is included with the recorded data and supplementary materials [28], [29].

Figure 57 shows the radar application during the experiment. In the subsequent plots, marker color indicates the selected Doppler-filter index, and marker size indicates the detected post-processing signal magnitude. Index $k = 0$ denotes the zero-Doppler channel.

The estimated radial distance is shown in Fig. 58. Two dominant target tracks are visible and correspond to the two

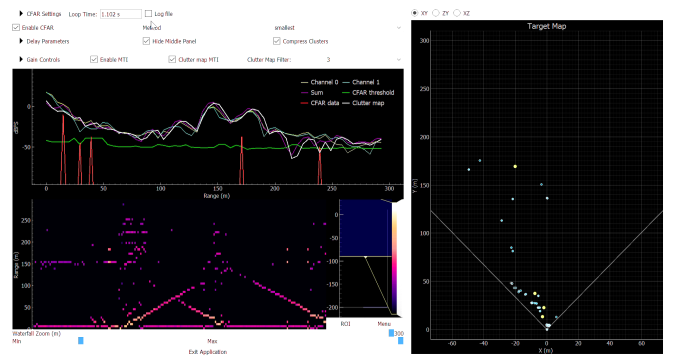


Fig. 57. Radar application display during Experiment #5.

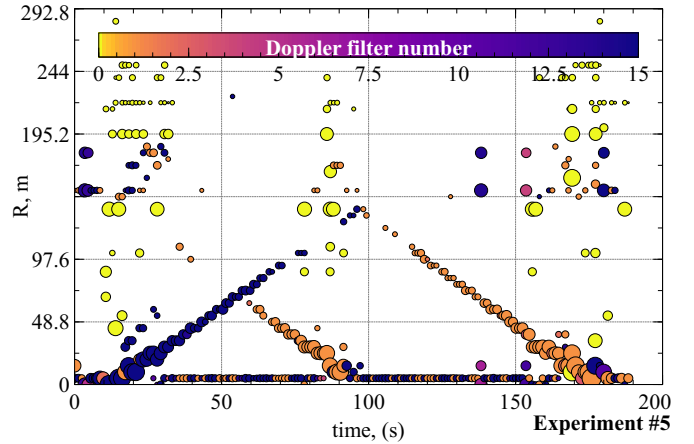


Fig. 58. Estimated radial distance R as a function of time.

walking persons. The Doppler-filter indices distinguish motion away from and toward the radar, with outbound detections concentrated primarily near $k = 15$ and inbound detections concentrated primarily near $k = 1$. The tracks remain continuous over most of the route, indicating reliable target detection despite the reduced angular margin associated with the selected route geometry.

The elevated radar position and the associated reduction in near-ground multipath appear to improve detection continuity compared with Experiment #2. While zero-Doppler detections associated with clutter remain visible, the pedestrian tracks are readily distinguishable from the stationary background responses.

Figures 59 and 60 show the corresponding Cartesian-coordinate estimates. The dominant trajectories form smooth outbound and inbound paths with substantially less scatter than those observed in Experiment #2. This improvement is attributed to the elevated radar installation, the H-plane calibration procedure, and the resulting improvement in phase-comparison monopulse azimuth estimation.

Although the walking route was intentionally positioned close to the horizontal-pattern crossover region, the coordinate tracks remain compact and continuous. The results therefore demonstrate stable operation of the azimuth-estimation algo-

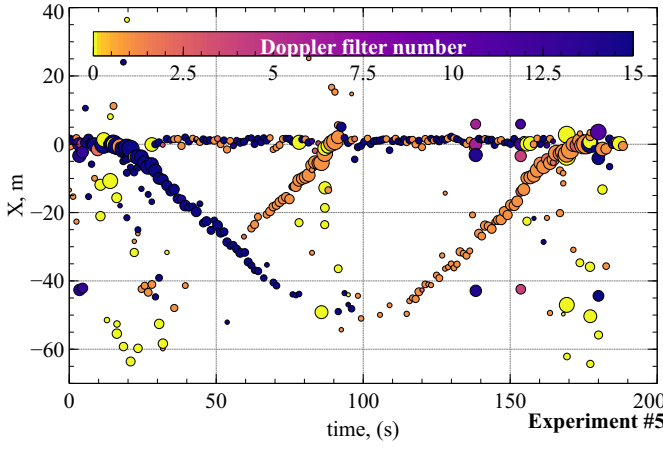


Fig. 59. Estimated target x coordinate as a function of time.

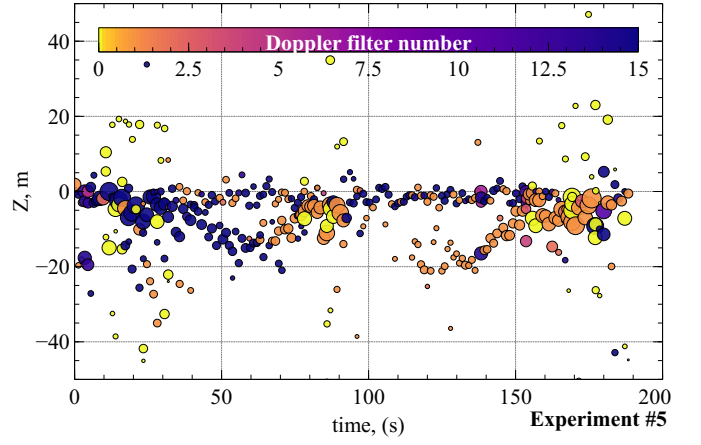


Fig. 61. Estimated target z coordinate as a function of time.

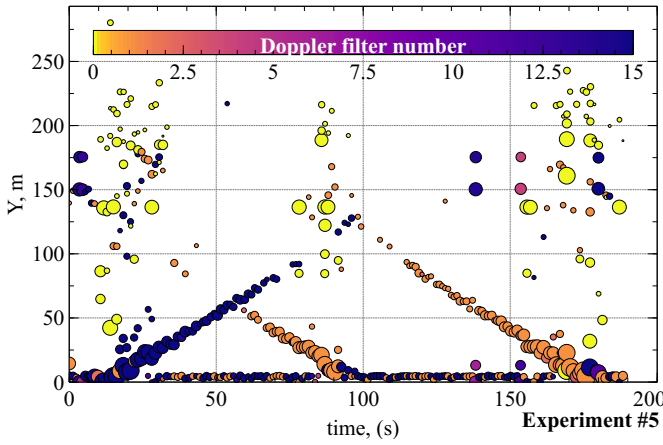


Fig. 60. Estimated target y coordinate as a function of time.

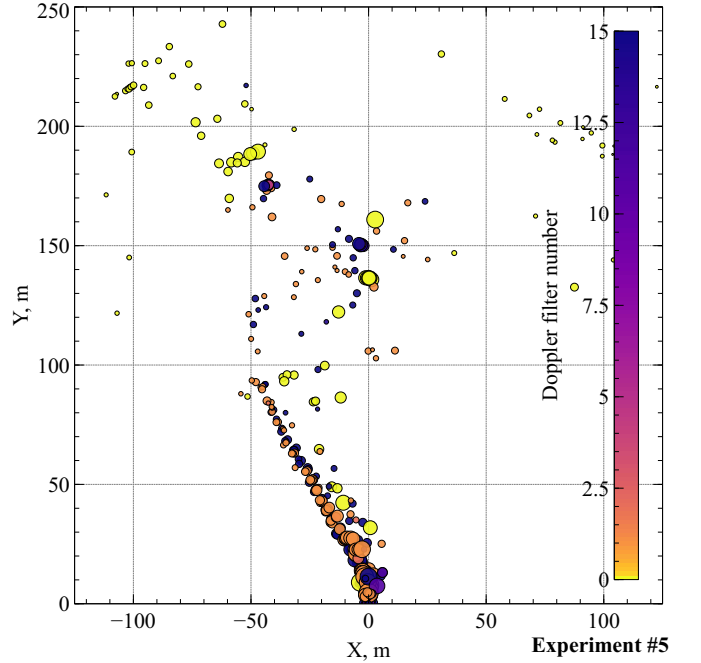


Fig. 62. Estimated target trajectory in the horizontal xy plane.

rithm under conditions that are more sensitive to calibration and angular-estimation errors than those encountered in Experiment #2.

The estimated z -coordinate exhibits noticeably less dispersion than in Experiment #2. The elevated observation point reduces the influence of direct ground reflections and provides a more favorable geometry for elevation estimation. Although scatter remains because of multipath propagation, antenna-pattern errors, and residual channel mismatch, the results indicate improved stability of the amplitude-comparison monopulse elevation estimate.

Because the radar was installed approximately 5 m above sea level, the measured target heights are consistent with targets located near the shoreline below the radar position. The measured height estimates are broadly consistent with the known geometry of the measurement site, in which the radar was located approximately 5 – 10 m above sea level. The results therefore provide qualitative support for the elevation-estimation method, although a quantitative validation of height accuracy was not performed.

Figure 62 presents the estimated target trajectory in the horizontal xy plane. The two pedestrian trajectories follow the

expected route and remain well localized near the shoreline path. Compared with Experiment #2, the trajectory is considerably more compact and exhibits fewer isolated outliers.

No evidence of phase wrapping is apparent in the recorded data from this experiment. The combination of H-plane calibration using the ferry reference object and the elevated radar position appears to improve the robustness of the phase-comparison monopulse azimuth estimate. The results demonstrate compact and qualitatively consistent trajectory formation even when the target route is positioned close to the horizontal-pattern crossover region.

Overall, Experiment #5 demonstrates continuous detection, Doppler discrimination, and two-plane monopulse localization of multiple pedestrian targets in a coastal environment. Compared with Experiment #2, the combination of elevated

radar placement and H-plane boresight calibration substantially reduced angular-estimation scatter and improved trajectory consistency. The experiment therefore provides additional qualitative validation of the proposed calibration procedure and demonstrates reliable pedestrian tracking under realistic coastal operating conditions.

F. Effect of Radar Height

Experiments #1–#3 were conducted with the radar installed approximately 0.6 m above ground level, whereas Experiments #4 and #5 used a radar position approximately 5 m above sea level. The experimental results indicate that the elevated radar position generally improves detection performance and angular-estimation stability.

Several propagation mechanisms contribute to this improvement.

