

Automated parametric ray-tracing analysis of fluence rate in a compound parabolic collector (CPC)-based solar Ultraviolet (UV) disinfection reactor

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Abstract

Limited access to safe drinking water motivates solar water disinfection (SODIS), but conventional batch methods are constrained by small volumes and long exposure times. Continuous-flow reactors with tracking compound parabolic collectors (CPCs) can substantially increase the Ultraviolet (UV) radiation delivered to water. Their disinfection potential depends on the fluence rate encountered by pathogens along their trajectories in the reactors. Spectral fluence rate is particularly important because solar irradiance and microbial sensitivity vary with wavelength. Total UV irradiance alone cannot represent the radiation relevant to inactivation. Yet the spatial and spectral fluence-rate field inside solar UV reactors has not been systematically characterized across operating conditions and design choices. This study develops an automated parametric Monte Carlo ray-tracing framework for CPCs to resolve that field and quantify its sensitivity to solar conditions, reactor geometry, wavelength, and reflector material. A reference configuration at 317 nm is used to demonstrate model capabilities. Optical gain is defined as the ratio of the mean fluence rate over the entire pipe to the ambient direct normal irradiance at the evaluated wavelength. For the reference case, the mean fluence rate was 1.325 W m^{-2} , corresponding to an optical gain of 7.14. The fluence field was strongly nonuniform in the tube cross section but the

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mean fluence rate over the tube cross section exhibited a coefficient of variation of less than 1% along the interior length. Under modeled solar tracking conditions, optical gain peaked at solar noon at a value independent of site and date, while its diurnal profile depended on hour angle and solar declination. Optical gain varied non-monotonically with wavelength owing to the combined spectral optical properties of the reflector, water, and borosilicate glass. Among the design variables examined, reflector reflectance had the greatest influence on optical gain at the selected wavelength. The resulting fluence fields from this work provide the spatial and spectral inputs needed for subsequent trajectory-based UV dose and microbial-inactivation predictions.

Keywords: solar water disinfection, continuous-flow reactor, tracking compound parabolic collector, spectral fluence rate, optical gain, ray tracing

Abbreviations

Al foil	aluminum foil
PET/VDAl	polyethylene terephthalate/vacuum-deposited aluminum
Al/PET/Al	aluminum/polyethylene terephthalate/aluminum
CRC/Al/PET	corrosion-resistant-coated aluminum/polyethylene terephthalate
CFD	computational fluid dynamics
CPC	compound parabolic collector
DO	discrete ordinates
GCI	grid convergence index
PET	polyethylene terephthalate
PTV	particle tracking velocimetry
SODIS	solar water disinfection
SPD	spectral power distribution

TUV	Tropospheric Ultraviolet and Visible
USDA	United States Department of Agriculture
UV	Ultraviolet
UTC	Coordinated Universal Time
UVT	Ultraviolet transmittance

Nomenclature

Symbol	Unit	Definition
Reactor geometry		
x, y, z	cm	Cartesian coordinates pointing east, north, and toward the zenith, respectively.
y_{rp}, z_{rp}	cm	Local coordinates of a recording plane.
W_r	cm	Width of the flat receiver.
W_a	cm	Aperture width of the full CPC.
$W_{a,tr}$	cm	Aperture width of the truncated CPC.
CR	–	Geometric concentration ratio, W_a/W_r .
θ_c	°	Half-acceptance angle of the ideal CPC.
f	cm	Focal length of each parabolic reflector arm.
t	cm	Parameter specifying position along a reflector arm.
$Y(t)$	cm	Auxiliary function used to construct the reflector profile.
$y(t), z(t)$	cm	Coordinates of the right reflector profile.
t_{start}, t_{full}	cm	Values of t at the receiver edge and top of the full reflector.
Z_{full}	cm	Height of the full CPC.
TF	–	Fraction of the full reflector height removed by truncation.
Z_{tr}	cm	Height of the truncated CPC.
D_i, D_o	cm	Inner and outer diameters of the absorber tube.
h	cm	Absorber-tube wall thickness.
ϵ_r	cm	Clearance added to the tube diameter to set W_r .
L	cm	Length of the CPC-pipe reactor.
Solar geometry and tracking		
α_s	°	Solar elevation angle.
γ_s	°	Solar azimuth measured eastward from north.
θ_z	°	Solar zenith angle.
φ	°	Site latitude.
δ	°	Solar declination angle.
ω	°	Solar hour angle.
d	–	Unit vector pointing toward the sun.
d_x, d'_E	–	East–west solar-direction components in the reactor and equatorial frames.
d_{yz}	–	Projection of d onto the yz plane.
α_p	°	Ideal tracking angle about the east–west axis.
β	°	Angle between the solar direction and the yz plane.
Δ	°	Angular increment of the discrete tracking mechanism.

Continued on next page

Nomenclature (continued)

Symbol	Unit	Definition
α_{tr}	$^\circ$	Realized tracking angle.
α_{res}	$^\circ$	Residual tracking misalignment.
n_{ap}	–	Unit normal of the tracked CPC aperture.
Solar-plane position and orientation		
C	cm	Reactor center after tracking.
r	cm	Distance from the reactor center to the solar plane.
C'	cm	Solar-plane insertion point.
n_0, n_{sun}	–	Initial and oriented unit normals of the solar plane.
ϕ	$^\circ$	Spin angle of the square solar plane.
ϕ^*	$^\circ$	Spin angle minimizing the larger footprint half-extent.
Φ	$^\circ$	Set of candidate spin angles.
α_x, α_y	$^\circ$	Rotations about the x and y axes used to orient the solar plane.
R_x, R_y, R_z	–	Rotation matrices about the x , y , and z axes.
P_i	cm	Tracked corners of the reactor bounding box.
u_1, u_2	–	Orthonormal basis vectors spanning the solar plane.
$a_1(\phi), a_2(\phi)$	cm	Projected-footprint half-extents along the solar-plane axes.
W_p	cm	Side length of the square solar plane.
Spectral irradiance and radiant power		
λ	nm	Wavelength.
$\Delta\lambda$	nm	Wavelength interval represented by a simulation.
$I_{\text{DNI}}(\lambda)$	$\text{W m}^{-2} \text{ nm}^{-1}$	Direct-normal spectral irradiance.
$P_{\text{source}}(\lambda)$	W	Radiant power assigned to the solar source.
A_{sun}	m^2	Area of the square solar plane.
$P_{\text{ap}}(\lambda)$	W	Direct-beam radiant power incident on the aperture.
A_{ap}	m^2	Truncated aperture area.
Optical properties and fluence rate		
n	–	Real part of the refractive index.
$\kappa(\lambda)$	–	Extinction coefficient.
λ_0	m	Vacuum wavelength used to calculate absorption.
$\alpha_{\text{abs}}(\lambda)$	m^{-1}	Spectral absorption coefficient.
$\rho(\lambda)$	–	Specular reflector reflectance.
$H_i^{(k)}$	W m^{-2}	Fluence rate in cell i at ray-count level k .
$\overline{H}_0$	W m^{-2}	Mean fluence rate over the interior recording planes.
$G(\lambda, \omega, \delta)$	–	Optical gain relative to incident direct-normal irradiance at wavelength λ , hour angle ω , and solar declination δ .

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Nomenclature (continued)

Symbol	Unit	Definition
G_{ref}	–	Optical gain of the reference configuration.
$G_{\text{max}}(\lambda)$	–	Maximum optical gain for wavelength λ .
Convergence and fitted correlations		
$E_{\text{avg}}^{(k)}$	%	Average cell-wise relative error at ray-count level k .
i	–	Index of reactor corner points or computational cells, depending on context.
k	–	Ray-count level index.
N	–	Number of valid computational cells.
N_{ray}	–	Number of traced rays.
ϵ_k	W m^{-2}	Regularization term in the cell-wise error calculation.
R^2	–	Coefficient of determination for the fitted ray-count power law.
a, b	%, –	Coefficient and exponent of the ray-count power law.
a_0	–	Constant in the optical gain correlation.
$a_\beta, a_{\text{CR}}, a_{\text{D}}, a_{\text{T}}, a_\rho$	–	Fitted exponents in the optical gain correlation.

1. Introduction

Limited access to safe drinking water continues to expose populations worldwide to waterborne diseases [1]. Solar water disinfection (SODIS) uses ambient solar radiation to inactivate pathogenic microorganisms without requiring an external energy input. However, conventional batch SODIS that relies on polyethylene terephthalate (PET) bottles or bags is limited by small treatment volumes and long exposure times. The use of non-imaging compound parabolic collectors (CPCs) can increase the solar radiation delivered to treated water and thus improve microbial inactivation and reduce treatment times compared to conventional SODIS [2, 3]. Solar heating can provide an additional inactivation mechanism, particularly when the water temperature exceeds approximately 45°C [4]. Continuous-flow CPC reactors extend SODIS beyond batch treatment by passing water through a receiver tube positioned within a concentrating reflector. Prototypes of this configuration have been constructed and evaluated under field conditions by Mbonimpa et al. [5] and Busse [6]. The disinfection performance of a continuous-flow reactor is governed by the Ultraviolet (UV) dose accumulated by microorganisms as they pass through the reactor. This dose depends jointly on the residence time of each microorganism and the local fluence rate encountered along its trajectory.

21 Consequently, spatially non-uniform fluence rate fields can produce
22 substantial differences in accumulated dose, meaning that some trajectories
23 might receive insufficient UV exposure [7, 8]. The radiation delivered to the
24 receiver is influenced not only by incident spectral irradiance but also by
25 solar position, CPC geometry, reflector material, and transmission of light
26 through the receiver tube and water. These factors may affect both the
27 magnitude and spatial distribution of the fluence rate within the pipe.

