Adaptive Event Triggereing Normed Observer Control of Nonlinear Time Delayed Systems

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Abstract

In this paper, event-triggering state-norm estimators are studied for nonlinear delayed systems also normed-observability concept is extended for these systems. Moreover, we established a Lyapunov–Krasovskii functional to obtain normed observability of the delayed nonlinear systems. Furthermore, an adaptive observer is developed such that convergence of the error system is guaranteed. The proposed event-triggering algorithm yields an event-based observer that ensures uniform ultimate boundedness of the tracking error.

1 Introduction

Sampling continuous-time signals plays an important role in modern engineering and various disciplines such as digital communications and signal processing benefit from it. Practically, there is an ever-increasing demand for removing expensive analog circuits and replacing them with cheap, multi-purpose and integrated digital frameworks which in turn aid us to prevent problems of frequency aliasing and so on. Besides many other numerous applications, make it forming a vital research direction in the modern technological world, [1-3].

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In control theory event-triggering strategy is based on a well-known sampling method called Lebesgue sampling, a none constant sampling law that occurs when a definite event takes place. This irregular control technique in the digital approach yields closed-loop system stability. Moreover, in this strategy, less mathematical operations are accomplished because the signal input is implemented when an event is triggering, which adds to the merits of it, [4].

Another outstanding and practical advantage of this technique is the designing of sensors. Sensors are the indispensable part of the control systems which valuate the system information and transfer it to the control, [5]. While transmitting data over a lossy channel, considering a control approach which requires minimal state transmission, is interesting. The event-triggering method addresses this issue and enables sensors to decide the transmission instant, [5-7].

Different control techniques have been applied with event-triggering control strategy, namely in [8-11]. In [8], the authors have considered an event-triggered controller together with delay conditions in a networked system. The subject of [9] is power systems, which are studied again under delays by introducing an event-triggering controller. The event-triggering technique also has studied through observer design, in [10] the main focus is studying linear multi-agent systems by applying an observer-based event-triggering strategy to achieve the consensus. Again in the case of linear systems in [11], an observer-based approach for event triggering control of the system is studied.

On many occasions, in the absence of the full system state, state estimation is implemented while only specific output information is available, but the access to the full state is required. The usual approach to handle this situation is introducing an observer that estimates the system state without observing outputs and inputs. Designing such an observer is a tough challenge for complicated nonlinear systems such as time-delayed systems. The manifestation of this concept was in [12] where authors provided the foundation of input-output-to-state stability (IOSS), for nonlinear systems. Also, the existence of these types of observers is proved in [12, 13]. Input-to-state stability (ISS) concepts have applied to an event-triggering feedback control method in [14, 15]. In [15], a discrete-time version of the ISS technique for the consensus problem of a multi-agent stochastic system is discussed.

Feedback event-triggering technique can be applied to overcome the over-provisioning problem as during the considered period may happen because of the system's performance. The proposed method involves a sensor transmitting the over-flowing information of the measurements and the information is transformed when the difference between measurements is observed so that the stability is verified. Consider the following systems

$$\dot{x}(t) = f(t, x, x_\tau, u), \qquad \text{a.e.} \quad t \geq t_0 \tag{1}$$

$$y = h(x(t)) \tag{2}$$

here Lipschitz function $f(t, x, u)$ on x is almost everywhere continuous on $(x, u) \in R^n \times R^m$, $f(0) = 0$ and h is a a real differentiable function.

$$\dot{x} = f(t, x, x_\tau, u) \tag{3}$$

In which here $x_\tau = x(t-\tau)$ for $\tau \in [t_0, t_f]$ and $t-\tau > 0$. If the computation of $k(x)$ is sufficiently fast, also if the sensors and actuators are sufficiently accurate, then the implementation will converge to the specification. Moreover, for the definite values of x we suppose $\aleph$-Assumption as follows

Assumption 1. *We assume*

$$\aleph = \aleph_0 \bigcup \acute{\aleph}_0 \tag{4}$$

$$where \quad \aleph_0 = Inf\{r \mid |x| \leq r|x_\tau|\}, \quad \acute{\aleph}_0 = Inf\{\acute{r} \mid |x_\tau| \leq \acute{r}|x|\}. \tag{5}$$

2 Problem Description and Required Prelimaniries

We investigate control problem (1), for $u \in R^m$ and $x \in R^m$ for which a feedback controller:

$$u = K(x) \tag{6}$$

Here $|.|$, $|.|_\infty$, $|.|_0$ are respectively the Euclidian norm, Supremum norm, and the absolute value. We need to remind some categories of functions as what follows

Definition 1. *A strictly increasing nonnegative function $\gamma : R^+ \bigcup\{0\} \to R^+ \bigcup\{0\}$ is said to be a $\mathcal{K}$- function if it is continuous, and $\gamma(0) = 0$.*