- For a target above a conductive ground plane, the received signal can be modified by constructive and destructive interference between the direct and reflected propagation paths. The corresponding propagation factor is given by [4, Eq. 12.5]

$$\eta^4 = 16 \sin^4 \left(\frac{2\pi h_a h_t}{\lambda_0 R} \right), \quad (88)$$

where h_a is the radar antenna height, h_t is the target height, λ_0 is the free-space wavelength, and R is the target range.

For small grazing angles,

$$\eta^4 \approx 16 \left(\frac{2\pi h_a h_t}{\lambda_0 R} \right)^4. \quad (89)$$

Within the small-argument region represented by Eq. (89), increasing the radar or target height increases the propagation factor. More generally, Eq. (88) predicts an oscillatory interference pattern, and changing either height shifts the locations of constructive and destructive propagation lobes. An elevated installation may nevertheless improve Fresnel-zone clearance and reduce obstruction by nearby objects.

- Another important propagation effect is associated with the first Fresnel zone, illustrated in Fig. 63. The first Fresnel zone defines a region around the direct propagation path that contributes significantly to the received signal. When the radar is installed close to the ground, as shown in Fig. 63a, a substantial portion of the first Fresnel zone intersects the ground or sea surface. In this situation, reflections from the surface and nearby obstacles can introduce additional attenuation, multipath propagation, and phase distortion. Objects intruding into the first Fresnel zone may also reduce the received signal level. When the radar is installed at a greater height, as shown in Fig. 63b, the first Fresnel zone is less obstructed by the ground surface and nearby objects. A larger fraction of the propagated energy therefore follows an unobstructed path, resulting in a more favorable propagation factor,

reduced multipath sensitivity, and improved signal-to-noise ratio.

The experimental results presented in this paper are consistent with these theoretical considerations. The elevated radar installations used in Experiments #4 and #5 produced generally more stable range, azimuth, and elevation estimates than the lower-mounted radar configurations used in Experiments #1–#3.

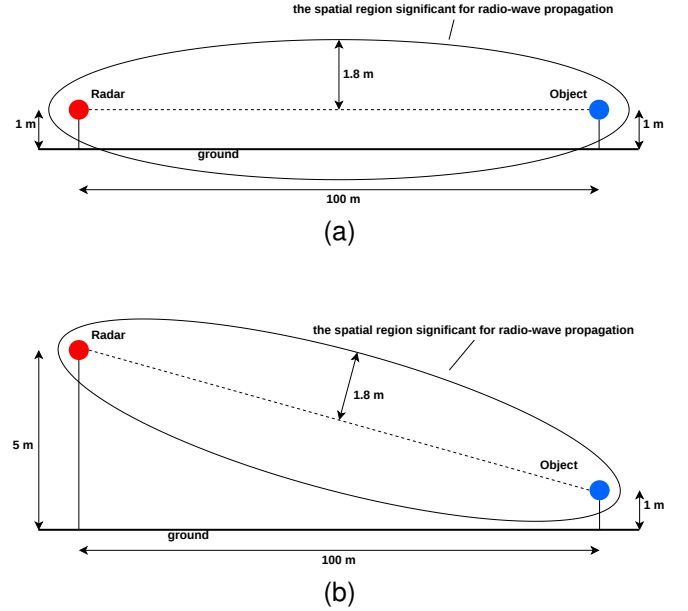


Fig. 63. Effect of radar height on the first Fresnel zone. (a) Low radar installation height. (b) Elevated radar installation height.

XIII. LIMITATIONS AND FUTURE WORK

The presented radar demonstrates pulse-compression ranging, Doppler processing, clutter-map subtraction, CFAR detection, two-plane monopulse localization, and qualitative trajectory formation using a low-cost SDR platform. However, the experimental results also reveal limitations associated with angular calibration, phase ambiguity, multipath propagation, antenna-pattern nulls, host-based processing, and the limited number of coherent receive channels. In particular, additional hardware, calibration, and tracking improvements are required before accurate and quantitatively validated 3D target tracking can be achieved. The following improvements are therefore considered for future development:

- **Four-channel 3D localization and digital beamforming.** Two synchronized ADALM-PLUTO SDRs will be used to provide four coherent receive channels. The additional spatial channels will support independent angle estimation in two orthogonal planes, improved phase-ambiguity resolution, and digital beamforming. This configuration is expected to improve cross-range and height accuracy and provide a more robust basis for quantitative 3D target tracking. The signal-to-spur ratio will also be improved through PlutoSDR hardware modifications

and the use of an external low-phase-noise reference oscillator.

- **Electronically squinted antenna arrays.** The mechanically squinted receive antennas will be replaced by patch-antenna arrays with electronically formed squinted beams implemented through fixed phase shifts or delays in the PCB feed network. This approach is expected to reduce the physical size of the antenna system, improve the repeatability of the monopulse characteristics, and provide greater flexibility in beam configuration. More symmetric antenna patterns and improved control of the phase-center positions should also reduce systematic errors in 3D coordinate estimation.
- **Weather-resistant mechanical design.** Weather-resistant antenna and radar enclosures will be developed to support reliable outdoor operation under varying weather conditions, including rain, wind, humidity, and temperature changes.
- **Improved monopulse calibration and phase-ambiguity resolution.** A calibration procedure will be developed to compensate for channel-dependent gain, phase, delay, antenna alignment, and phase-center errors over the complete field of view. Reference targets at known positions will be used to obtain calibrated two-dimensional monopulse error surfaces rather than relying only on the principal E- and H-plane characteristics. Phase-unwrapping and multi-baseline methods will also be investigated to extend the unambiguous angular range of the phase-comparison monopulse estimator.
- **Target acquisition and temporal 3D tracking.** Track initiation, measurement association, and 3D trajectory estimation will be implemented using Kalman-filter-based methods. Extended or unscented Kalman filters may be used to account for the nonlinear transformation from range and angular measurements to Cartesian coordinates. Machine-learning and artificial-intelligence-based tracking methods will also be investigated. Temporal tracking is expected to reduce coordinate scatter, reject phase-wrapped measurements and isolated false detections, and improve trajectory continuity during temporary target fading or passage through antenna-pattern nulls.
- **Multipath mitigation and measurement-quality evaluation.** Methods for detecting and suppressing multipath-induced angle and range errors will be investigated. The tracking algorithm will assign a confidence or covariance estimate to each 3D position measurement based on the received SNR, monopulse slope, antenna gain, Doppler-filter response, and proximity to phase ambiguities or radiation-pattern nulls. Measurements with poor angular observability can then be rejected or assigned a lower weight during trajectory estimation.
- **Advanced clutter suppression and adaptive Doppler processing.** Further development of clutter-map subtraction, MTI, and adaptive Doppler-processing techniques will be pursued to improve the detection of weak, slowly moving, and partially obscured targets in complex envi-

ronments. Adaptive selection of Doppler filters and CFAR parameters will also be investigated to maintain target tracks in the presence of stationary clutter and varying multipath conditions. In the current implementation, detections from the nonzero-Doppler MTI channels and the zero-Doppler clutter-map MTI channel are compared, and the channel with the highest peak magnitude is selected. Future work will investigate a selection criterion based on the amount by which each detection exceeds its corresponding CFAR threshold. This approach may provide more reliable channel selection when the MTI and clutter-map processing paths have different noise and interference levels.

- **FPGA-based signal processing.** Selected DSP functions currently executed on the host PC will be migrated to the PlutoSDR FPGA. Candidate functions include pulse compression, Doppler-filter-bank processing, channel calibration, and CFAR detection. This migration is expected to reduce data-transfer and processing latency, lower host-computer requirements, and increase the update rate required for real-time 3D target tracking.
- **Quantitative experimental validation of 3D tracking.** Future experiments will evaluate 3D localization and tracking accuracy using an independent position-reference system. The evaluation will include stationary and moving reference targets at known ranges, azimuth angles, and elevation angles. Detection probability, false-alarm probability, coordinate bias, RMS position error, track continuity, update latency, and the effects of SNR, multipath propagation, and antenna-pattern nulls will be measured systematically. These experiments will distinguish qualitative trajectory formation from quantitatively validated 3D tracking performance.

XIV. CONCLUSION

This paper presented a low-cost pulse-compression radar based on a modified ADALM-PLUTO SDR. The implemented processing chain combines a 30.72 MHz-bandwidth LFM waveform, matched filtering, coherent Doppler processing, clutter-map subtraction, CFAR detection, target-cluster compression, and two-plane monopulse localization. Amplitude-comparison monopulse is used to estimate elevation, while phase-comparison monopulse is used to estimate azimuth. These angular estimates are combined with the measured range to calculate Cartesian target coordinates and enable qualitative trajectory formation.

The implementation demonstrates that a compact SDR can support an integrated coherent radar-processing chain while also revealing important limitations associated with full-duplex operation. In particular, TX leakage does not need to saturate the receiver to degrade detection performance. Transmitter-induced spurious components and compressed-pulse sidelobes can raise the post-compression interference floor and mask weak targets. The prototypes mitigate these effects through antenna separation, RF-absorbing material, transmit-waveform weighting, reduced DAC excitation, and

reference-signal subtraction. Software procedures additionally compensate for channel-dependent delays and sample-level timing offsets between coherent time intervals.

Complementary detection paths extend radar operation across different target-motion conditions. Nonzero-Doppler channels provide coherent integration and suppress stationary clutter, whereas the zero-Doppler clutter-map path detects changes associated with newly appearing, displaced, or slowly varying targets. CFAR processing is applied independently to the Doppler channels before filter selection, and adjacent detections are compressed into target clusters to avoid reporting a single extended response as multiple targets.

Experimental evaluation included five outdoor measurements involving pedestrian, vehicle, and maritime targets in urban and coastal environments. The measured detections were consistent with the target routes and enabled qualitative trajectory formation for walking and running persons, a passenger car, and the ferry vessel *Egholm II*. The selected Doppler-filter indices varied consistently with the direction and magnitude of the target radial velocity, and the vehicle experiment produced a broader Doppler-index distribution than the pedestrian experiments because of the higher target speed.

In all experiments, range estimates were generally more stable than cross-range and height estimates. The observed scatter in the calculated Cartesian coordinates is attributed to angular-estimation uncertainty, antenna-pattern nulls, multipath propagation, residual channel mismatch, antenna-alignment errors, and, in some experiments, the absence of reference-object calibration. Phase wrapping in the phase-comparison monopulse azimuth estimate was observed in Experiment #1, demonstrating a fundamental ambiguity of the two-channel azimuth estimator outside its unambiguous angular range. However, this effect was not observed in the subsequent pedestrian, vehicle, and ferry experiments. The measurements therefore demonstrate target detection, Doppler-based radial-velocity discrimination, two-plane monopulse localization, and qualitative trajectory formation under realistic operating conditions, but do not constitute a quantitative validation of three-dimensional tracking accuracy. The ferry experiment additionally provided a qualitative comparison between the estimated trajectory and a known vessel route, offering further support for the localization method.