28 In the continuous-flow configuration considered here, a previous study
29 modeled radiation transport using the discrete ordinates (DO) method
30 under a single representative solar condition and reported the
31 cross-sectionally averaged irradiance amplification. The same study also
32 found that DNA-weighted UV dose governed reactor performance [5].
33 However, irradiance describes radiant power incident perpendicularly on a
34 surface, while fluence rate accounts for radiation arriving at a point from all
35 directions and represents the local radiation exposure experienced by
36 microorganisms [9]. Since DNA-weighted dose depends on
37 wavelength-dependent microbial response and the local fluence rate
38 encountered along each trajectory, cross-sectionally averaged irradiance
39 alone cannot fully characterize disinfection performance. Therefore, the
40 spatial and spectral fluence rate field within the receiver must be resolved.

41 Substantial work has been done to study CPCs for other applications
42 such as energy collection, but these systems require far less precision when
43 directing radiation [10]. Thus, no previous study has systematically
44 quantified how variations in solar position and spectral irradiance across
45 locations, seasons, and times of day combine with reactor characteristics
46 such as collector geometry, wavelength, and reflector material to affect the
47 spatial and spectral fluence-rate field throughout the receiver tube. In
48 water treatment applications, this is crucial for ensuring water quality and
49 safety. This study develops a parametric Monte Carlo ray-tracing
50 framework to address this gap. The framework is implemented in Photopia
51 for Rhino using a Python-based workflow that generates fully configured
52 simulation files, including all geometry, optical properties, source
53 specifications, recording surfaces, and ray-tracing settings and outputs
54 fluence rate fields. Herein, we apply the developed framework to a
55 representative CPC to study the sensitivity of the reactor’s optical
56 performance to the model parameters.

2. Methodology

A Python-based workflow was developed to automatically generate fully configured, simulation-ready Photopia files. The workflow:

1. constructs the CPC and tube geometry;
2. determines the tracking orientation and solar-source position;
3. specifies the spectral power distribution (SPD) and source power;
4. modifies the corresponding Photopia lamp-library entries;
5. calculates the wavelength-dependent optical properties;
6. creates, adds, and assigns the corresponding material-library definitions;
7. configures the recording planes; and
8. specifies the ray count and other ray-tracing settings.

The files were subsequently processed in batch using Photopia Queue [11] to perform the Monte Carlo ray-tracing simulations. After the simulations were completed, the output files were automatically post-processed to extract and organize the spatial fluence rate data. This workflow enables a large number of optical configurations to be evaluated consistently and provides the fluence-rate fields required for subsequent particle-trajectory, dose, and microbial-inactivation analyses. The following sections describe each component of the framework and the convergence and sensitivity analyses used to evaluate the model.

2.1. Reactor Geometry and Optical Configuration

The reactor is a CPC with flat absorber configuration with a transmissive tube resting on its receiver plane. The two-dimensional reflector cross-section is defined in the yz -plane and extruded along the x -axis. When implemented, the CPC is oriented east-west. The cross-sectional profile and the three-dimensional extruded assembly are shown in Fig. 1(a) and (b).

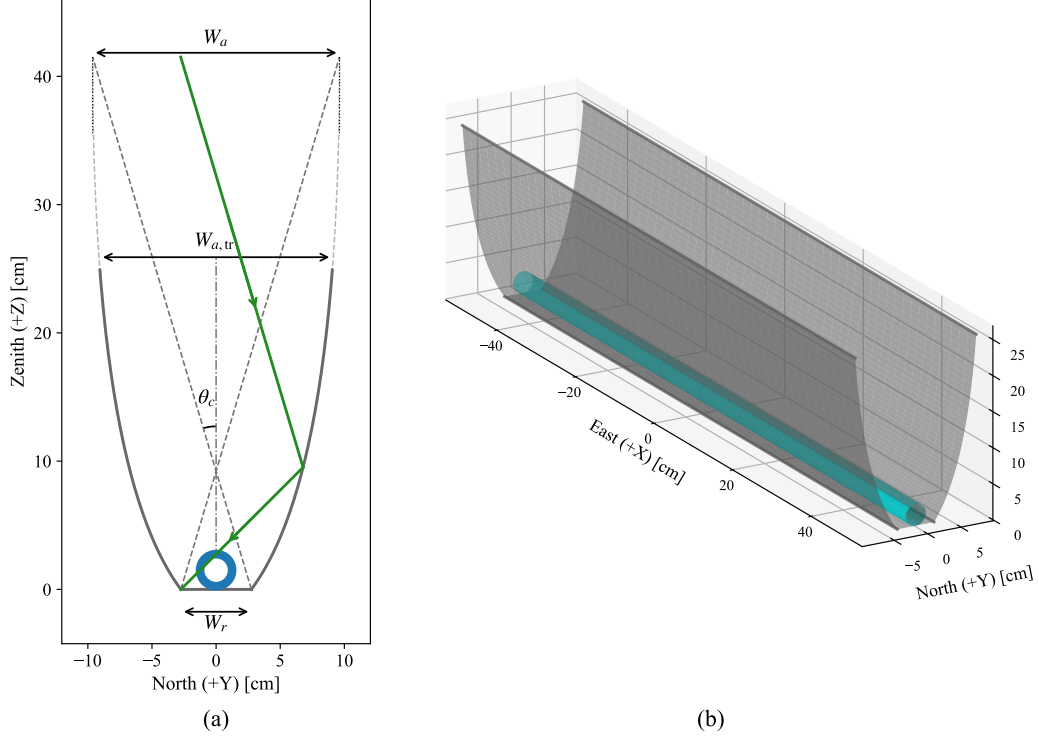


Figure 1: Schematic of the CPC-pipe reactor; (a) the two-dimensional view of full-height and truncated CPC; (b) the three-dimensional view of truncated CPC.

2.1.1. Concentration ratio and acceptance angle

For a flat receiver of width W_r and aperture width W_a , the geometric concentration ratio (CR) is defined in Eq. (1) [12].

$$CR = \frac{W_a}{W_r}. \quad (1)$$

According to the edge-ray principle, concentration ratio determines the acceptance half-angle, θ_c , following Eq. (2) [12, 13]. This is shown in Fig. 1(a).

$$\sin \theta_c = \frac{W_r}{W_a} = \frac{1}{CR}, \quad \theta_c = \arcsin\left(\frac{1}{CR}\right). \quad (2)$$

All rays incident within $\pm\theta_c$ of the aperture normal are delivered to the receiver. Rays outside this range are rejected. A higher concentration ratio narrows the angular acceptance [12].

93 *2.1.2. Reflector profile*

94 Each reflector wall is a parabola whose focus lies on the opposite edge of
 95 the flat receiver and whose axis is tilted by the acceptance angle θ_c from the
 96 aperture normal [13]. The focal length (f) of each parabolic arm is calculated
 97 in Eq. (3).

$$f = \frac{W_r}{2} \times (1 + \sin \theta_c). \quad (3)$$

98 The right reflector arm is described using the parameter t . First, t and $Y(t)$
 99 define the unrotated parabolic profile. The profile is then rotated by θ_c and
 100 shifted by $-W_r/2$ in the y -direction to obtain the global coordinates $y(t)$ and
 101 $z(t)$ using Eqs. (4)–(6).

$$Y(t) = \frac{t^2}{4f} - f, \quad (4)$$

$$y(t) = -\frac{W_r}{2} + t \cos \theta_c - Y(t) \sin \theta_c, \quad (5)$$

$$z(t) = t \sin \theta_c + Y(t) \cos \theta_c, \quad (6)$$

104 The left arm is the mirror image of the right arm. For an ideal CPC, the arm
 105 begins at the receiver edge, at $t_{\text{start}} = W_r \cos \theta_c$ and ends where the reflector
 106 becomes parallel to the optical axis, at $t_{\text{full}} = 2f/\tan \theta_c$. The two symmetric
 107 reflector arms are then extruded at a distance L along the x -axis. The full
 108 height of the CPC is calculated in Eq.(7).

$$Z_{\text{full}} = \frac{W_r (1 + \sin \theta_c) \cos \theta_c}{2 \sin^2 \theta_c}. \quad (7)$$

109 *2.1.3. Truncation*

110 A full CPC can become impractically tall, increasing both reflector
 111 material requirements and manufacturing cost. The upper part of the
 112 reflector provides only a small increase in optical efficiency [13]. Therefore,
 113 the reflector is truncated by removing a fraction of its upper height. A
 114 truncation fraction, $TF \in [0,1)$, is defined so that the built height is
 115 calculated in Eq. (8).

$$Z_{\text{tr}} = (1 - TF) Z_{\text{full}}, \quad (8)$$

116 This truncation reduces the effective CPC concentration ratio slightly but
 117 cuts the reflector material requirements and cost.

118 2.1.4. Absorber tube

119 The transmission tube is a borosilicate tube of inner diameter D_i and
 120 wall thickness h , giving an outer diameter $D_o = D_i + 2h$. The tube wall
 121 thickness was fixed at $h = 0.5$ cm for all simulations. The receiver width is
 122 set slightly wider than the tube, $W_r = D_o + \varepsilon_r$, to prevent direct contact. The
 123 reflector profile, full height, truncated height, and truncated aperture are all
 124 determined by receiver width W_r , concentration ratio CR , and truncation
 125 fraction TF through Eqs. (2)–(8).

126 2.2. Solar Geometry

127 The direction of the incident solar radiation is described by the solar
 128 elevation angle α_s , measured from the horizontal plane, and the solar azimuth
 129 angle γ_s , measured eastward starting from due north. The solar zenith angle
 130 θ_z is the complement of the elevation, $\theta_z = 90^\circ - \alpha_s$ [14]. Fig. 2 illustrates
 131 the reactor relative to the solar trajectory with the diurnal variations in solar
 132 elevation and azimuth for a representative location and date.