Definition 2. *A strictly increasing $\mathcal{K}$-function $\gamma : R^+ \bigcup\{0\} \to R^+ \bigcup\{0\}$, is said to be $\mathcal{K}_\infty$ function if $\lim_{s\to\infty} \gamma(s) = \infty$.*

Definition 3. *A strictly decreasing nonnegative function $\mu : R^+ \bigcup\{0\} \to R^+ \bigcup\{0\}$ is said to be an $\mathcal{L}$- function if it is continuous, and $\lim_{s\to\infty} \mu(s) = \infty$.*

Definition 4. *A strictly decreasing nonnegative function $\beta(s,t) : R^+ \bigcup\{0\} \times R^+ \bigcup\{0\} \rightarrow R^+ \bigcup\{0\}$ is said to be an $\mathcal{KL}$- function if if, for each fixed $t \geq 0$, the function $\beta(.,t)$ is a $\mathcal{K}$-function, and for each fixed $s \geq 0$, the function $\beta(s,.)$ is $\mathcal{L}$- function.*

Definition 5. *The system (1) is Input-to State Stable (ISS) if there exist $\mathcal{KL}$- function β and $\mathcal{K}$- functions γ so that*

$$|x(t)| \leq \max\{\beta(|x_\tau|, t - t_0), \gamma(|y|))\} \tag{7}$$

Definition 6. *The system (1) is Input/Output-to State Stable (IOSS) if there exist $\mathcal{KL}$- function β and $\mathcal{KL}$- functions γ_1, γ_2 so that*

$$|x(t)| \leq \max\{\beta(|x_\tau|, t - t_0), \gamma_1(\|u\|), \gamma_2(\|y(t,x,u)\|))\} \tag{8}$$

the control u is considered on $t \in [t_0, t_f)$, which $t_f = t_f(x_\tau, u)$. Now we define the difference functional D a real continuous function Lipschitz Φ as

Definition 7.

$$\dot{\Phi}(t, x, x_\tau) = \limsup_{h \to 0^+} \frac{\Phi_h(x) - \Phi(x)}{h} \tag{9}$$

where

$$\Phi_h(x) = \Phi(t + h, x + h, x_{\tau+h}) \tag{10}$$

For $t \in [t_0, t_f)$.

Definition 8.

Assumption 2. *An absolutely continuous function V which holds the following condition is said to be an Lyapunov candidate for system (1)-(2):*

1. *$max\{\alpha_1(|x_\tau(0)|), \alpha_1(|x(t)|\} < V(t, \Phi) < \alpha_2(|x_\tau|)$.*
2. *There are $\mathcal{K}_\infty$- functions σ_1, σ_2 so that*

$$\dot{V}(t, \Phi) \leq \rho(t)V(t, \Phi) + \sigma_1(|u|) + \sigma_2(|y|) - \alpha_3(|x_\tau|) \tag{11}$$

Where $y = y(t, x, u)$, $u = u(t, x)$, $\rho(t_0)\alpha_2(|x_\tau|) < \alpha_3(|x_\tau|)$.

Now we introduce the IOSS Lyapunov-Krasovskii functional as

$$\mathcal{W}(t, \Phi) = \exp(\int_{t_0}^{t} \rho(s)ds)V(s, \Phi) \tag{12}$$

In the above definition we consider restriction of positive none-increasing function $\rho(t)$ on $[t_0, t]$ so that $0 < \rho_m \leq \rho(t) \leq \rho_M$ and $\rho_m t - \lambda < \int_{t_0}^{t} \rho(s)ds < \rho_B$ in which $\rho_B = \rho_M l$, $\rho_m t_0 < \lambda$ and l is the length of $[t_0, t_f]$.

Without loss of generality, gently we will consider the following Assumption

Assumption 3. *Consider all of the previous assumption conditions, are verified. Moreover, for positive integers q_0, q_1, q we have*

$$\dot{V}(t, \Phi) \leq q_2 \rho(t) V(t, \Phi) + \sigma_1(|u|) + \sigma_2(|y|) - \alpha_3(|x_\tau|) \tag{13}$$

and

$$\mathcal{W}(t, \Phi) = q_1 \exp\Big(\int_{t_0}^{t} \rho(s)ds\Big) V(s, \Phi) \tag{14}$$

Such that $q = q_1 + q_1 q_2$, $q_0 = q_1 q_3$, $q < q_0$, $q\rho(t) - q_0\rho(t_0) < 0$, $q\rho(t_0)\alpha_2(|x_\tau|) < \alpha_3(|x_\tau|)$.