Analytical models were developed for the antenna patterns, monopulse response, detection range, and thermal-noise-limited localization accuracy. For a target radar cross section of 1 m^2 , 16 coherently integrated chirps, and a total system loss of 20 dB, the model predicts a maximum detection range of approximately 1000 m for the higher-power prototype. This value is a theoretical system-level estimate rather than an experimentally verified detection range and depends on the assumed target characteristics, detection criterion, interference level, propagation conditions, and antenna alignment.

The results also demonstrate a fundamental limitation of fixed-beam amplitude- and phase-comparison monopulse localization using wide-beamwidth antennas. The resulting angle-of-arrival estimates can exhibit substantial uncertainty

in the presence of thermal noise, interference, multipath propagation, and antenna-pattern nulls. Greater angular accuracy may be achieved using narrow-beam antenna arrays, multiple coherent receive channels, and digital multibeam beamforming [34, Chapter 4.5] [6, Chapter 13]. With sufficient computational resources, multiple receive beams can be processed simultaneously without mechanical scanning. This approach can reduce the processing-loop interval, increase the position-update rate, and improve target-localization and tracking performance.

The presented radar and accompanying open-source implementation demonstrate that a low-cost SDR platform can support pulse-compression ranging, coherent Doppler processing, adaptive clutter suppression, CFAR detection, and two-plane monopulse localization within a unified architecture. Together, the analytical, simulation, and experimental results establish commercially available SDR hardware as a useful and accessible platform for radar research, education, and rapid prototyping of advanced sensing and localization systems.

DATA AVAILABILITY

The Python source code for the radar application, together with the supplementary materials supporting the results reported in this paper, is publicly available through IEEE DataPort (DOI: <https://dx.doi.org/10.21227/a9nd-4x08>) and Figshare (DOI: <https://doi.org/10.6084/m9.figshare.33325134>).

APPENDIX A

RADAR ANTENNA CHARACTERISTICS AND MONOPULSE PROCESSING

Each RX or TX channel of the presented radar is connected to a 2×2 patch antenna array. The PAA is practically implemented as printed circuit board (PCB) where power from all microstrip antennas are combined using transmission lines (TL). In the receiver, the two such antennas can form equivalent sum-pattern (antenna array 2×4) and delta-pattern for monopulse target tracking and angle-of-arrival measurement. The RX-channel antennas are squinted in E-plane by angles $\theta_{sq}^{\circ E} \approx \pm \theta_{0.5P}^{\circ E, RX} / 2$ to implement close to optimal amplitude comparison monopulse (ACM) in E-plane and separated by distance $L_x / 2$ to implement optimal phase comparison monopulse (PCM) in H-plane ($z = 0$) [9]–[12]. In the transmitter part, the two TX channels form similar equivalent antenna array 2×4 .

The radar antenna design parameters are provided in Table A.1. The corresponding antenna design files are available online [27].

We can start from single microstrip patch antenna (MPA) which is demonstrated in Fig. A.1. Normalized radiation

TABLE A.1
MPA AND PAA DESIGN PARAMETERS (FIGS. 1, A.1–A.3).

Parameter	Value
f_0	2.425 ± 0.025 GHz
W	36.3 mm
L	27.3 mm
L_e	29.3 mm
h	1.524 mm
ϵ_r	4.7
d_x	61.2 mm
d_y	61.2 mm
d	122.4 mm
L_x	244.9 mm
θ_{sq0}^E (RX)	$\pm 19^\circ$
θ_{sq0}^E (TX)	0°

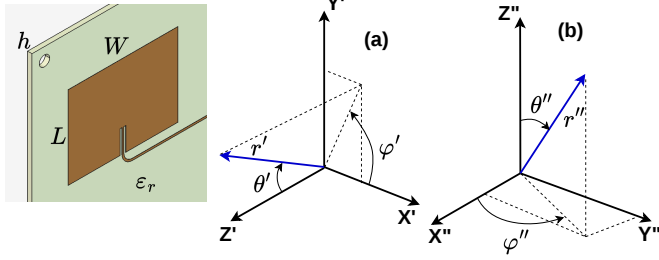


Fig. A.1. Microstrip patch antenna (MPA) with the coordinate system from [37, Fig. 14.16] (a) and with the coordinate system similar to the radar coordinate system except squinted configuration (b).

pattern for MPA can be derived using [37, Eq. 14.44] as follows:

$$E_{MPA}(\theta', \varphi') \approx \sin \theta' \frac{\sin(k_0 \frac{W}{2} \cos \theta')}{k_0 \frac{W}{2} \cos \theta'} \times \cos\left(k_0 \frac{L_e}{2} \sin \theta' \sin \varphi'\right), \text{ if } \sin \theta' \cos \varphi' > 0, \\ E_{MPA}(\theta', \varphi') = 0, \text{ if } \sin \theta' \cos \varphi' \leq 0, \quad (\text{A.1})$$

where angles θ' , φ' are used from original [37, Fig. 14.16] coordinate system which is repeated in Fig. A.1a; $k_0 = 2\pi/\lambda_0$ is an angular wave number; $\lambda_0 = c/f_0$ is a wavelength in free space at frequency f_0 ; W and L_e are width and effective length of the MPA. The MPA is implemented over the dielectric substrate which has the height h and the dielectric constant ϵ_r . The metal ground plane is placed on the bottom side of the PCB. Generally, normalized radiation pattern E_{MPA} can be interpreted as a E-field strength in far-field in TX mode or as voltage induced in a RX mode from far-field source using reciprocal principle [12].

Assuming for this moment that the RX antennas are not squinted, the angles θ' , φ' must be expressed as a function of angles θ'' , φ'' as in Fig. A.1b where the coordinate system

is similar to the radar coordinate system except squinted configuration.

The following rotations must be performed to obtain coordinate system X'' , Y'' , Z'' from X' , Y' , Z' :

- The system X' , Y' , Z' is rotated around X' axis for the angle $\alpha = -\pi/2$. The new axis Z is now on the previous axis Y' position and becomes Z'' .
- Now the updated system is rotated around the new Z position for the angle $\gamma = -\pi/2$. The system X' , Y' , Z' is now fully transferred to the X'' , Y'' , Z'' system.

Thus, the vector $(x'', y'', z'')^T$ defined in the coordinate system X'' , Y'' , Z'' can be expressed in X' , Y' , Z' using passive transformation (vector is fixed in space and coordinate system is transformed [38, Section 14.10-6(b)]) as follows:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = (R_x R_z) \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}, \quad (\text{A.2})$$

where

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}, \quad R_z = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (\text{A.3})$$

where $\alpha = -\pi/2$ and $\gamma = -\pi/2$. Using Eq. (A.2), the following functions can be obtained:

$$\theta'(\theta'', \varphi'') = \arccos(\sin \theta'' \cos \varphi'') \quad (\text{A.4})$$

$$\varphi'(\theta'', \varphi'') = \text{atan2}(\cos \theta'', \sin \theta'' \sin \varphi''), \quad (\text{A.5})$$

which can be substituted in Eq. (A.1) to calculate the radiation pattern of a MPA in the X'' , Y'' , Z'' (r'' , θ'' , φ'') coordinate system (Fig. A.1b). We can continue with patch antenna array (PAA) for one RX or TX channel which is demonstrated in Fig. A.2. Array factor (AF) for a planar PAA can be derived

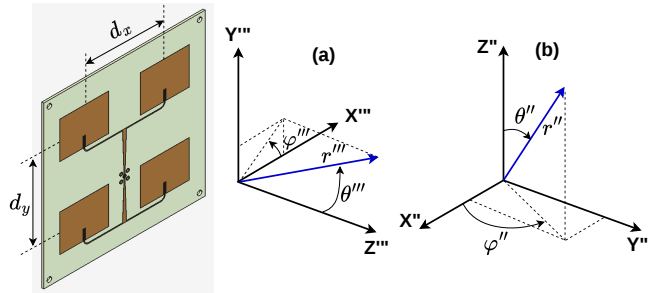


Fig. A.2. Patch antenna array (PAA) with coordinate system from [37, Fig. 6.28] (a) and with the coordinate system similar to the radar coordinate system except squinted configuration (b).

using [37, Eq. 6.88] as follows:

$$AF(\theta''', \varphi''') = \frac{\sin(\frac{M}{2} \Phi_x''') \sin(\frac{N}{2} \Phi_y''')}{\sin(\frac{1}{2} \Phi_x''') \sin(\frac{1}{2} \Phi_y''')}, \quad (\text{A.6})$$

where

$$\Phi_x''' = k_0 d_x \sin \theta''' \cos \varphi''' + \beta_x, \quad (\text{A.7})$$

$$\Phi_y''' = k_0 d_y \sin \theta''' \sin \varphi''' + \beta_y, \quad (\text{A.8})$$

where angles θ''' , φ''' are used from original [37, Fig. 6.26] coordinate system which is repeated in Fig. A.2a; M is an array size in x-direction and N is an array size in y-direction (Fig. A.2a); β_x and β_y are progressive phase shift between elements in x- and y- directions, respectively. Particularly, for the channel's PAA $M = 2$, $N = 2$, $\beta_x = 0$ and $\beta_y = 0$ since phase of a separate MPA in 2×2 array on the PCB cannot be controlled.

Again, assuming for this moment that the RX antennas are not squinted, the angles θ''' , φ''' must be expressed as a function of angles θ'' , φ'' as in Fig. A.2b where the coordinate system is similar to the radar coordinate system except squinted configuration.

The following rotations must be performed to obtain coordinate system X''' , Y''' , Z''' from X'' , Y'' , Z'' :

- The system X'' , Y'' , Z'' is rotated around X'' axis for the angle $\alpha = +\pi/2$. The new axis Y is now on the previous axis Y'' position and becomes Z'' .
- Now the updated system is rotated around the new Y position for the angle $\beta = +\pi$. The system X'' , Y'' , Z'' is now fully transferred to the X''' , Y''' , Z''' system.