133 In the reactor frame, with $+x$ toward east, $+y$ toward north, and $+z$
 134 toward the zenith, the unit vector pointing from the origin toward the sun is
 135 calculated in Eq. (9).

$$d = \begin{pmatrix} \cos \alpha_s \sin \gamma_s \\ \cos \alpha_s \cos \gamma_s \\ \sin \alpha_s \end{pmatrix}. \quad (9)$$

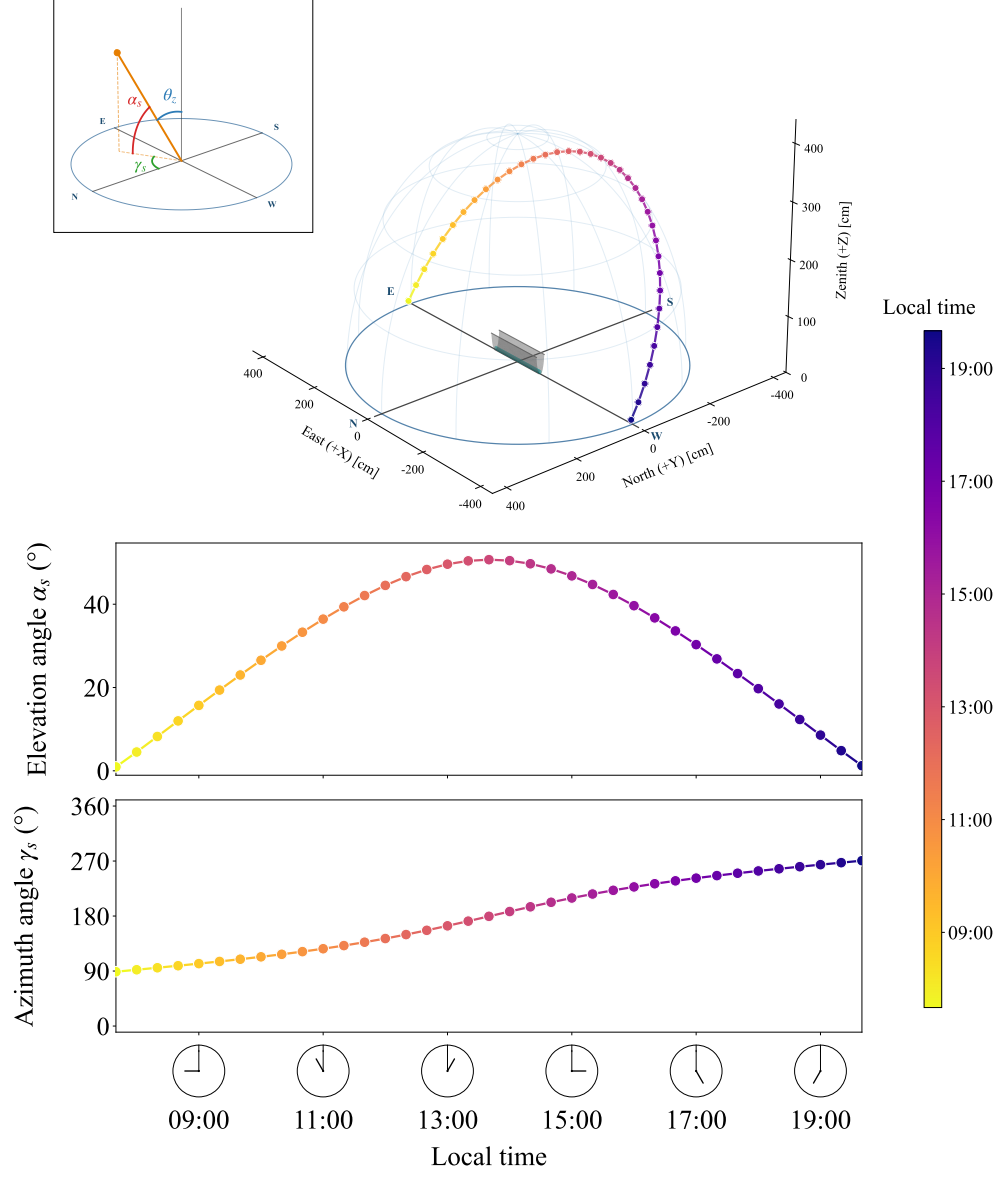


Figure 2: Apparent solar trajectory and corresponding solar angles for an actual site and date. The inset defines the solar zenith (θ_z), elevation (α_s), and azimuth (γ_s) angles. The lower panels show the variations of the solar elevation and azimuth angles throughout the day. Color bar indicates the local time.

136 2.2.1. CPC Tracking

137 The CPC trough is aligned along the east–west direction, thus,
 138 single-axis tracking is achieved by rotating the reactor about this axis. This
 139 configuration corresponds to horizontal east–west-axis tracking with
 140 continuous north–south adjustment to minimize the solar incidence angle
 141 described by [15]. Accordingly, the ideal tracking angle α_p is defined as the
 142 angle between the zenith direction and the projection of the solar beam
 143 onto the transverse yz -plane given by Eq. (10).

$$\alpha_p = \arctan(-\tan \theta_z \cos \gamma_s) \quad (10)$$

144 To reflect a physically realistic tracking mechanism, the tracking system is
 145 constrained to discrete angular increments rather than a continuous range.
 146 We assume a practical actuator resolves the tracking angle to within $\Delta = 5^\circ$.
 147 The tracking angle, α_{tr} , defined between the aperture normal and the zenith
 148 axis, is rounded to the nearest tracking increment following Eq. (11).

$$\alpha_{tr} = \Delta \text{round}\left(\frac{\alpha_p}{\Delta}\right). \quad (11)$$

149 This discretisation leaves a residual angular misalignment between the ideal
 150 and realized tracking angles as shown in Eq. (12).

$$\alpha_{res} = |\alpha_{tr} - \alpha_p|. \quad (12)$$

151 Consequently, the normal direction of the tracked aperture is given by
 152 Eq. (13).

$$n_{ap} = (0, -\sin \alpha_{tr}, \cos \alpha_{tr}). \quad (13)$$

153 However, the longitudinal component of the solar beam d_x may not be
 154 compensated for by single-axis tracking. The solar direction makes an angle
 155 of β with the transverse yz -plane, which is calculated in Eq. (14).

$$\beta = \arcsin(|\cos \alpha_s \sin \gamma_s|), \quad (14)$$

156 The tracking geometry and the relevant angles are illustrated in Fig. 3.

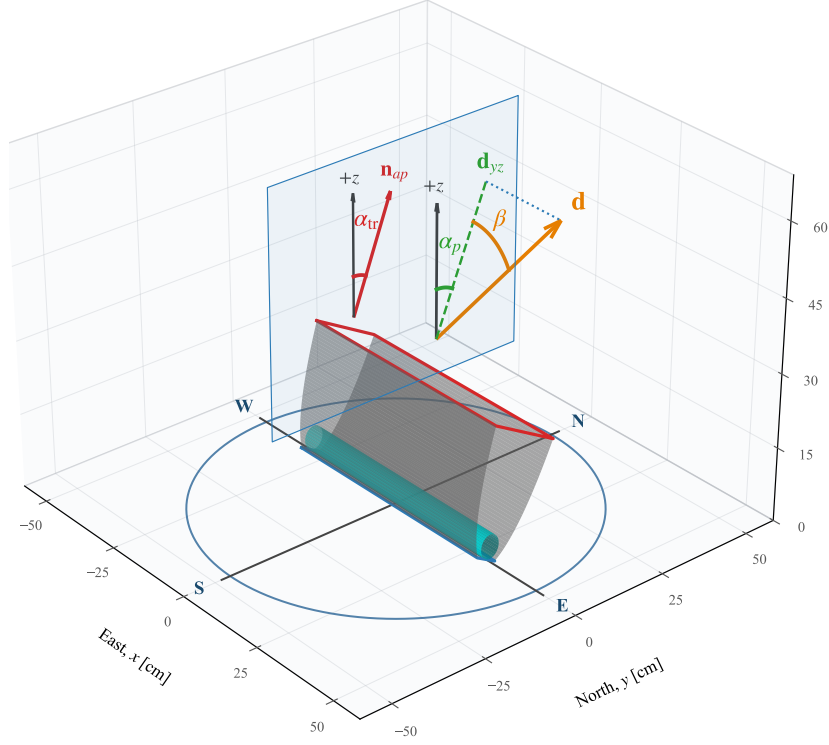


Figure 3: Three-dimensional CPC solar tracking geometry. The solar vector, d , forms an angle, β , with the transverse yz plane. Its transverse projection, d_{yz} , and the aperture normal, n_{ap} , form the angles α_p and α_{tr} with the zenith axis, respectively.

157 2.3. Solar Source Geometric Modeling

158 The sun model in Photopia is a square planar lamp that produces a
 159 relatively collimated beam with an angular spread of 0.53° to represent the
 160 apparent size of the solar disk [16]. By default, the normal to the solar
 161 plane points in the $-z$ direction. The source plane is positioned and
 162 oriented to direct the beam along $-d$ and fully cover the projected footprint
 163 of the tracked CPC. Its dimensions were constrained by the $5\text{ ft} \times 5\text{ ft}$ sun
 164 lamp model adopted in the Photopia simulations. This section determines
 165 the source insertion point and orientation required for complete
 166 illumination across the considered reactor geometries. The Photopia lamp
 167 settings described in the following sections were implemented according to
 168 the Photopia for Rhino instructions provided by LTI Optics [17].

169 2.3.1. Sun insertion point

170 The solar plane is centered on the projection of the reactor center onto
 171 a plane normal to the solar direction vector, d . As parallel projection is
 172 an affine map, it preserves barycenters. Thus, the projected reactor center
 173 coincides with the center of the projected reactor footprint [18]. The reactor
 174 center, C , is defined at the mid-height of the truncated reflector. Following
 175 the tracking rotation, its position is calculated using Eq. (15).

$$C = \left(0, -\frac{Z_{\text{tr}}}{2} \sin \alpha_{\text{tr}}, \frac{Z_{\text{tr}}}{2} \cos \alpha_{\text{tr}}\right). \quad (15)$$

176 Placing the solar plane at a fixed distance, $r = 400$ cm, from point C along
 177 d gives the insertion point, C' , as represented in Eq (16).

$$C' = C + r d. \quad (16)$$

178 The distance $r = 400$ cm was selected to position the solar plane sufficiently
 179 far from, and entirely outside the reactor geometry.