Lemma 1. *Suppose that n is non-negative and x verifies the following inequality*

$$x(t) \leq m(t) + \int_{t_0}^{t} n(s)x(s)ds, \tag{15}$$

for $t \in [t_0, t]$. Then

$$x(t) \leq m(t) + \int_{t_0}^{t} m(s)n(s)exp\Big(\int_{s}^{t} n(r)dr\Big)ds. \tag{16}$$

Definition 9. *A normed observer for system (1)-(2) is*

$$\dot{z} = g(t, z, z_\tau, u, y) \tag{17}$$

here inputs and outputs of the systems coincide with each others, moreover the following conditions should be verified

1. System (1)-(2) should be Input to State Stable (ISS) .
2. There must be found respectively $\mathcal{K}$ and $\mathcal{KL}$ functions as θ, β such that

$$|x(t)| \leq \beta(|x_\tau| + |z_\tau|, t - t_0) + \theta(t, z, u, y(t, x, u)) \tag{18}$$

for all $t \in [t_0, t_f)$. In this text arbitrary real function z is restricted to the interval $[t_0, t]$.

Definition 10. ***(Normed Observer Stabilization)***

The system (1)-(2) is said to be Normed Observer Stabilized if there exist a K function $\gamma(s)$, a KL function $\beta(s,t)$, and a real function $\sharp$ such that $\sharp(z) \subseteq Im(z)$, then

$$|e(t)| \leq \beta(|e(t)| + |z_\tau|, t - t_0) + \theta(t, z - \sharp(z), u, y(t, x, u)) \tag{19}$$

here $e(t)$ is the error of the proposed system.

Assumption 4. *In system (1)-(2) the function f satisfies the following inequality*

$$f(t, x, x_\tau, u) \leq f_0(t, x, x_\tau) + g_0(t, x, x_\tau)u \tag{20}$$

where g_0, f_0 are none-zero continously differentiable monotone increasig functions.

3 Existence of the Normed Observer

Theorem 2. *The following statements for the system (1)-(2) are equivalent:*

1. *The system is IOSS.*
2. *There is a normed observer for the system.*

Proof. Suppose that the system is IOSS we claim that $\dot{z} = -Az - Bz_\tau + \sigma_1(|u|) + \sigma_2(|y|)$ is a state normed observer. Input-State-Stability immediadely is achieved, as it can be considered as a linear stable system with the input $\sigma_1(|u|) + \sigma_2(|y|)$. We need to prove that the condition (16) is valid. Taking the first derivative of the $\mathcal{W}(t)$ we have

$$\dot{\mathcal{W}}(t) = \exp(\int_{t_0}^{t} \rho(s)ds)\dot{V}(t, x_t) + \rho(t)\exp(\int_{t_0}^{t} \rho(s)ds)V(t, x_t) \tag{21}$$

By applying inequalities in Assumption (2), we will have

$$\dot{\mathcal{W}}(t) \leq 2\rho(t)\mathcal{W}(t) + \exp(\int_{t_0}^{t} \rho(s)ds)(\dot{z} + z + z_\tau) \tag{22}$$

Also by considering Lemma 1 as well as the upper bound on $\exp(\int_{t_0}^{t} \rho(s)ds)$ on we will lead to

$$\mathcal{W}(t) \leq \mathcal{W}(t_0) + \int_{t_0}^{t} e^{\rho_B}\big(z + z_\tau + \dot{z}\big) + 2\int_{t_0}^{t} \rho(s)\big[\mathcal{W}(t_0) + \int_{t_0}^{t} e^{\rho_B}\big(z + z_\tau + \dot{z}\big)\big]\exp(\int_{s}^{t} 2\rho(v)dv)ds \tag{23}$$

For $\aleph$-property in Assumption 1, the integral involving $z + z_\tau + \dot{z}$, converts to $z_\tau + rz_\tau + r\dot{z}_\tau$. Besides, restriction of the functions on $[t_0, t]$ gives an approporiate positive constant upper bound M for the terms including z_τ.

$$
\begin{aligned}
\mathcal{W}(t) \leq \big(\mathcal{W}(t_0) + e^{\rho_B}(Ml(Ar+B) + re^{\rho_B}|z_\tau|\big) + 2\int_{t_0}^{t} \rho(s) \\
\exp(\int_s^t 2\rho(v)dv)\big[\mathcal{W}(t_0) + \int_{t_0}^{t} e^{\rho_B}\big(Az + Bz_\tau + \dot{z}\big)\big]ds
\end{aligned}
\tag{24}
$$

Again according to the Assumption 2,

$$
\mathcal{W}(t) \quad \leq \quad \big(\kappa_1 + re^{\rho_B}|z_\tau|\big) + 2\rho_B e^{2\rho_B}\Big[\mu_0(|z_\tau| + \alpha_2(|x_\tau|)) + \kappa_2\Big] \tag{25}
$$