Thus, the vector $(x'', y'', z'')^T$ defined in the coordinate system X'' , Y'' , Z'' can be expressed in X''' , Y''' , Z''' using passive transformation (vector is fixed in space and coordinate system is transformed [38, Section 14.10-6(b)]) as follows:

$$\begin{bmatrix} x''' \\ y''' \\ z''' \end{bmatrix} = (R_x R_y) \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}, \quad (\text{A.9})$$

where

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}, \quad R_y = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \quad (\text{A.10})$$

where $\alpha = +\pi/2$ and $\beta = +\pi$. Using Eq. (A.9), the following functions can be obtained:

$$\theta'''(\theta'', \varphi'') = \text{acos}(\sin \theta'' \sin \varphi'') \quad (\text{A.11})$$

$$\varphi'''(\theta'', \varphi'') = \text{atan2}(\cos \theta'', -\sin \theta'' \cos \varphi''), \quad (\text{A.12})$$

which can be substituted in Eqs. (A.6) – (A.8) to calculate the radiation pattern of a PAA in the X'' , Y'' , Z'' (r'' , θ'' , φ'') coordinate system (Fig. A.2b).

Finally, the PAA for RX channels are squinted in E-plane ($x = 0$) as shown in Figs. 1 and A.3. The PAA in Fig. A.2b is rotated around X'' axis by angle $-\theta_{sq}^E$. Equivalently, the coordinate system X'' , Y'' , Z'' is rotated in opposite direction (by angle $\alpha = +\theta_{sq}^E$). In this case, the system X'' , Y'' , Z''

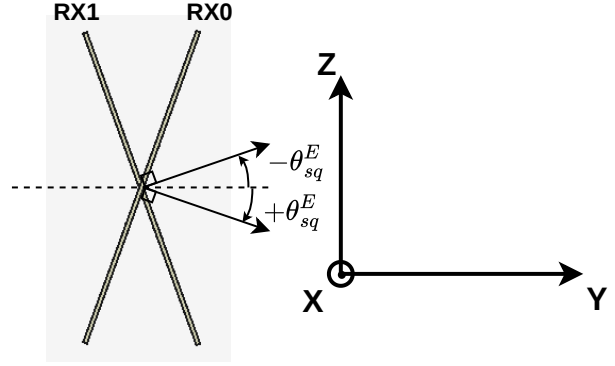


Fig. A.3. Squinted patch antenna arrays (PAA) with the radar coordinate system.

from Fig. A.2b is transformed to the radar coordinate system X, Y, Z from Figs. 1 and A.3. Thus, the vector $(x, y, z)^T$ defined in the coordinate system X, Y, Z can be expressed in X'' , Y'' , Z'' using passive transformation (vector is fixed in space and coordinate system is transformed [38, Section 14.10-6(b)]) as follows:

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = R_x \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad (\text{A.13})$$

where

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}, \quad (\text{A.14})$$

where $\alpha = \theta_{sq}^E$. Using Eq. (A.13), the following functions can be obtained:

$$\theta''(\theta, \varphi, \theta_{sq}^E) = \text{acos}(\sin \theta_{sq}^E \sin \theta \sin \varphi + \cos \theta_{sq}^E \cos \theta) \quad (\text{A.15})$$

$$\varphi''(\theta, \varphi, \theta_{sq}^E) = \text{atan2}(\cos \theta_{sq}^E \sin \theta \sin \varphi - \sin \theta_{sq}^E \cos \theta, \sin \theta \cos \varphi). \quad (\text{A.16})$$

The full radiation pattern of the PAA in Figs. 1 and A.3 with respect to the coordinate system X, Y, Z (r, θ, φ) can now be obtained as in Eq. (A.17) where functions Eqs. (A.15), (A.16), (A.11), (A.12), (A.6), (A.4), (A.5), and (A.1) are also used. Comparison between analytical approach (Eq. (A.17)) and COMSOL Multiphysics® simulation of radiation pattern $E_{PAA}(\theta, \varphi, 0)$ for the one channel non-squinted PAA (Fig. A.2) is demonstrated in Fig. A.4. As shown, Eq. (A.17) can be used to predict antenna characteristics and further monopulse parameters with good agreement.

Sum- and delta- voltage patterns of the RX0 and RX1 antennas in receiving mode in Fig. 1 can now be expressed as follows:

$$E_{\Sigma}^{RX}(\theta, \varphi) = E_{PAA}(\theta, \varphi, +\theta_{sq0}^E) e^{jk_0 \frac{d}{2} \sin \theta \cos \varphi} + E_{PAA}(\theta, \varphi, -\theta_{sq0}^E) e^{-jk_0 \frac{d}{2} \sin \theta \cos \varphi}, \quad (\text{A.18})$$

$$E_{PAA}(\theta, \varphi, \theta_{sq}^E) = E_{MPA}(\theta'(\theta, \varphi, \theta_{sq}^E), \varphi''(\theta, \varphi, \theta_{sq}^E), \varphi'(\theta''(\theta, \varphi, \theta_{sq}^E), \varphi''(\theta, \varphi, \theta_{sq}^E))) \\ \times AF(\theta'''(\theta''(\theta, \varphi, \theta_{sq}^E), \varphi''(\theta, \varphi, \theta_{sq}^E)), \varphi'''(\theta''(\theta, \varphi, \theta_{sq}^E), \varphi''(\theta, \varphi, \theta_{sq}^E))). \quad (A.17)$$

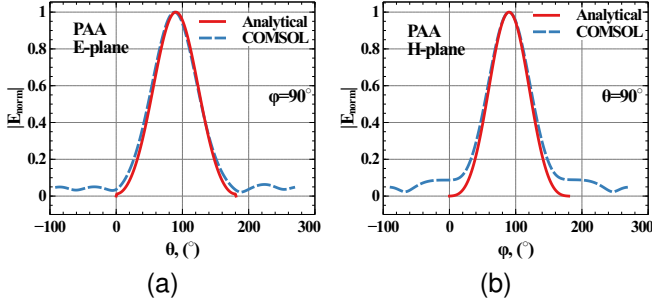


Fig. A.4. Comparison between analytical approach (Eq. (A.17)) and COMSOL Multiphysics® simulation of radiation pattern $E_{PAA}(\theta, \varphi, 0)$ for one channel PAA (Fig. A.2); non-squinted case $\theta_{sq}^E = 0$; (a) In E-plane ($x = 0$); (b) In H-plane ($z = 0$).

$$E_{\Delta}^{RX}(\theta, \varphi) = E_{PAA}(\theta, \varphi, +\theta_{sq0}^E) e^{jk_0 \frac{d}{2} \sin \theta \cos \varphi} \\ - E_{PAA}(\theta, \varphi, -\theta_{sq0}^E) e^{-jk_0 \frac{d}{2} \sin \theta \cos \varphi}, \quad (A.19)$$

where $\pm\theta_{sq0}^E$ are some fixed squint angles so RX0 and RX1 antennas are symmetrically squinted in E-plane ($x = 0$) as demonstrated in Figs. 1 and A.3; d is a distance between phase centers of RX0 and RX1 antennas along the X-axis; $j = \sqrt{-1}$.

The radiation patterns of the squinted RX0/RX1 antennas (Figs. 1 and A.3) obtained using Eqs. (A.17) and (A.18) are demonstrated in Figs. A.5 and A.6.

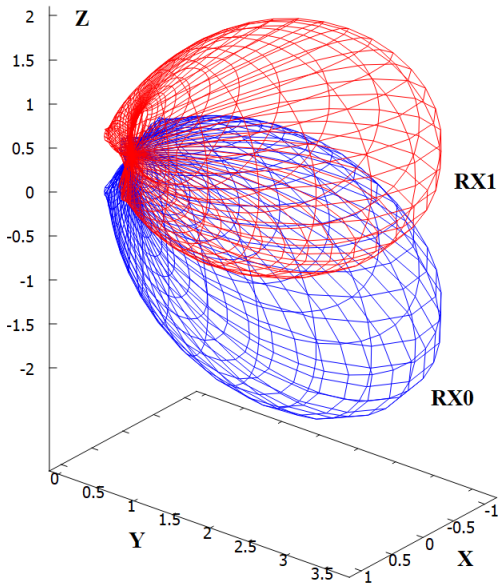


Fig. A.5. Radiation patterns of the RX antennas in Fig. 1.

In principal E-plane ($x = 0$ or $\varphi = \pi/2$ in Fig. 1), where the phase centers of the RX0 and RX1 antennas are coincident, the

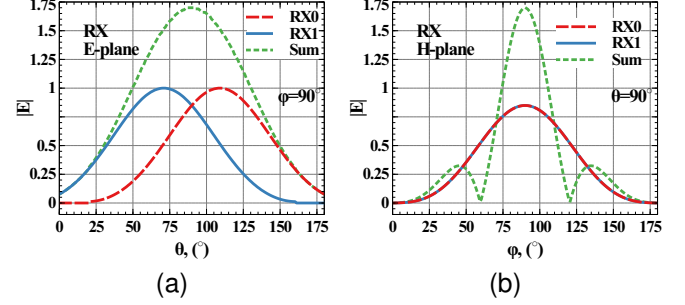


Fig. A.6. Radiation patterns of the squinted RX antennas from Fig. 1; (a) In E-plane ($x = 0$); (b) In H-plane ($z = 0$).

angle-of-arrival is determined by amplitude radiation patterns of antennas [9]–[11] and the following error signal can be used:

$$\varepsilon_E(\theta, \pi/2) = \text{Re} \left(\frac{E_{\Delta}^{RX}(\theta, \pi/2)}{E_{\Sigma}^{RX}(\theta, \pi/2)} \right), \quad (A.20)$$

where for the ideal case and for $\varphi = \pi/2$, the ratio $\frac{E_{\Delta}^{RX}(\theta, \varphi)}{E_{\Sigma}^{RX}(\theta, \varphi)}$ is purely real.