180 2.3.2. Sun orientation

181 The solar plane is first rotated by a spin angle ϕ about the global z -axis.
 182 The selection of ϕ is described in Section 2.3.3. Then, it is rotated about the
 183 x and y axes to align its normal with $-d$. The three rotations are applied to
 184 the initial unit normal of the lamp, $n_0 = (0, 0, -1)$, according to Eq. (17).

$$n_{\text{sun}} = R_y(\alpha_y) R_x(\alpha_x) R_z(\phi) n_0 = \begin{pmatrix} -\sin \alpha_y \cos \alpha_x \\ \sin \alpha_x \\ -\cos \alpha_y \cos \alpha_x \end{pmatrix}, \quad (17)$$

185 where, R_x , R_y , and R_z denote the rotation matrices about the corresponding
 186 global axes. As the initial normal is parallel to the z axis, the rotation $R_z(\phi)$
 187 spins the square solar plane without changing its normal direction. The spin
 188 must be applied first when the normal still lies along z because it rotates the
 189 square about its own normal. Setting $n_{\text{sun}} = -d$ yields the rotation angles
 190 calculated in Eq. (18).

$$\alpha_x = -\arcsin(\cos \alpha_s \cos \gamma_s), \quad \alpha_y = \text{atan2}(\cos \alpha_s \sin \gamma_s, \sin \alpha_s), \quad (18)$$

191 where, atan2 uses the signs of both arguments to determine the rotation
 192 angle in the correct quadrant. Photopia provides a discrete library of sun

193 lamp models at solar elevation increments of 10° . Therefore, the closest
 194 available model to the actual solar elevation is selected. The selected lamp
 195 follows the naming convention *SUNYY5X5*, where *YY* denotes the solar
 196 elevation in degrees and *5X5* denotes a $5 \text{ ft} \times 5 \text{ ft}$ solar plane [16].

197 2.3.3. Coverage Condition

198 The projected footprint of the reactor is determined using the eight corner
 199 points that define the outer extent of the CPC geometry. After rotation about
 200 the x axis by α_{tr} , these points are projected onto the solar plane. The convex
 201 hull of the projected points defines the complete footprint used to evaluate
 202 source coverage. The solar plane is then spun such that its emitting area
 203 fully contains this footprint. An orthonormal basis (u_1, u_2) is constructed
 204 on the solar plane using the Gram-Schmidt procedure [19], such that both
 205 basis vectors are perpendicular to the solar direction d . The basis is then
 206 spun through an angle ϕ about d to align the coordinate directions with
 207 the edges of the square solar plane. Here, ϕ is the spin angle of the plane
 208 about its normal, introduced in Section 2.3.2. For a given value of spin angle,
 209 Eq. (19) calculates the footprint half-extents by projecting the eight tracked
 210 CPC corners, P_i , relative to the footprint center, C' , onto the two in-plane
 211 basis vectors, $u_1(\phi), u_2(\phi)$, and taking the maximum absolute projection in
 212 each direction.

$$a_1(\phi) = \max_i |(P_i - C') \cdot u_1(\phi)|, \quad a_2(\phi) = \max_i |(P_i - C') \cdot u_2(\phi)|, \quad (19)$$

213 For each simulation, the spin angle is selected to minimize the larger of the
 214 two footprint half-extents. The candidate angles are defined in Eq. (20).

$$\Phi = \{0^\circ, 0.5^\circ, \dots, 90^\circ\}, \quad \phi^* = \arg \min_{\phi \in \Phi} \max\{a_1(\phi), a_2(\phi)\}. \quad (20)$$

215 The search is restricted to Φ because the square solar plane is invariant under
 216 rotations of 90° about its normal. The reactor is fully covered when the larger
 217 of optimized footprint half-extent satisfies Eq. (21).

$$\max(a_1(\phi^*), a_2(\phi^*)) \leq \frac{1}{2}W_p \quad (21)$$

218 where, $W_p = 5 \text{ ft} = 152.4 \text{ cm}$ is the side length of the square solar plane. If
 219 Eq. (21) is not satisfied at ϕ^* , the reactor cannot be fully covered at any of the

220 evaluated orientations. Such cases are excluded from the simulations. Fig. 4
 221 illustrates the projected reactor footprint and the corresponding orientation
 222 of the solar plane for a representative solar position.

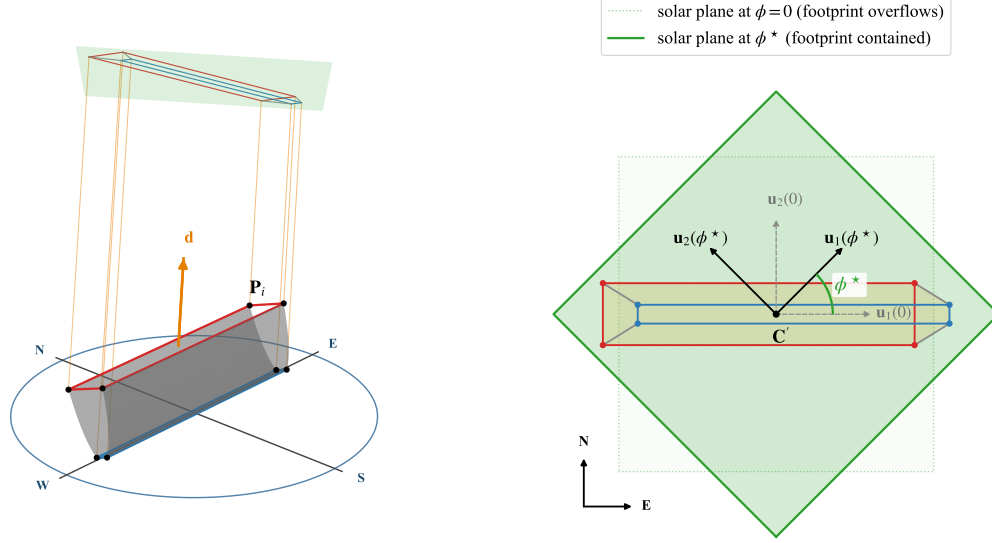


Figure 4: Projection of the eight CPC corner points, P_i , along solar direction, d , on the solar plane with respect to the plane's insertion point, C' . Left is the three-dimensional reactor projection and right is the top view of the solar plane and reactor projected footprint. $u_1(\phi^*)$ and $u_2(\phi^*)$ represent the orthonormal bases on the solar plane.

223 2.4. Spectral Irradiance and Radiant Power

224 The optical model was evaluated over the wavelength range 300–332 nm,
 225 covering the most biologically effective portion of the solar UV spectrum that
 226 reaches the ground level. Solar irradiance below ~ 300 nm is negligible at
 227 ground level [20]. UV-induced microbial inactivation generally becomes less
 228 effective toward longer UVB wavelengths, as shown for a range of bacterial,
 229 viral, and eukaryotic microorganisms [21, 22, 23].

230 Only the direct component of solar irradiance was considered in the
 231 optical simulations. During tracking, the direct solar beam falls within the
 232 CPC acceptance range and can be concentrated onto the receiver. Diffuse
 233 radiation arrives from the entire surrounding area of the sky, and only a
 234 fraction lies within this range. This assumption represents the intended

235 clear-sky operating conditions, under which the direct component is
 236 dominant [13]. The spectral direct normal irradiance used in the optical
 237 simulations was obtained directly from the United States Department of
 238 Agriculture (USDA) UVB Monitoring and Research Program [24, 25, 26].
 239 The dataset provides solar geometry parameters, including the azimuth,
 240 zenith, and elevation angles, together with direct normal, diffuse, and total
 241 spectral irradiance measured at selected UV wavelengths (300, 305, 311,
 242 317, 325, 332, and 368 nm) reported at 3-min intervals. Photopia requires
 243 the radiant power of the source to be specified using the *Radiant Watts*
 244 setting [17]. For a wavelength interval centered at λ , the radiant power
 245 assigned to the sun model is calculated using Eq. (22).

$$P_{\text{source}}(\lambda) = I_{\text{DNI}}(\lambda) A_{\text{sun}} \Delta\lambda, \quad (22)$$

246 where, $I_{\text{DNI}}(\lambda)$ is the direct normal spectral irradiance, A_{sun} is the area of
 247 the solar plane, and $\Delta\lambda$ is the wavelength interval represented by the
 248 simulation. The source SPD is defined with a single nonzero entry at the
 249 selected wavelength so that each simulation represents monochromatic
 250 irradiation [17]. Before reflection or concentration, the direct beam radiant
 251 power incident on the reactor aperture is calculated in Eq. (23).

$$P_{\text{ap}}(\lambda) = I_{\text{DNI}}(\lambda) \Delta\lambda \cos \beta \cos \alpha_{\text{res}} A_{\text{ap}}, \quad (23)$$

252 where, $A_{\text{ap}} = W_{\text{a,tr}} \cdot L$ is the truncated aperture area. The angles α_{res} and β
 253 are defined in Eqs. (12) and (14), respectively.

254 2.5. Optical Properties and Material Definitions

255 The optical materials were defined following the Photopia for Rhino
 256 instructions provided by LTI Optics [27]. The interior surface of the CPC
 257 was modeled as a specular reflector with angle-independent reflectance.
 258 The transmission tube was modeled using two refractive interfaces, where
 259 the outer surface formed the air–glass interface, and the inner surface
 260 formed the glass–water interface. The outer interface was defined using the
 261 refractive index and absorption coefficient of the glass, with air specified as
 262 the adjacent medium. Similarly, the inner interface was defined using the
 263 refractive index and absorption coefficient of water, with glass specified as
 264 the adjacent medium. The tube glass was SCHOTT N-BK7 borosilicate
 265 crown glass. Its wavelength-dependent refractive index was calculated using
 266 the Sellmeier relation and the SCHOTT N-BK7 coefficients [28], accessed

267 through the refractiveindex.info database [29]. The spectral absorption
 268 coefficient, $\alpha_{abs}(\lambda)$, was calculated from the dimensionless extinction
 269 coefficient, $\kappa(\lambda)$, using Eq. (24) [30].

$$\alpha_{abs}(\lambda) = \frac{4\pi\kappa(\lambda)}{\lambda_0}, \quad (24)$$

270 where, λ_0 is the vacuum wavelength expressed in meters, yielding $\alpha_{abs}(\lambda)$ in
 271 m^{-1} . The resulting values were converted to in^{-1} , as required by Photopia
 272 [27].