For the more general case, when angle φ stays close to the plane $x = 0$ so $\varphi \approx \pi/2$ but varies in some range (for the presented radar: $70^\circ \lesssim \varphi \lesssim 110^\circ$), the following equation can be used by implementing the non-coherent processing [11, Section 8.10]:

$$\varepsilon_E(\theta, \varphi) \approx \frac{|E_{\Delta}^{RX}(\theta, \varphi) + E_{\Sigma}^{RX}(\theta, \varphi)| - |E_{\Delta}^{RX}(\theta, \varphi) - E_{\Sigma}^{RX}(\theta, \varphi)|}{|E_{\Delta}^{RX}(\theta, \varphi) + E_{\Sigma}^{RX}(\theta, \varphi)| + |E_{\Delta}^{RX}(\theta, \varphi) - E_{\Sigma}^{RX}(\theta, \varphi)|} \\ = \frac{|E_{PAA}(\theta, \varphi, +\theta_{sq0}^E)| - |E_{PAA}(\theta, \varphi, -\theta_{sq0}^E)|}{|E_{PAA}(\theta, \varphi, +\theta_{sq0}^E)| + |E_{PAA}(\theta, \varphi, -\theta_{sq0}^E)|}, \quad (A.21)$$

where the non-coherent processing is more appropriate due to the phase and amplitude distortions in two-channel monopulse used for direction finding in two planes (3D surveillance radar) [9, Section 4.2] [10, Section 1.5]. Four channels are normally necessary for direction finding in two planes. It should be noted that Eq. (A.21) gives the zero error signal at boresight direction in E-plane ($x = 0$) which corresponds to $\theta = \pi/2$ with respect to the radar coordinate system in Fig. 1.

Due to the phase and amplitude distortions in two-channel monopulse used for direction finding in two planes, when angle θ stays close to the plane $z = 0$ in Fig. 1, so $\theta \approx \pi/2$ but varies in some range (for the presented radar: $65^\circ \lesssim \theta \lesssim 115^\circ$), and when the phase centers of the RX0 and RX1 antennas are separated by distance d , the angle-of-arrival

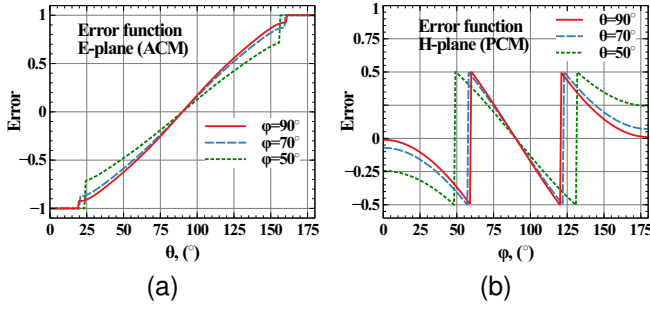


Fig. A.7. Monopulse error signals for the radar from Fig. 1; (a) In E-plane ($x = 0$), ACM, Eq. (A.21); (b) In H-plane ($z = 0$), PCM, Eq. (A.22).

is determined by phase radiation patterns of antennas [9]–[11] and the following error signal can be used:

$$\begin{aligned} \varepsilon_H(\theta, \varphi) &= \frac{1}{k_0 d} \text{Arg} \left(\frac{E_{\Sigma}^{RX}(\theta, \varphi) + E_{\Delta}^{RX}(\theta, \varphi)}{E_{\Sigma}^{RX}(\theta, \varphi) - E_{\Delta}^{RX}(\theta, \varphi)} \right) \\ &= \frac{1}{k_0 d} \text{Arg} \left(\frac{E_{PAA}(\theta, \varphi, +\theta_{sq0}^E) e^{jk_0 \frac{d}{2} \sin \theta \cos \varphi}}{E_{PAA}(\theta, \varphi, -\theta_{sq0}^E) e^{-jk_0 \frac{d}{2} \sin \theta \cos \varphi}} \right) \\ &= \frac{((k_0 d \sin \theta \cos \varphi) \bmod 2\pi - \pi)}{k_0 d}, \quad (\text{A.22}) \end{aligned}$$

assuming the typical monopulse processor argument measurement will wrap the phase in range $(-\pi, \pi \text{ rad}]$. It should be noted that Eq. (A.22) gives the zero error signal at boresight direction in H-plane ($z = 0$) which corresponds to $\varphi = \pi/2$ with respect to the radar coordinate system in Fig. 1.

Error signals obtained by Eqs. (A.21) and (A.22) for the presented radar are demonstrated in Fig. A.7 where phase and amplitude distortions in two-channel monopulse used for direction finding in two planes can be observed in direction different from the principal planes ($x = 0$ and $z = 0$).

Practically, if V_{RX0} and V_{RX1} are channels' complex voltage signals received from the target, the angle-of-arrival (AOA) can be estimated using Eqs. (A.21) and (A.22) as follows. First, we can estimate elevation angle θ_{AOA} of the target using amplitude comparison monopulse in E-plane ($x = 0$):

$$\theta_{AOA} \approx \frac{1}{(k_m / \theta_{0.5P}^{\circ E, \Sigma, RX})} \left(\frac{|V_{RX0}| - |V_{RX1}|}{|V_{RX0}| + |V_{RX1}|} \right) + \frac{\pi}{2}, \quad (\text{A.23})$$

where $\theta_{0.5P}^{\circ E, \Sigma}$ is a sum-pattern half-power beamwidth of the RX antenna; and

$$k_m = \frac{d(\Delta/\Sigma)}{d(\theta/\theta_{0.5P}^{\circ E, \Sigma})} \quad (\text{A.24})$$

is a *normalized monopulse slope* in E-plane ($x = 0$) [11, Section 5.1.3] [35, Eq. 8.5] which can be estimated from Fig. A.7a.

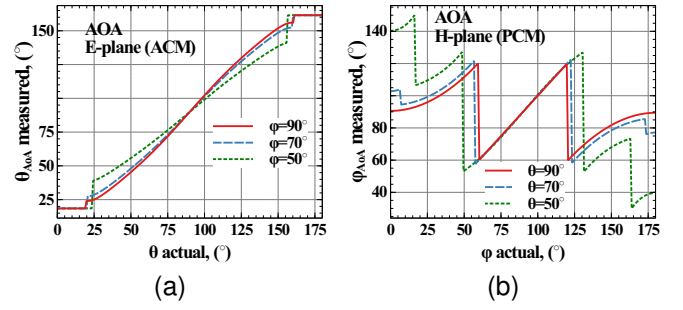


Fig. A.8. Angles-of-Arrival (AOA) which are measured by monopulse radar from Fig. 1 depending on actual target position angles; (a) In E-plane ($x = 0$), ACM, Eq. (A.23); (b) In H-plane ($z = 0$), PCM, Eq. (A.25).

Now, elevation angle θ_{AOA} can be used to correct azimuth angle φ_{AOA} which is estimated using phase comparison monopulse in H-plane ($z = 0$):

$$\begin{aligned} \varphi_{AOA} &\approx \text{acos} \left(\frac{\text{Arg} \left(\frac{V_{RX0}}{V_{RX1}} \right)}{k_0 d \sin \theta_{AOA}} \right) \\ &= \text{acos} \left(\frac{\text{atan2} \left(\text{Im} \left(\frac{V_{RX0}}{V_{RX1}} \right), \text{Re} \left(\frac{V_{RX0}}{V_{RX1}} \right) \right)}{k_0 d \sin \theta_{AOA}} \right) \quad (\text{A.25}) \end{aligned}$$

Figure A.8 demonstrates the measured AOA θ_{AOA} and φ_{AOA} as functions of the actual target-coordinate angles θ and φ . As shown in Fig. A.8b, φ_{AOA} can be estimated with higher precision independently from θ_{AOA} in some range $60^\circ \lesssim \varphi \lesssim 120^\circ$ due to $1/\sin \theta_{AOA}$ term in Eq. (A.25). Ambiguity outside the range $60^\circ \lesssim \varphi \lesssim 120^\circ$ is caused by V_{RX0}/V_{RX1} phase difference $|\Delta\phi| = |k_0 d \sin \theta \cos \varphi|$ which becomes more than π for incident wave so phase wrapping occurs.

Knowing AOA θ_{AOA} and φ_{AOA} , the estimated Cartesian coordinates of the target can be obtained:

$$x_t = r_t \sin(\theta_{AOA}) \cos(\varphi_{AOA}), \quad (\text{A.26})$$

$$y_t = r_t \sin(\theta_{AOA}) \sin(\varphi_{AOA}), \quad (\text{A.27})$$

$$z_t = r_t \cos(\theta_{AOA}), \quad (\text{A.28})$$

where r_t is radial distance to the target measured by the radar.

The calculated characteristics obtained using Eqs. (A.1), (A.17) and (A.18) are summarized in Table A.2. The TX antenna characteristics are calculated in the same way as for RX antennas but assuming the nonsquinted antennas for TX0 and TX1 so $\theta_{sq}^E = 0$ as can be seen in Fig. 1. The simulated characteristics obtained using COMSOL Multiphysics® are summarized in Table A.3. Antenna maximum gain is obtained by multiplying directivity by efficiency of antenna. Efficiency is obtained by simulation of one channel PAA array in COMSOL Multiphysics® solver. The directivity of the RX antenna sum pattern is calculated by substituting Eq. (A.18) into the

TABLE A.2
ANTENNAS CALCULATED CHARACTERISTICS. ANALYTICAL APPROACH.

Parameter	Value
Single MPA beam width, E-plane $\theta_{0.5P}^{\circ E, MPA}$	90°
Single MPA beam width, H-plane $\theta_{0.5P}^{\circ H, MPA}$	82°
Single MPA maximum directivity D_{max}^{MPA}	6 dB
One channel PAA beam width, E-plane $\theta_{0.5P}^{\circ E, PAA}$	56°
One channel PAA beam width, H-plane $\theta_{0.5P}^{\circ H, PAA}$	51°
One channel maximum directivity D_{max}^{PAA}	11.3 dB
RX sum-pattern beam width, E-plane $\theta_{0.5P}^{\circ E, \Sigma, RX}$	66°
RX sum-pattern beam width, H-plane $\theta_{0.5P}^{\circ H, \Sigma, RX}$	26°
RX sum-pattern maximum directivity D_{max}^{RX}	12.8 dB
TX sum-pattern beam width, E-plane $\theta_{0.5P}^{\circ E, \Sigma, TX}$	56°
TX sum-pattern beam width, H-plane $\theta_{0.5P}^{\circ H, \Sigma, TX}$	26°
TX sum-pattern maximum directivity D_{max}^{TX}	14.1 dB
Normalized monopulse slope in E-plane k_m	≈ 0.92

TABLE A.3
ANTENNAS CALCULATED CHARACTERISTICS. COMSOL MULTIPHYSICS®
SIMULATION.

Parameter	Value
Single MPA beam width, E-plane $\theta_{0.5P}^{\circ E, MPA}$	92.7°
Single MPA beam width, H-plane $\theta_{0.5P}^{\circ H, MPA}$	88.7°
Single MPA maximum directivity D_{max}^{MPA}	6.5 dB
Single MPA maximum gain G_{max}^{MPA}	5.7 dB
Single MPA efficiency η^{MPA}	82%
One channel PAA beam width, E-plane $\theta_{0.5P}^{\circ E, PAA}$	56°
One channel PAA beam width, H-plane $\theta_{0.5P}^{\circ H, PAA}$	52.5°
One channel PAA maximum directivity D_{max}^{PAA}	10.8 dB
One channel PAA maximum gain G_{max}^{PAA}	9.9 dB
One channel PAA efficiency η^{PAA}	81%

following expression and performing numerical integration [12, Eq. (2.24)]:

$$D^{RX}(\theta, \varphi) = \frac{4\pi |E_{\Sigma}^{RX}(\theta, \varphi)|^2}{\int_0^{2\pi} \int_0^\pi |E_{\Sigma}^{RX}(\theta, \varphi)|^2 \sin \theta d\theta d\varphi}, \quad (\text{A.29})$$

The directivity of the TX antenna sum pattern is calculated analogously, with the squint angle set to zero.

Design and simulation files for the antennas are available online [28] [29].

APPENDIX B

EFFECT OF ANTENNA MISMATCH ON TRANSMITTED AND RECEIVED POWER

A mismatch between the transmitter and the TX antenna reduces the power delivered to the antenna and therefore introduces an additional loss factor in the transmitted signal path. As illustrated on the left-hand side of Fig. B.1, the

normalized power delivered by the amplifier or driver to the TX antenna can be expressed as

$$p_{TX} = \frac{4r}{(1+r)^2}, \quad (\text{B.1})$$

where p_{TX} is the TX power normalized to the maximum available power, such that $p_{TX} = 1$ for $r = 1$; $r = R_{\Sigma}/R_0$; R_{Σ} is the antenna radiation resistance in the transmitting mode, referred to the antenna feed terminals; and R_0 is the output resistance of the amplifier stage, assumed to be equal to the characteristic impedance of the connected transmission line.

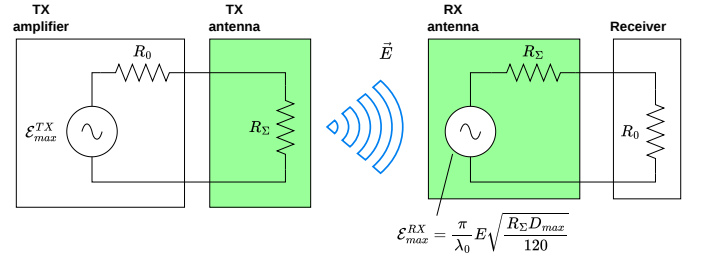


Fig. B.1. Equivalent circuits illustrating the EMFs and power transfer in the TX and RX antenna modes.

The following assumptions are made for both the TX antenna model described above and the subsequent analysis of the same antenna in the receiving mode:

- The antennas operate at resonance; therefore, the reactive component of the antenna input impedance is zero.
- The antennas are lossless; therefore, the radiation resistance is equal to the real part of the antenna input impedance.
- The transmission line connected to the antenna is lossless and has a purely real characteristic impedance R_0 .
- The amplifier output resistance and the load impedance of the receiver are matched to the characteristic impedance R_0 of the transmission line.

At first glance, one might attempt to derive an expression for the receiving mode analogous to Eq. (B.1) by simply interchanging R_{Σ} and R_0 in the equivalent circuit shown on the left-hand side of Fig. B.1. However, this substitution alone leads to a different expression. To obtain the correct result, it is also necessary to account for the dependence of the antenna electromotive force (EMF) in the receiving mode on R_{Σ} [12, Eq. (3.9a)]:

$$\mathcal{E}_{max}^{RX} = \frac{\pi}{\lambda_0} E \sqrt{\frac{R_{\Sigma} D_{max}}{120}}, \quad (\text{B.2})$$

where E is the incident electric-field strength and D_{max} is the maximum directivity of the antenna in the transmitting mode.

Taking this dependence into account, the normalized power delivered by the RX antenna to the receiver load, as illustrated on the right-hand side of Fig. B.1, can be expressed as

$$p_{RX} = \frac{4r}{(1+r)^2}. \quad (\text{B.3})$$

Thus, the dependence of the normalized power-transfer efficiency on the impedance ratio r is identical in the transmitting and receiving modes, as shown by Eqs. (B.1) and (B.3).

APPENDIX C ANGLE ACCURACY VS. SNR FOR PHASE COMPARISON MONOPULSE

Equations (39) and (40) are derived in [4], [34], [35] for an angle-tracking system employing two receive (RX) channels, amplitude-comparison monopulse (ACM) processing, and a sum-difference error function. The derivation assumes that the two antennas have identical phase radiation patterns. Consequently, the angle estimate is obtained from the ratio of the difference (Δ) pattern to the sum (Σ) pattern of the received signals.

In [34], [35], phase-comparison monopulse is further treated as a special case of amplitude-comparison monopulse, in which the radiation patterns of the two receive channels also differ in phase. The same sum-difference error function is used; therefore, the angular errors due to thermal noise have the same form as those given by Equations (39) and (40).

However, in the present work, the H-plane angle is estimated from the phase difference between the RX channels using Equation (74), as derived in Appendix A. Therefore, in this section, the angular error due to thermal noise is derived for the proposed phase-difference method and compared with the errors given by Equations (39) and (40).

First, consider a two-channel receiver intended for phase-comparison monopulse (PCM) tracking. The receiver employs two wide-beamwidth antennas separated by a distance d . For the optimum monopulse configuration, the total aperture size is assumed to be $L_x = 2d$ [9]–[12], as illustrated in Fig. C.1.

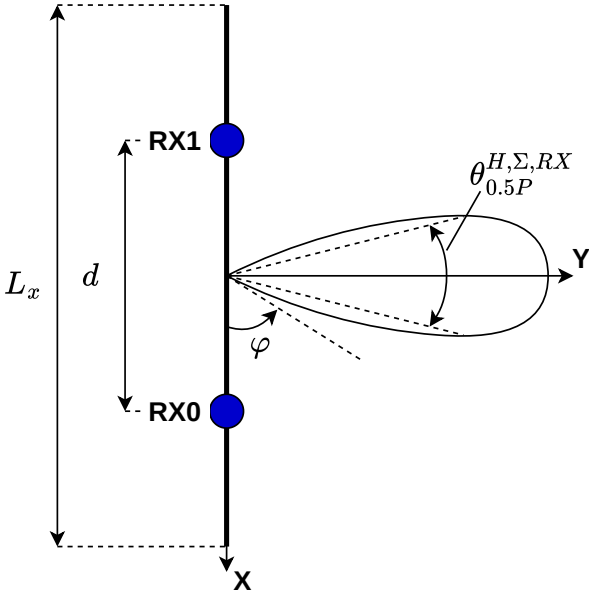


Fig. C.1. Two-channel receiver configuration for PCM angle tracking.

The angle of arrival φ in the principal H-plane can be expressed similarly to Equation (74) as

$$k_0 d \cos \varphi = \phi_0 - \phi_1, \quad (\text{C.1})$$

where ϕ_0 and ϕ_1 are the respective signal phases obtained from the complex I/Q data of channels RX0 and RX1, k_0 is the free-space wavenumber, and d is the separation between the two antennas.

The angular error caused by the phase-measurement errors can be obtained by differentiating Equation (C.1):

$$-k_0 d \sin \varphi \delta \varphi = \delta \phi_0 - \delta \phi_1, \quad (\text{C.2})$$

where $\delta \phi_0$ and $\delta \phi_1$ are the phase-measurement errors in channels RX0 and RX1, respectively.

Assuming that the phase errors in the two RX channels are statistically independent, have equal variances, and are caused by thermal noise, the variance of their difference is

$$\text{var}(\delta \phi_0 - \delta \phi_1) = 2\sigma_\phi^2, \quad (\text{C.3})$$

where σ_ϕ is the RMS phase-measurement error in one RX channel. Consequently, the RMS angular error is

$$\sigma_\varphi = \frac{\sqrt{2}}{k_0 d |\sin \varphi|} \sigma_\phi. \quad (\text{C.4})$$

For measurements near the boresight direction, $\varphi \approx \pi/2$, and hence $|\sin \varphi| \approx 1$. Therefore, Equation (C.4) reduces to

$$\sigma_\varphi \approx \frac{\sqrt{2}}{k_0 d} \sigma_\phi. \quad (\text{C.5})$$

The relationship between the SNR in one RX channel, S/N , and the RMS phase-measurement error is given in [39] as

$$\sigma_\phi = \frac{1}{\sqrt{2(S/N)}}. \quad (\text{C.6})$$

Using the SNR referenced to the sum-channel signal, $(S/N)_1^\Sigma$, as defined in Section VII, with

$$(S/N)_1^\Sigma = 2(S/N), \quad (\text{C.7})$$

the phase-measurement error can equivalently be written as

$$\sigma_\phi = \frac{1}{\sqrt{(S/N)_1^\Sigma}}. \quad (\text{C.8})$$

The factor $k_0 d$ in Equation (C.5) can be expressed in terms of the half-power beamwidth of the RX sum-channel pattern in the H-plane, denoted by $\theta_{0.5P}^{H, \Sigma, RX}$.