273 The wavelength-dependent refractive index of water was calculated
 274 using the formulation of Schiebener et al. [31], which accounts for
 275 wavelength, temperature, and water density. The relation was evaluated at
 276 20°C and a density of 998.2 kg m^{-3} . The spectral absorption coefficient of
 277 water was obtained from the pure-water UV absorption measurements
 278 reported by Mason et al. [32] over 250–550 nm. The reported values,
 279 originally given in m^{-1} , were converted to in^{-1} for use in Photopia. The
 280 refractive index of the surrounding air was calculated using Edlén’s
 281 dispersion relation for dry standard air at 15°C and 760 Torr [33].
 282 Absorption by air was neglected over the modeled 300–332 nm wavelength
 283 range. For each wavelength-specific simulation, two refractive material files
 284 corresponding to the tube interfaces and one reflective material file
 285 corresponding to the CPC surface were created and added to the Photopia
 286 material libraries. The materials were then assigned to their respective
 287 geometric surfaces in Rhino.

288 *2.6. Ray-Tracing Simulation and Fluence Rate Evaluation*

289 Monte Carlo ray tracing is well suited to this optical system because it
 290 directly follows collimated solar rays through specular reflections and
 291 refractive interfaces. In the photometric settings, a Type C photometric
 292 test was defined using horizontal angles from 0° to 360° in 10° increments
 293 and vertical angles from 0° to 180° in 1° increments [34]. In the analysis
 294 settings, the lamp-flux energy units were set to radiant watts appropriate
 295 for radiometric UV disinfection simulations [35]. The internal spectral
 296 resolution was set to 1 nm [35]. To characterize the axial variation in
 297 fluence rate, 51 square recording planes were positioned inside the absorber
 298 tube following the Photopia for Rhino procedure [36]. Each plane was
 299 perpendicular to the reactor axis and spanned $\pm D_i/2$ from the tube center
 300 in both the y and z directions. The planes were uniformly spaced at 2-cm

intervals over the $L = 100$ cm reactor length. Each recording plane was discretized using a 40×40 grid and configured in Fluence mode described in LTI Optics, LLC [37]. In this mode, Photopia places a virtual spherical recorder at the center of each grid cell. Each recorder collects the total radiant power arriving from all directions, which is divided by its cross-sectional area to obtain the local fluence rate. As the Photopia recording grid is square, a circular mask of radius $D_i/2$ was applied during post-processing to retain only the cells located within the tube cross-section.

2.7. Grid and Ray Count Convergence

Grid and ray-count convergence studies were performed to assess the sensitivity of the computed fluence rate fields to the spatial resolution of the recording planes and the number of Monte Carlo rays.

For the grid study, the cross-sectional recording planes were discretized using 20×20 , 40×40 , and 80×80 grids. The convergence metric was the aggregate mean fluence rate calculated over all valid in-pipe cells of the 49 interior recording planes. The grid convergence index (GCI) procedure of Celik et al. [38] was applied, with an acceptance criterion of 0.5% for the aggregate mean. The aggregate mean fluence rates was obtained using the 20×20 , 40×40 , and 80×80 grids. The GCI associated with the 40×40 result was 0.449%, which was below the adopted criterion. Therefore, the 40×40 grid was selected for the subsequent simulations.

Ray count convergence was evaluated using 10^7 , 3×10^7 , 10^8 , 3×10^8 , 10^9 , and 3×10^9 rays, corresponding to ~ 0.5 -decade increments. At each level, the cell-wise fluence rate field was compared with that obtained at the preceding level using the average relative error, E_{avg} , defined in Eq. (25).

$$E_{\text{avg}}^{(k)} = \frac{1}{N} \sum_{i=1}^N \frac{|H_i^{(k)} - H_i^{(k-1)}|}{\max(|H_i^{(k-1)}|, \epsilon_k)} \times 100, \quad (25)$$

where, $H_i^{(k)}$ is the fluence rate in cell i at ray count level k , and N is the total number of cells. The regularization term, ϵ_k , prevents division by near-zero values in low-fluence rate regions and is calculated in Eq. (26).

$$\epsilon_k = 10^{-6} \max_i |H_i^{(k-1)}| \quad (26)$$

330 E_{avg} accounts for all cell values, allowing detection of local discrepancies in
 331 fluence rate even when the mean fluence rate seems converged. The relative
 332 difference between the results obtained using 3×10^8 and 10^9 rays was 0.089%.
 333 Meanwhile, E_{avg} decreased monotonically from 24.28% to 2.17%. As the
 334 primary objective of this study was to quantify changes in spatially averaged
 335 fluence rate, 3×10^8 rays were selected for subsequent simulations. This
 336 selection is specific to the aggregate mean. Analysis focused on local fluence
 337 rate values or steep spatial gradients would require a larger ray count and an
 338 additional local convergence assessment. The ray count convergence results
 339 are shown in Fig. S1. The relationship between E_{avg} and ray count was
 340 quantified by fitting the power law following Eq. (27).

$$E_{\text{avg}} = aN_{\text{ray}}^b. \quad (27)$$

341 Linear least-squares regression of $\log_{10}(E_{\text{avg}})$ against $\log_{10}(N_{\text{ray}})$ yielded
 342 $b = -0.523$ and $R^2 = 0.999$. This is consistent with the theoretical $N_{\text{ray}}^{-0.5}$
 343 reduction in Monte Carlo statistical uncertainty according to Rogers and
 344 Bielajew [39], and demonstrates that the sampling error decreases at the
 345 expected rate.

346 *2.8. Sensitivity Analysis Design*

347 A series of parametric analyses was conducted to evaluate the effects of
 348 solar conditions, CPC geometry, wavelength, and reflector material on the
 349 reactor fluence rate. The reference configuration had $D_i = 3$ cm, $CR = 7$,
 350 and $TF = 0.45$, with an Al/PET/Al film used as the reflector. The
 351 wavelength-dependent reflectance of this film was obtained from the
 352 measurements of Mbonimpa et al. [5]. The reference solar conditions were
 353 based on USDA measurements recorded in Holtville, California, on June 21,
 354 2023, at 11:45 local standard time. The solar elevation and azimuth were
 355 80° and 180° , respectively, and the reference wavelength was 317 nm.
 356 Within each analysis, the parameter or parameter group of interest was
 357 varied while other inputs were held fixed. Solar elevation, azimuth, and
 358 direct-normal spectral irradiance were varied as a coupled set to preserve
 359 physically consistent solar conditions. Similarly, when wavelength was
 360 varied, the corresponding solar irradiance and wavelength-dependent
 361 optical properties were updated consistently.

3. Results

3.1. Reference Case Spatial Distribution of Fluence Rate

The spatial fluence rate distribution was first evaluated for the reference case defined in Section 2.8. The ideal tracking angle was $\alpha_p = 10.0^\circ$, which was exactly attainable by the discrete tracking mechanism. Fig. 5(a) illustrates the arrangement of the recording planes within the tracked reactor and the relationship between the local recording plane and the global coordinate systems. Fig. 5(b) shows the fluence rate distribution on the cross-section of the pipe in the local recording plane coordinate system. This shows a strongly non-uniform fluence rate distribution on the pipe cross-section. The mean fluence rate over all of the 49 interior recording planes denoted as $\overline{H}_0$, was 1.325 W.m^{-2} . The mean fluence rate, $\overline{H}_0$, is proportional to the incident spectral irradiance because the ray-tracing model is linear in source power. Dividing by that irradiance isolates the reactor's optical response. Accordingly, we define the optical gain at each wavelength in Eq. (28).

$$G(\lambda) = \frac{\overline{H}_0}{I_{\text{DNI}}(\lambda) \Delta\lambda}, \quad (28)$$

where, $\Delta\lambda = 1 \text{ nm}$ is the wavelength interval represented by each monochromatic simulation. For the reference configuration, G is 7.14, compared to a truncated effective concentration ratio of $W_{\text{a,tr}}/W_r = 6.48$. This higher optical enhancement occurs because fluence rate integrates the radiation arriving from all directions rather than the flux crossing a single plane. Discrete fluence values ranged from 0.300 to 5.333 W.m^{-2} , with a coefficient of variation of 0.598. As the incident rays from the source lamp propagated in the transverse plane, reflection from the two CPC arms followed by refraction through the tube wall produced a symmetric, V-shaped high-fluence region in the receiver-facing half of the tube. The mean fluence rate in the receiver-facing half of the tube was 2.14 times that in the aperture-facing half. The mean fluence rate calculated for each recording plane exhibited a coefficient of variation of only 0.66% across the 49 interior recording planes. Thus, the contribution of axial variations to the accumulated dose is negligible, and the dose variation depends primarily on the transverse particle trajectory. The two end planes were excluded because their boundary positions reduced the radiation collected by the recording surfaces. This end effect should be considered by

396 extending the CPC and tube beyond the active treatment length. The
 397 required extension should be determined for the selected geometry.