For the two-element array shown in Fig. C.1, the sum-channel array factor can be written as [12], [37]

$$AF_2(\varphi) = \frac{\sin(k_0 d \cos \varphi)}{\sin(\frac{1}{2} k_0 d \cos \varphi)} = 2 \cos\left(\frac{1}{2} k_0 d \cos \varphi\right). \quad (\text{C.9})$$

The array factor is normalized to its maximum value of two. Therefore, at the half-power points,

$$\frac{1}{\sqrt{2}} = \cos\left[\frac{1}{2} k_0 d \sin\left(\frac{\theta_{0.5P}^{H, \Sigma, RX}}{2}\right)\right], \quad (\text{C.10})$$

where $\theta_{0.5P}^{H,\Sigma,RX}$ is the full half-power beamwidth of the RX sum-channel pattern in the H-plane. It follows that

$$\frac{\pi}{4} = \frac{1}{2} k_0 d \sin\left(\frac{\theta_{0.5P}^{H,\Sigma,RX}}{2}\right). \quad (C.11)$$

Thus, the exact relation between the electrical element spacing and the half-power beamwidth is

$$k_0 d = \frac{\pi}{2 \sin\left(\frac{\theta_{0.5P}^{H,\Sigma,RX}}{2}\right)}. \quad (C.12)$$

For a sufficiently narrow beam, using $\sin(\theta/2) \approx \theta/2$, where the beamwidth is expressed in radians, Equation (C.12) becomes

$$k_0 d \approx \frac{\pi}{\theta_{0.5P}^{H,\Sigma,RX}}. \quad (C.13)$$

Substituting Equations (C.13) and (C.8) into Equation (C.5) gives the following relationship between the RMS angular error and the sum-channel SNR:

$$\sigma_\varphi \approx \frac{\sqrt{2}}{\pi} \frac{\theta_{0.5P}^{H,\Sigma,RX}}{\sqrt{(S/N)_1^\Sigma}} \approx \frac{0.45 \theta_{0.5P}^{H,\Sigma,RX}}{\sqrt{(S/N)_1^\Sigma}}. \quad (C.14)$$

Equation (C.14) has the same form as the angle-error relations presented in [4], [34], [35] and given by Equations (39) and (40). Therefore, under the stated assumptions, the RMS angular error due to thermal noise has the same dependence on the sum-channel SNR and the sum-channel half-power beamwidth for both amplitude-comparison monopulse (ACM) and phase-comparison monopulse (PCM). In particular, the angular error is proportional to the sum-channel beamwidth and inversely proportional to the square root of the sum-channel SNR.

APPENDIX D

RADAR APPLICATION INTERFACE AND STARTUP PROCEDURE

The graphical user interface (GUI) used to configure the radar, monitor the signal-processing chain, and display detected targets is shown in Fig. D.1. The principal interface regions are identified by numbered rectangles. The controls and displays contained in each region are described below. The *CFAR Settings*, *Delay Parameters*, and *Gain Controls* panels are expandable. Selecting a panel heading displays or hides the controls contained within that panel. While the application is running, it prevents the PC from activating the screen saver or entering sleep mode.

1) CFAR detection controls

This region contains the parameters used by the range-domain constant-false-alarm-rate detector.

Bias: Sets the detection-threshold offset, in decibels, above the background level estimated from the CFAR reference cells.

Clutter Bias: Sets the detection-threshold offset for the clutter-map processing path. This parameter is independent of the bias used for the nonzero-Doppler MTI channels.

Guard: Sets the number of guard cells placed on each side of the cell under test. These cells prevent the target response and nearby range sidelobes from contributing directly to the background estimate.

Ref: Sets the number of reference cells used on each side of the cell under test to estimate the local background level.

Enable CFAR: Enables or disables CFAR-based target detection. Disabling CFAR also disables target-data logging.

Method: Selects the statistic used to combine the two CFAR reference windows. The available options are *average*, *greatest*, *smallest*, and *none*. The *greatest* and *smallest* methods use, respectively, the larger or smaller estimate obtained from the reference cells on the two sides of the cell under test.

2) Channel-delay and processing controls

This region contains the interchannel delay-calibration controls and two general processing options.

Delay RX1 (ps): Applies a carrier-phase correction corresponding to the specified delay in RX1. This control compensates for the additional cable and hardware delay of RX1 and calibrates the receive-channel phase difference.

Delay TX1 (ps): Applies the specified delay correction to the TX1 waveform. Changing this parameter regenerates the cyclic transmit buffer using the updated TX-channel delay.

3) RF gain, MTI, and clutter-map controls

RX Gain 0: Sets the hardware gain of receive channel RX0. The permitted application range is -3 to 70 dB.

RX Gain 1: Sets the hardware gain of receive channel RX1. The permitted application range is -3 to 70 dB.

TX Gain 0: Sets the hardware attenuation parameter of transmit channel TX0. The permitted range is -88 to 0 dB, where 0 dB corresponds to the maximum configured output level.

TX Gain 1: Sets the hardware attenuation parameter of transmit channel TX1. The permitted range is -88 to 0 dB.

Changing any RX or TX gain reinitializes the CTI-alignment reference using the first RX0 chirp of the most recently acquired CTI. Subsequent CTIs are aligned with this reference by cross-correlation, thereby preventing gain-dependent waveform changes from affecting the sample-shift estimate.

4) Transmit-reference compensation

These controls adjust the stored reference chirp subtracted from the received signals to reduce the deterministic transmit-leakage component.

TX Ref Mag: Sets the magnitude scaling factor applied to the stored transmit reference before subtraction. The selectable range is 0.00 to 1.00 .

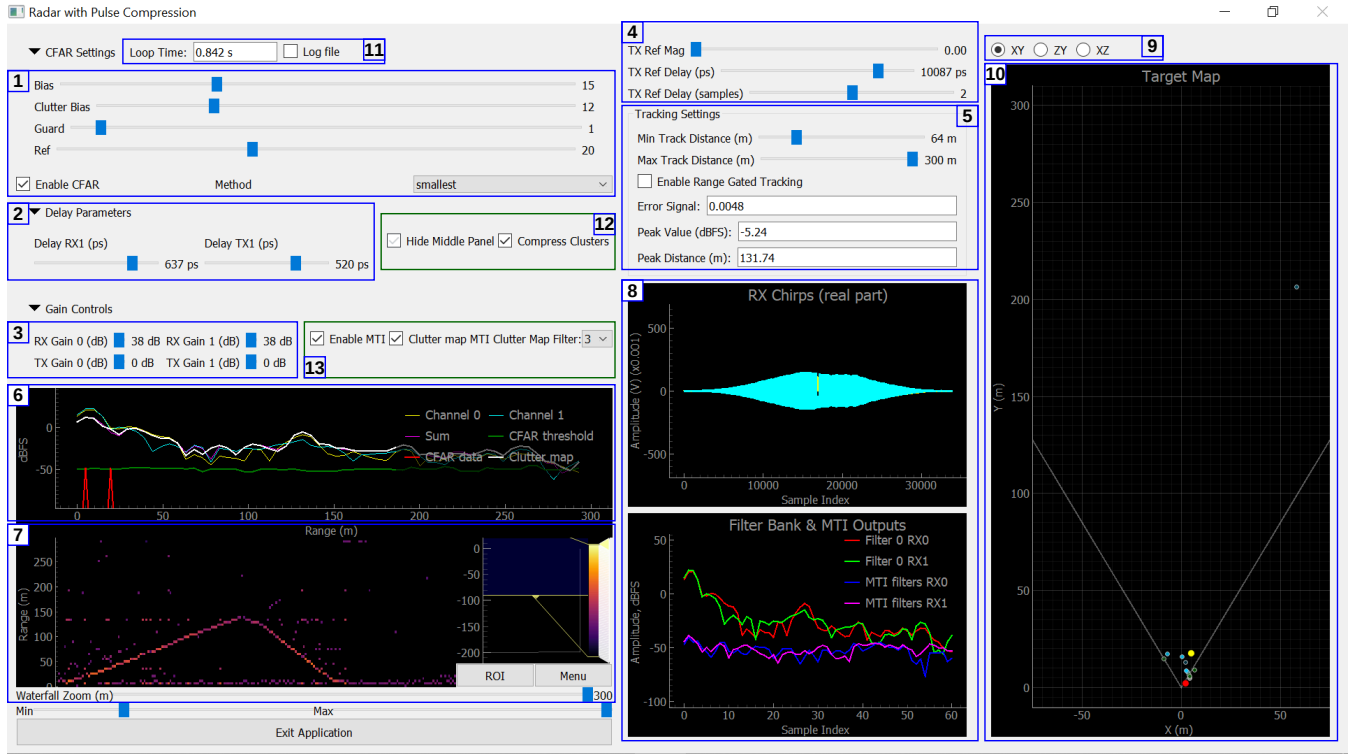


Fig. D.1. Radar application GUI. The numbered regions identify the principal controls and displays described in Appendix D. The intermediate diagnostic panel can be hidden to enlarge the target-position display.

TX Ref Delay (ps): Applies a fine carrier-phase correction to the stored transmit-reference chirp before subtraction.

TX Ref Delay (samples): Provides coarse alignment of the transmit reference by shifting it by an integer number of samples. The implemented range is -200 to $+200$ samples.

The sample-delay control provides coarse temporal alignment, whereas the picosecond control provides fine phase-related adjustment.

5) Reference-target tracking controls and status

Min Track Distance: Defines the lower boundary of the reference-target range gate. If it exceeds the current maximum, the maximum-distance setting is increased automatically.

Max Track Distance: Defines the upper boundary of the reference-target range gate. If it falls below the current minimum, the minimum-distance setting is reduced automatically.

Enable Range Gated Tracking: Enables automatic tracking of the strongest valid reference target detected within the selected range gate.

Error Signal: Displays the normalized interchannel phase error used by the phase-comparison monopulse tracking loop.

Peak Value (dBFS): Displays the magnitude of the selected reference-target response in dBFS.

Peak Distance (m): Displays the measured radial distance to the selected reference target.

6) Range-profile display

The range-profile plot displays processed signal magnitude as a function of target range.

Channel 0: Displays the processed range response of RX0.

Channel 1: Displays the processed range response of RX1.

Sum: Displays the combined response of the two receive channels.

CFAR threshold: Displays the adaptive range-dependent detection threshold calculated by the CFAR processor.

CFAR data: Displays the signal supplied to the CFAR detector or the corresponding detection response.

Clutter map: Displays the persistent-background estimate obtained from the zero-Doppler processing channel.

The horizontal axis represents range in metres, and the vertical axis represents signal magnitude in dBFS.

7) Range-time waterfall display and display controls

The waterfall contains a history of 200 processed range profiles. Each update adds one profile, allowing target range variation and detection history to be observed. When CFAR is disabled, the waterfall is formed from the combined receive-channel response; when CFAR is enabled, it is formed from the CFAR output.

Waterfall Zoom (m): Sets the maximum range displayed in the waterfall. The displayed interval extends from zero to the selected range.

Min: Sets the lower magnitude limit of the waterfall color scale in dBFS.

Max: Sets the upper magnitude limit of the waterfall color scale in dBFS.

ROI: Activates the region-of-interest controls provided by the `ImageView` widget.

Menu: Opens the display menu provided by the `ImageView` widget.

The *Waterfall Zoom* control changes the displayed range, whereas *Min* and *Max* change only the color mapping.

8) Intermediate diagnostic displays

This region is updated only when the intermediate panel is visible.

RX Chirps (real part): Displays the real parts of the most recently acquired chirps from RX0 and RX1. The horizontal axis is the sample index, and the vertical axis is ADC-input-equivalent amplitude in volts. The displayed range is -0.625 to $+0.625$ V.

Filter 0 RX0: Displays the zero-Doppler filter output for RX0.

Filter 0 RX1: Displays the zero-Doppler filter output for RX1.

MTI filters RX0: Displays the combined response of the enabled nonzero-Doppler filters for RX0.

MTI filters RX1: Displays the combined response of the enabled nonzero-Doppler filters for RX1.

The filter-bank and MTI responses are shown in dBFS as functions of sample index.

9) Target-map plane selector

XY: Displays horizontal cross-range position X against down-range position Y . This is the default view.

ZY: Displays vertical position Z against down-range position Y .

XZ: Displays horizontal position X against vertical position Z .

10) Target-position display

The target map displays the estimated Cartesian target coordinates in the plane selected in Region 9. The display history is currently hard-coded to 20 CTIs for the XY, ZY, and XZ views. Target-marker size remains constant. The detections from the newest CTI are shown in yellow, whereas detections from the preceding CTIs are shown in blue with progressively fading brightness as their age increases. In the XY view, the two grey lines indicate the approximate boundaries of the unambiguous angular region determined by the receive-antenna spacing. Detected targets approaching these angular boundaries are displayed using a gradual yellow-to-red color transition, whereas targets beyond the boundaries are marked in red. The visible coordinate ranges are

derived from the configured maximum range and the approximate H-plane and E-plane antenna beamwidths.

11) Timing and data-logging controls

Loop Time: Displays the measured duration of the preceding acquisition, signal-processing, detection, tracking, and display-update cycle.

Log file: Enables writing detected-target data to a timestamped CSV file. This control is available only while CFAR is enabled. A new file is created whenever logging is enabled.

Exit Application: Stops the update timer, closes the application, restores the computer sleep settings, and terminates the Qt application.

For each logged target, the application stores the timestamp, range sample index, estimated X , Y , and Z coordinates, selected Doppler-filter index, and sum-channel magnitude in dBFS.

12) Middle-panel and target-cluster controls

Hide Middle Panel: Hides or displays the middle diagnostic panel, which contains the transmit-reference compensation controls, reference-target tracking controls, received-chirp display, and Doppler-processing displays. Hiding the panel provides additional space for the target-position display.

Compress Clusters: Enables target-cluster compression. Within each sequence of adjacent detected range bins, only the bin having the greatest signal magnitude is retained. This prevents an extended target response or range sidelobes from being reported as multiple targets. However, unresolved targets occupying adjacent range bins may be merged into a single detection.

13) MTI and clutter-map controls

Enable MTI: Enables the nonzero-Doppler moving-target-indication processing path. In this mode, CFAR detection is applied independently to the enabled nonzero-Doppler filter outputs to detect targets having nonzero radial velocity.

Clutter map MTI: Enables the complementary clutter-map processing path. This path detects newly appearing, displaced, or slowly varying targets whose responses occur in or near the zero-Doppler channel.

Clutter Map Filter: Selects one of the three implemented clutter-map subtraction methods described in Section IX.

The GUI is updated after every coherent time interval. Each update acquires and processes the received data, updates the range profile and waterfall, estimates target coordinates, updates the tracking quantities, and optionally writes detected-target information to the log file.

The following procedure provides an example of how the principal radar parameters may be adjusted after starting the application. The exact settings depend on the radar hardware, antenna configuration, and operating environment.

- 1) Set *TX Gain 0* to its maximum value of 0 dB.

- 2) If TX1 is used to increase the transmitted power, set *TX Gain 1* to its maximum value of 0 dB.
- 3) Adjust *Delay TX1* manually to obtain coherent addition of the TX0 and TX1 signals and thereby maximize the combined transmitted-signal level. The resulting signal level can be monitored using the RX0 and RX1 traces in the range-profile display in Region 6. During this adjustment, the received signals should also be monitored to ensure that the ADC is not saturated.
- 4) For the presented radar prototypes, the required *Delay TX1* setting includes compensation for the additional TX1 cable delay. The optimum startup setting was observed to alternate between approximately 500 and 750 ps. This variation appears to depend on the transmitter phase state after startup, although its relationship to the PLL state has not been verified experimentally.
- 5) Changing *Delay TX1* or any RX or TX gain automatically reinitializes the reference chirp used for CTI alignment. The first RX0 chirp of the newly acquired CTI is stored as the new reference, and subsequent CTIs are aligned with this reference by cross-correlation.
- 6) Adjust *RX Gain 0* and *RX Gain 1* so that the receiver noise floor remains sufficiently above the intrinsic ADC noise floor, while the transmit-leakage signal and strong target responses remain below the ADC saturation level. For the presented radar, an RX gain of approximately 30–40 dB provides a practical initial setting.
- 7) Adjust *Delay RX1* to compensate for the additional RX1 cable and hardware delay. This adjustment provides the initial calibration of the relative phase between RX0 and RX1 required for phase-comparison monopulse processing.
- 8) For further receive-channel calibration, use a suitable reference object at a known direction. Clear the *Hide Middle Panel* checkbox to display the tracking controls. Set the minimum and maximum tracking distances to define a range gate containing the reference object, and then enable *Range Gated Tracking* in Region 5. The application automatically adjusts *Delay RX1* using the measured phase-comparison monopulse error until the error approaches zero, thereby steering the digital RX sum-channel beam toward the reference object.
- 9) If transmit-leakage compensation is required, clear the *Hide Middle Panel* checkbox to display the corresponding controls. First, adjust *TX Ref Delay (samples)* to coarsely align the stored reference chirp with the leakage signal. Then adjust *TX Ref Delay (ps)* and *TX Ref Mag* to obtain fine delay and magnitude matching.
- 10) Select the required detection paths using *Enable MTI* and *Clutter map MTI*. Select the appropriate clutter-map processing variant using *Clutter Map Filter*.
- 11) Enable CFAR processing and adjust the CFAR bias, clutter-map bias, numbers of guard and reference cells, and the CFAR estimation method. The processed data and corresponding detection thresholds can be monitored in the range-profile display in Region 6.
- 12) Adjust *Waterfall Zoom*, *Min*, and *Max* to obtain the required displayed range and color-scale limits. *Waterfall Zoom* controls the displayed range, whereas *Min* and *Max* control the lower and upper limits of the waterfall color scale.
- 13) Enable *Log file* to record detected-target data. Data logging is available only when CFAR processing is enabled. Each time the checkbox is selected, a new timestamped log file is created and recording begins. Clear the checkbox to stop recording. Selecting it again creates another log file and starts a new recording session. When MTI is enabled, each detection is supplemented with the corresponding Doppler-filter index, $k = 1, \dots, n - 1$. When clutter-map MTI is enabled, detections from the clutter-map channel are assigned the equivalent Doppler-filter index $k = 0$.

The principal hardcoded global parameters of the radar software are:

- *radar_number*: Selects the radar hardware unit (Prototype #1 or #2) and its corresponding carrier frequency, DAC scaling, and channel-delay calibration. The default value is

$$\text{radar_number} = 1.$$

- *num_chirps*: Defines the number of chirps contained in one coherent processing interval. The configured value is

$$\text{num_chirps} = 16.$$

- *max_dist*: Defines the maximum configured radar processing and display distance in metres. The configured value is

$$\text{max_dist} = 300 \text{ m.}$$

- *K_DAC*: Defines the digital amplitude scaling of the transmitted chirp. Its value depends on the selected radar unit:

$$K_{\text{DAC}} = \begin{cases} 2^{14}, & \text{for Prototype \#1,} \\ 2^{13}, & \text{for Prototype \#2.} \end{cases}$$

- *alpha_clutter_map*: Defines the recursive update coefficient of the clutter map. The configured value is

$$\text{alpha_clutter_map} = 0.15.$$

- *XY_history_length*: Defines the number of previous target-coordinate frames stored and displayed in the target-map history. The configured value is

$$\text{XY_history_length} = 20.$$

- *chirp_time_ms*: Defines the duration of one transmitted chirp in milliseconds. The configured value is

$$\text{chirp_time_ms} = 1 \text{ ms.}$$

APPENDIX E

PLUTO SDR CONFIGURATION AND TOOLS

A. PlutoSDR hardware modifications

- The hardware revision of the ADALM-PLUTO SDR must be Rev. C or later.
- The ADALM-PLUTO SDR must be modified to enable an additional RX channel (RX1) and TX channel (TX1) [40], [41], making them available for use, Fig E.1.

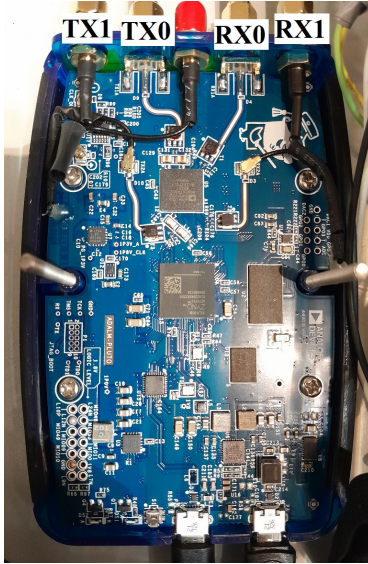


Fig. E.1. The ADALM-PLUTO SDR must be modified to enable an additional RX channel (RX1) and TX channel (TX1).

B. PlutoSDR firmware configuration

- PlutoSDR FW version: 0.39 [42].
- PlutoSDR two RX and two TX channels (2r2t) mode must be activated [43] using commands:
 - `fw_setenv attr_name compatible`
 - `fw_setenv attr_val ad9361`
 - `fw_setenv compatible ad9361`
 - `fw_setenv mode 2r2t`
- USB-Ethernet IP address should be configured in *con-fig.txt* of PlutoSDR [44].

C. PC tools and configuration (Windows OS)

- Drivers for PlutoSDR *PlutoSDR-M2k-USB-Drivers.exe* [45].
- IIO-Oscilloscope *adi-osc-setup.exe* [46].
- Libiiio release v0.26 *libiiio-0.26.ga0eca0d2-setup.exe* [47].
- Anaconda3 package to use Spyder 6.1.0 as Python 3.13 environment [48]. Additional packages to install (in Anaconda Prompt):
 - `pip install pyqtgraph`
 - `pip install matplotlib`
 - `pip install scipy`
 - `pip install pyadi-iio`
 - `pip install pyftdi`

- Ethernet adapter IPv4 options to match with PlutoSDR IP address [44].

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