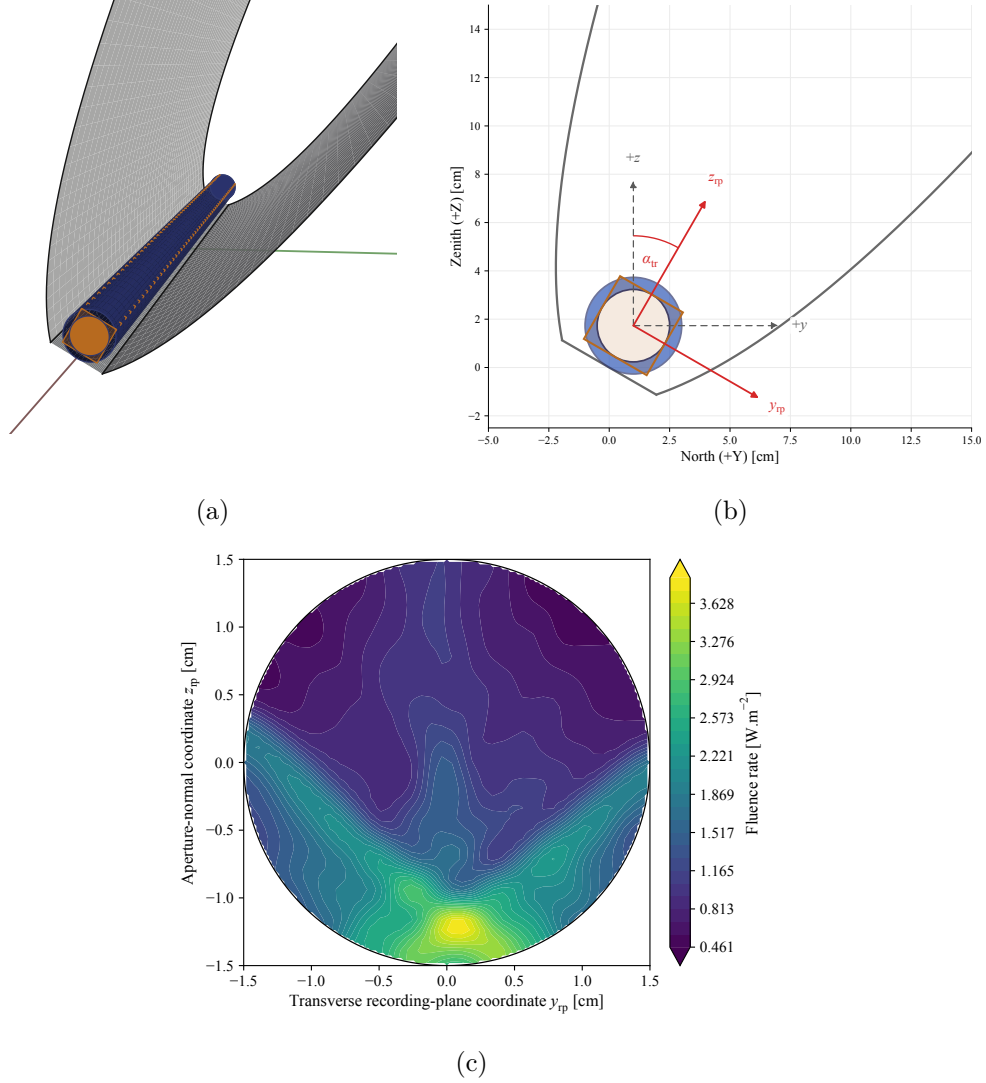


Figure 5: Recording plane configuration within the tracked CPC-pipe reactor: (a) three-dimensional view showing the square recording planes distributed along the transmission tube and (b) two-dimensional view showing a representative plane and its local axes y_{rp} and z_{rp} . α_{tr} is the tracking angle. (c) Spatial distribution of fluence rate across the tube cross-section at the central axial recording plane ($x = 0$ cm) for the reference case. Contours are smoothed and interpolated for visualization; therefore, the displayed color scale may not exactly reproduce the extrema of the original discrete fluence-rate values.

3.2. Sensitivity analysis results

3.2.1. Effects of site, season, and time of day

The combined effects of site, season, and time of day were evaluated using diurnal solar conditions for two USDA monitoring sites: Holtville, California, and Poplar, Montana. For each site, measurements from June 21 and December 13, 2023, were used to represent summer and winter conditions, respectively. Holtville and Poplar were selected because their substantial difference in latitude produces contrasting solar positions and UV irradiance conditions [40]. For each measurement time step, the solar elevation, solar azimuth, and direct normal spectral irradiance at 317 nm were obtained from the corresponding USDA record. These parameters were only varied all together to preserve physically realistic combinations of solar position and irradiance.

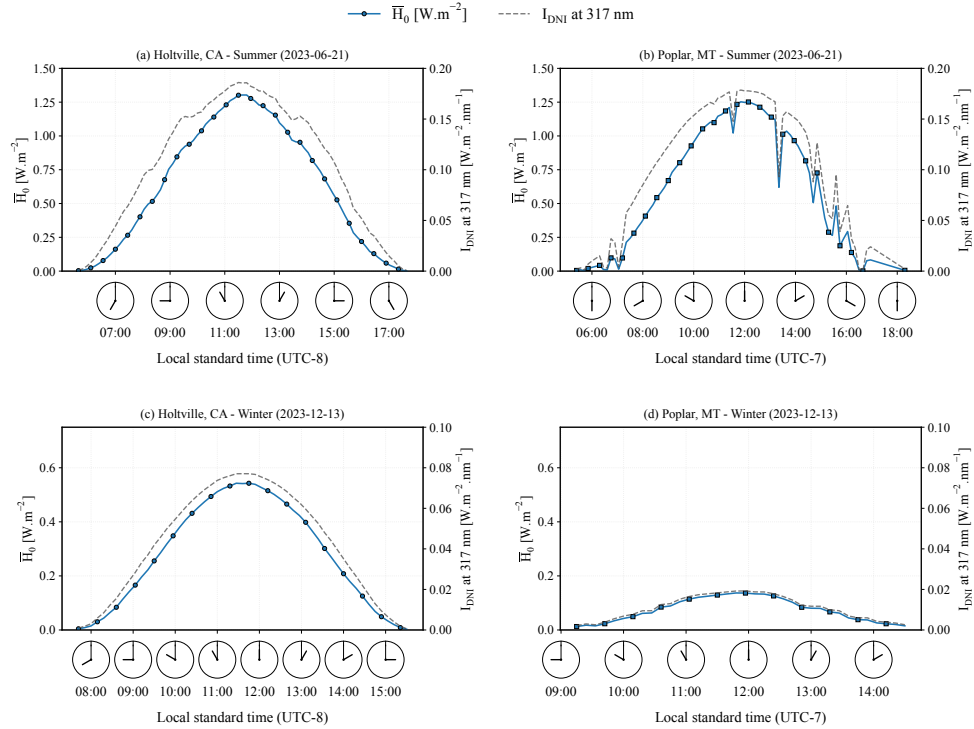


Figure 6: Diurnal variation in the mean fluence rate over 49 interior recording planes, $\overline{H}_0$ and the direct normal spectral irradiance, I_{DNI} , at 317 nm for all cases. Local standard times are shown with their offsets from Coordinated Universal Time (UTC)

Fig. 6 compares the diurnal variation in I_{DNI} with the mean fluence rate over all interior recording planes, $\overline{H}_0$. In all four cases, $\overline{H}_0$ closely followed the measured irradiance, including the fluctuations in the Poplar records. The fluctuations are likely attributable to cloud cover blocking the irradiance from reaching the sensor. The maximum mean fluence rates were ~ 1.33 and 1.28 W.m^{-2} for the Holtville and Poplar summer cases, respectively. The corresponding winter maximums decreased to ~ 0.55 and 0.14 W.m^{-2} . The particularly low winter fluence rate at Poplar resulted from its lower direct normal irradiance during the selected period. These results show that the absolute fluence rate was primarily governed by the available incident direct irradiance. To isolate the effect of solar position, the diurnal variation of the optical gain at 317 nm, $G(317\text{nm})$, is plotted in Fig. 7.

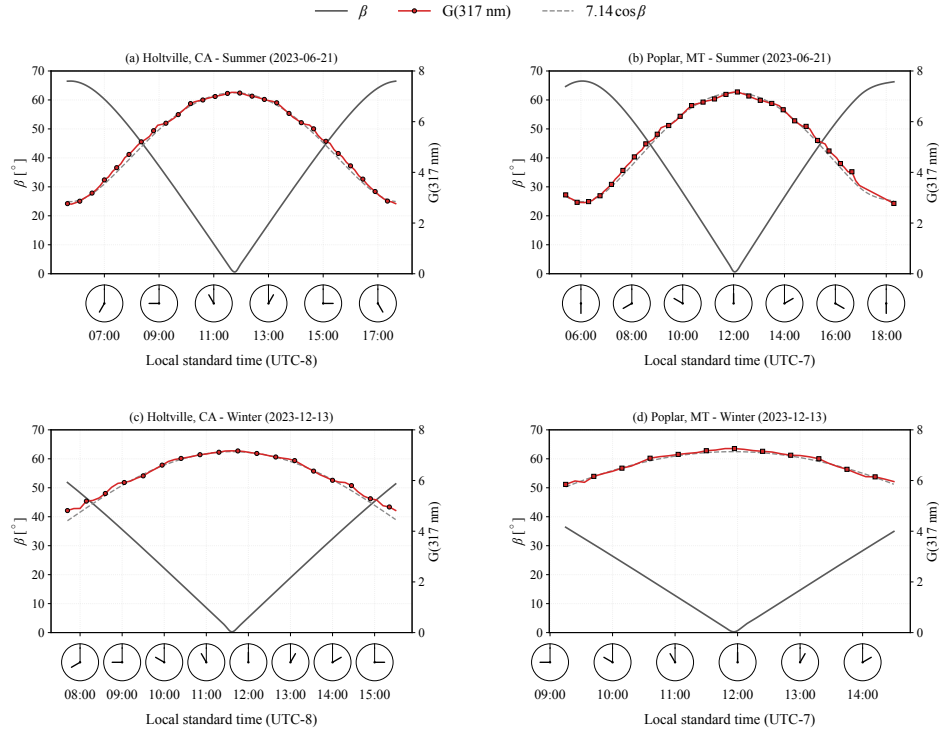


Figure 7: Diurnal variation in the angle, β , and the optical gain, for all cases. Local standard times are shown with their offsets from UTC.

423 The optical gain is directly related to $\cos\beta$, which accounts for the
 424 longitudinal component of the solar direction lying outside the transverse
 425 plane after single-axis tracking according to Eq. (14). Consequently, the
 426 gain reaches its maximum at solar noon, when $\beta = 0^\circ$ and the sun lies in
 427 the transverse plane, eliminating this misalignment. For the reference
 428 system, the maximum gain is 7.14. The solar-noon gain is identical across
 429 locations and dates, and only the absolute fluence rate varies with the
 430 available direct normal irradiance. This is because the transformation from
 431 equatorial to horizon coordinates can be represented by a rotation about
 432 the east-west-axis through an angle of $90^\circ - \varphi$, where φ is the local
 433 latitude. As a rotation does not change the vector component along its
 434 axis, the east-west direction component of the solar vector is identical in
 435 both coordinate systems. In equatorial coordinates, the east-west
 436 component, d'_E is given by [41] in Eq. (29).

$$d'_E = -\cos\delta \sin\omega, \quad (29)$$

437 where, δ is the solar declination and ω is the hour angle. The declination
 438 defines the angular position of the sun relative to the equatorial plane, and the
 439 hour angle describes the angular displacement of sun from the local meridian
 440 at a rate of 15° per hour. Substituting Eq. (29) into Eq. (14) yields Eq. (30).

$$\cos\beta = \sqrt{1 - (\cos\delta \sin\omega)^2}. \quad (30)$$

441 Thus, β is independent of latitude, φ , and depends only on solar
 442 declination, δ , and hour angle, ω . Latitude affects solar elevation and the
 443 available I_{DNI} , but not the reactor's optical gain under the modeled tracking
 444 conditions. This explains why the peak mean fluence rates at Holtville in
 445 summer and Poplar in winter differ by ~ 9.5 -fold in Fig. 6, while their
 446 optical gains follow nearly the same relationship with hour angle in Fig. 7.
 447 At solar noon, $\omega = 0^\circ$ and therefore $\beta = 0^\circ$, regardless of declination.
 448 Consequently, for a given wavelength, the maximum optical gain depends
 449 only on the reactor's geometry and optical properties and is independent of
 450 site and date. Away from solar noon, the gain profile varies with date
 451 through solar declination. For the selected dates, June 21 and December
 452 13, 2023, the declinations were $+23.45^\circ$ and -23.09° , respectively. These
 453 values yield only a 0.3% difference in $\cos\delta$, explaining the nearly identical
 454 profiles of β and optical gain versus hour angle in summer and winter.

455 However, this close agreement does not extend to all dates. Solar
 456 declination varies annually between -23.45° at the December solstice and
 457 $+23.45^\circ$ at the June solstice, passing through zero at both equinoxes
 458 (March 22 and September 22) [42]. Accordingly, $\cos \delta$ ranges from ~ 0.9174
 459 at the solstices to 1 at the equinoxes. For example, at $\omega = 45^\circ$, the gain of
 460 the reference reactor at 317 nm decreases from 5.46 at the solstices to 5.08
 461 at the equinoxes, a reduction of 7.1%. Thus, the diurnal optical gain profile
 462 of an east–west tracking CPC reactor with a given geometry and optical
 463 properties can be predicted for any site and date by modeling its maximum
 464 gain at solar noon and applying Eq. (31).

$$G(\lambda, \omega, \delta) = G_{\max}(\lambda) \times \sqrt{1 - (\cos \delta \sin \omega)^2}. \quad (31)$$

465 3.2.2. *Effect of CPC geometry*

466 The effects of reactor geometry including the inner tube diameter,
 467 concentration ratio, and truncation fraction were evaluated by varying one
 468 parameter at a time from the reference geometry. The resulting optical
 469 gains are presented in Fig. 8. The tube inner diameter had a comparatively
 470 small and non-monotonic effect on the gain. The maximum gain was
 471 achieved at $D_i = 2.0$ cm, although the overall variation across the evaluated
 472 range was only about 5.4%. In contrast, the concentration ratio had a
 473 strong and monotonic effect. Increasing CR from 2 to 9 increased the gain
 474 from 2.60 to 9.52. However, the optical enhancement was accompanied by a
 475 substantial increase in the reflector size. For example, increasing CR from 2
 476 to 9 increased Z_{tr} from 6.43 to 110.69 cm. The gain decreased
 477 monotonically with increasing truncation. It decreased from 7.68 for the
 478 untruncated geometry to 6.57 at $TF = 0.60$, corresponding to a reduction
 479 of $\sim 14.45\%$. By comparison, this truncation reduced the reflector height
 480 from 124.71 to 49.88 cm. The corresponding optical loss compared with the
 481 reduction in reflector height indicates that the upper reflector sections
 482 contributed less to the fluence rate than the lower sections. For the
 483 reference geometry, $TF = 0.45$ reduced the reflector height by 45% while
 484 decreasing the optical gain by only $\sim 7\%$ relative to the untruncated
 485 geometry.

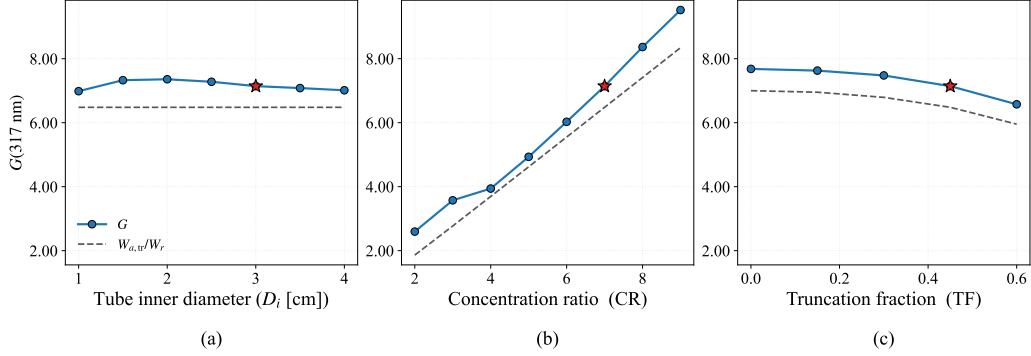


Figure 8: Effects of CPC geometry on the optical gain, $G(317 \text{ nm})$: (a) receiver-tube inner diameter, D_i ; (b) concentration ratio, CR ; and (c) truncation fraction, TF . The red stars indicate the common reference geometry ($D_i = 3 \text{ cm}$, $CR = 7$, and $TF = 0.45$).

3.2.3. Effect of wavelength

The effect of solar irradiance wavelength was evaluated at the six USDA UVB Monitoring Network wavelengths within the spectral range of interest: 300, 305, 311, 317, 325, and 332 nm. Each simulation used the direct normal spectral irradiance and reflector specular reflectance corresponding to the evaluated wavelength. Reflectance values were obtained by linear interpolation between the available measurements. As shown in Fig. 9(a), the mean fluence rate over all planes, $\overline{H}_0$, increased monotonically with wavelength, closely following the increase in the direct normal spectral irradiance, $I_{\text{DNI}}(\lambda)$. The value of $\overline{H}_0$ increased from $\sim 0.02 \text{ W.m}^{-2}$ at 300 nm to 3.20 W.m^{-2} at 332 nm.

To separate the effect of the incident solar irradiance from the wavelength-dependent optical response of the reactor, $G(\lambda)$ is used. Fig. 9(b) shows that the optical gain increased from ~ 3.6 at 300 nm to 7.3 at 311 nm, decreased slightly to 7.14 at 317 nm, and reached ~ 8.6 at 325 and 332 nm. Thus, the increase in $\overline{H}_0$ with wavelength was not solely caused by the increase in incident solar irradiance. The normalized response was also dependent on the wavelength-dependent optical properties of the reactor components, including the reflector reflectance, $\rho(\lambda)$, the refractive index and radiation transmission through the borosilicate tube, and the refractive index and absorption coefficient of water. Although $\rho(\lambda)$ reached its maximum near 311 nm and decreased slightly at longer wavelengths, the optical gain continued to increase after 317 nm, indicating that the reflectance of the reflector was not the only

parameter driving the wavelength-dependent optical performance.

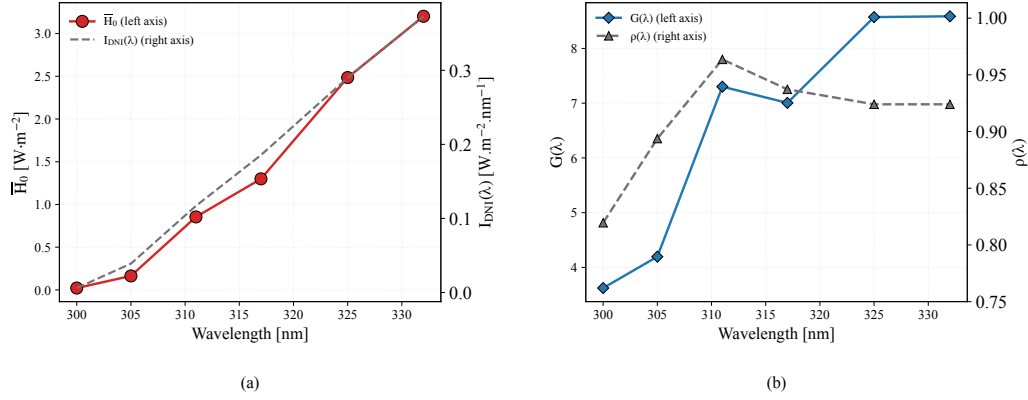


Figure 9: Wavelength-dependent optical response of the CPC reactor. (a) Mean fluence rate, $\bar{H}_0$ (left axis) and direct normal spectral irradiance, $I_{DNI}(\lambda)$ (right axis). (b) Optical gain, $G(\lambda)$ (left axis), and wavelength-dependent reflector reflectance, $\rho(\lambda)$ (right axis).

3.2.4. Effect of reflector material

The effect of reflector material was evaluated by comparing four representative materials characterized by Mbonimpa et al. [5]:

- Aluminum foil (Al foil)
- Polyethylene terephthalate/vacuum-deposited aluminum (PET/VDAI)
- Aluminum/polyethylene terephthalate/aluminum (Al/PET/Al)
- Corrosion-resistant-coated aluminum/polyethylene terephthalate (CRC/Al/PET)

As shown in Fig. 10(a), the Al/PET/Al film produced the highest mean fluence rate from 305 to 332 nm, followed by the CRC/Al/PET film. However, at 300 nm, the CRC/Al/PET film provided the highest optical gain. The Al foil and PET/VDAI film produced substantially lower and generally similar fluence rates throughout the transmission tube. Figure 10(b) shows that the differences among the materials persisted in optical gain. The optical gain generally increased between 300 and 332 nm, although its variation with wavelength was non-monotonic. The Al/PET/Al film exhibited the highest gain from 305 to 332 nm, whereas

the CRC/Al/PET film had the highest gain at 300 nm. For a given reflector material, the variation in optical gain with wavelength reflects the combined wavelength-dependent optical properties of the reflector, glass, and water. However, at a fixed wavelength, the differences among the four materials arise from their different reflectances.

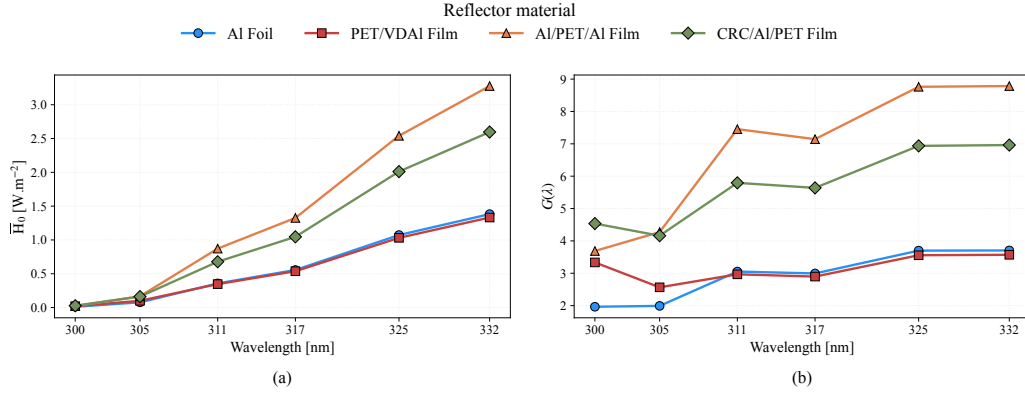


Figure 10: Effect of reflector material on (a) the mean fluence rate, $\bar{H}_0$ and (b) Optical gain, $G(\lambda)$, at selected wavelengths from 300 to 332 nm.

3.3. Relative importance of model inputs

Empirical equations can summarize how engineering responses vary with their inputs [43]. Here, a power-law correlation was used to describe how the optical gain varies with the model inputs. Using a logarithmic form of the correlation gives a linear relation in which each power-law exponent is a fitted slope as described in Eq. (32).

$$\ln G = \ln a_0 + a_\beta \ln \cos \beta + a_{CR} \ln CR + a_D \ln D_i + a_T \ln(1 - TF) + a_\rho \ln \rho. \quad (32)$$

Each exponent, $a_{var} = \partial \ln G / \partial \ln var$, represents the fractional change in optical gain per fractional change in its corresponding variable. Truncation is expressed through $(1 - TF)$, which is proportional to the retained reflector height. Each exponent was fit by least squares using simulations that varied one input at a time, with all other inputs held at their reference values. Fitting the correlation relative to the reference data at 317 nm removes the fitted constants as described in Eq. (33).

$$\frac{G}{G_{\text{ref}}} = (\cos \beta)^{0.984} \left(\frac{CR}{7} \right)^{0.861} \left(\frac{D_i}{3 \text{ cm}} \right)^{-0.006} \left(\frac{1 - TF}{0.55} \right)^{0.172} \left(\frac{\rho}{0.937} \right)^{1.230}, \quad (33)$$

where, $G_{\text{ref}} = 7.14$. Collector design determines how effectively the reactor uses the available irradiance. The evaluated geometry changes cannot compensate for low direct UV irradiance. Among the design variables, the reflectance of the reflector has the largest fit exponent. Its value exceeds unity, indicating that gain increases more than proportionally with reflectance. This behavior is consistent with observed behavior that some rays undergo multiple reflections before reaching the receiver. The concentration ratio exponent is below unity, meaning that optical gain increases more slowly than geometric concentration. The small positive truncation exponent indicates that removing the upper reflector sections has a limited effect on gain. The fit diameter exponent is close to zero. Moreover, the simulated diameter response is non-monotonic, with an interior maximum near 2.0 cm. Diameter has little effect on optical gain over the evaluated range, although it remains important for dose. At fixed flow rate, residence time scales with the tube cross-sectional area and thus with D_i^2 .

4. Conclusions

This study provides the first systematic spatial and spectral characterization of fluence rate within a continuous-flow solar CPC reactor across both environmental conditions and key design variables. The fluence-rate field was strongly nonuniform across the tube cross-section but varied negligibly along the interior reactor length, indicating that dose variability will depend primarily on transverse microbial trajectories. Normalizing the mean fluence rate by the incident direct-normal spectral irradiance separated the optical response of the reactor from the available solar resource. The maximum optical gain occurred at solar noon and was independent of site and date, while its diurnal variation was governed by hour angle and solar declination. Optical gain also varied non-monotonically with wavelength because of the combined spectral properties of the reflector, borosilicate glass, and water. Among the evaluated design variables, reflectance of the reflector had the greatest

influence on optical gain at the selected wavelength. Increasing the concentration ratio also improved gain but increased collector height and reflector requirements, whereas greater truncation reduced reflector size with only a modest reduction in gain.

The analysis conducted here identifies the effects of individual inputs but does not capture their interactions. A global sensitivity analysis could quantify both, as demonstrated for UV reactor performance by Naghavi et al. [44]. An accurate surrogate model of CPC optical performance could also reduce the need for repeated ray-tracing simulations. Such a model could support optimization and uncertainty quantification, as surrogate-based approaches have in UV disinfection reactors [45] and other engineering systems [46, 47, 48]. Future uncertainty analyses could include atmospheric variability, which affects the direct irradiance available at each site. The Tropospheric Ultraviolet and Visible (TUV) radiation model could provide modeled direct-beam spectral irradiance for wavelengths, sites, and dates beyond the available USDA measurements. Future optical studies could also examine alternative transmission-tube glasses, variations in water Ultraviolet transmittance (UVT), and scattering. Diffuse reflector reflection warrants particular attention. The present model assumed purely specular reflection, although a diffuse component may affect optical performance as shown by Seyedin et al. [45] and Hassanpour et al. [49]. Finally, dose measurements in a prototype could test the ray-tracing predictions under practical conditions. Ferrioxalate actinometry has been used to measure photon flux in a static batch configuration of solar CPC water disinfection reactor by Martínez-García et al. [50]. Spherical actinometry in a medium with matched optical properties could also help assess spatial optical variations [51].

This framework can ultimately support the design of continuous-flow CPC reactors for the solar conditions at any location, date, and time of day, while balancing optical performance. Coupling these results with fluid dynamic analyses such as computational fluid dynamics (CFD) and particle tracking velocimetry (PTV), and with wavelength-dependent inactivation kinetics will enable evaluation of system disinfection performance.

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5. Appendix

The ray-count convergence results are presented in Fig. S1, showing a decrease in E_{avg} from 24.28% to 2.17% as the number of traced rays increased from 3×10^7 to 3×10^9 .

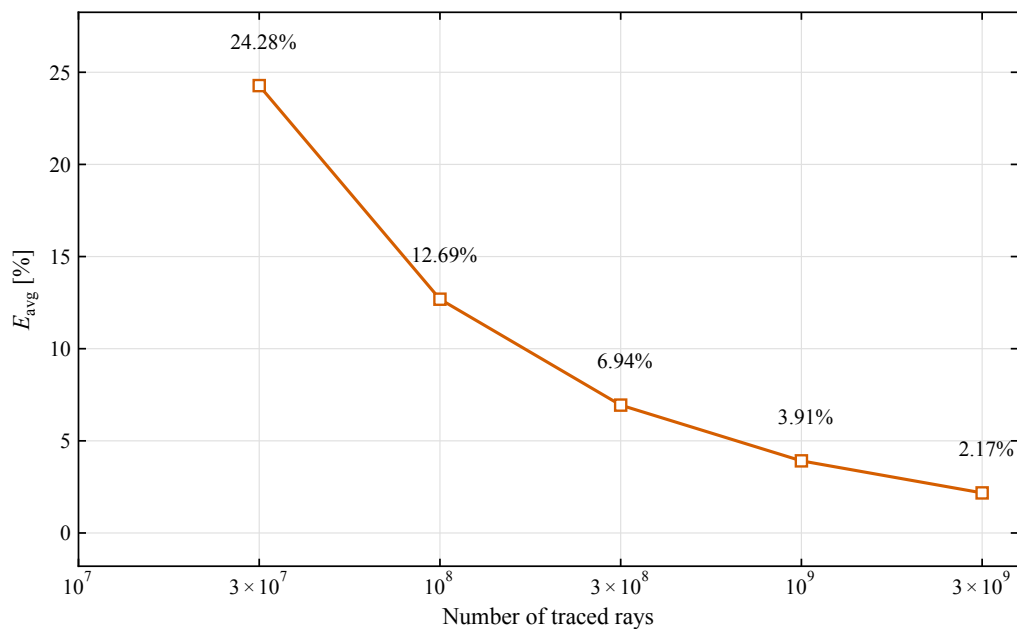


Figure S1: Average cell-wise relative error, E_{avg} , between fluence rate fields obtained at successive Monte Carlo ray count levels. The error was calculated over all valid in-pipe cells of the 49 interior recording planes.