Here $\kappa_1 = \mathcal{W}(t_0) + Ml(Ar+B)e^{\rho_B}$, $\kappa_2 = 2l\rho_B e^{2\rho_B}\mathcal{W}(t_0) + 2l\rho_B e^{3\rho_B}Ml(Ar+B)$, $\mu_0 \geq max\{le^{\rho_B}, lre^{\rho_B}\}$. Obviously there are strict increasing functions $\gamma_0(t)$, $\gamma_1(t)$ such that $\kappa_1 + re^{\rho_B}|z_\tau| \leq \gamma_0(|z_\tau|)$ and $\mu_0(|z_\tau| + \alpha_2(|x_\tau|)) + \kappa_2 \leq \gamma_1(|z_\tau| + \alpha_2(|x_\tau|))$, so the relation (24) can be stated as

$$
\mathcal{W}(t) \quad \leq \quad \big(\gamma_0(|z_\tau|)\big) + 2\rho_B e^{2\rho_B}\Big[\gamma_1(||z_\tau| + \alpha_2(|x_\tau|)\Big] \tag{26}
$$

Finally applying the definition of $\mathcal{W}(t)$, and the Assumpsion 1 on the bounds of $V(t, x_\tau)$

$$
|x(t)| \quad \leq \quad \alpha_1^{-1}\big(2\gamma_0(|z_\tau|)\big) + \alpha_1^{-1}(4l_0 e^{-\rho_m t}\Big[\gamma_1(||z_\tau| + \alpha_2(|x_\tau|)\Big]) \tag{27}
$$

By taking $\theta := \alpha_1^{-1}(2(.))$ and $\beta := \alpha_1^{-1}(4l_0 e^{-\omega t}\gamma_1(.))$, in which $l_0 = 2\rho_B e^{2\rho_B + \lambda}$ the existence of the norm observer is proved. Now, Suppose that there is a normed observer (16), and we need to show that the system is IOSS. By setting $z_\tau = 0$ and inserting the following relation in (16)

$$
|z(t)| \leq \max\{\beta(0, t - t_0), \gamma_1(\|u\|), \gamma_2(\|y(t, z, u)\|))\} \tag{28}
$$

We will obtain the following inequality

$$
|x(t)| \leq \beta(|x_\tau|, t - t_0) + \theta(t, z, u, y(t, x, u)) \tag{29}
$$

The equations (22),(23) complete the proof. $\square$

4 Event Triggering Policy

In this section, event-triggering algorithm in the presence of delay is discussed. Most of the times the delay role in the control input is neglected so its presence it seems unrealistic in some positions Despite mainly because one can. Meanwhile if he delay is large enough in comparison with the sampling periods then certainly it must be regarded in the computations.

By applying of the feedback law (3) on an embedded processor and sampling the state at time instants we get $u(t_i) = Kx(t_i)$ for the samples t_i which $i \in Z^+ \bigcup\{0\}$. Let δ_i be the delay associated with the triggering instant t_i . Besides we take into account updating the actuator values at introduced time nodes as $\{t_i + \delta_i\}$ here δ_i is the delay time to load the state information from the sensors, which is a sequence of instants that updates control input associated with the sequence t_i. Consequently, this delay will increase the error of the system till applying the next control. Here at first step we will determine the tolerance of the proposed delay and its effect on the error of the system. It must mentioned that each time interval the previous index is acting for instance on $[t_i, t_i + \delta_i)$ so the input $u(t_{i-1})$ is considered. To sum up the control input is defined as

$$u(t) = K(x(t_{i-1})), \quad \text{for } t \in [t_i, t_i + \delta_i) \tag{30}$$

$$u(t) = K(x(t_i)), \quad \text{for } t \in [t_i + \delta_i, t_{i+1} + \delta_{i+1}). \tag{31}$$

The strategy considered here for the sampling instants is as

$$t_{i+1} = min\{t \geq t_i + T \mid |e| > \frac{1}{\acute{r}}\alpha_2^{-1}(\frac{q_1\left[\sigma_1(|u|) + \sigma_2(|y|)\right]}{-\Pi}) \tag{32}$$

In sequel, the boundedness of the triggering method for the delayed case is studied.

Here σ_1, σ_2 are as in Definition 8, $\Pi(t) := q\rho(t) - q_0\rho(t_0)$. Also, we apply this notation $\Pi^+_{\max} := q_0\rho(t_0) - q\rho_m$, $\Pi^+_{\min} := (q_0 - q)\rho_M$, μ is a constant tuning positive parameter and finally

$$e(t) = x(t) - x(t_i). \tag{33}$$

Theorem 3. *Let us consider the system (1)–(2) and the control law (24)-(25).*

Then, the error in the plant remains bounded by provided (3.38) holds.

1. *For positive constant parameters ζ, a, b, and each $t \in [t_0, t_f]$ suppose that* $\pi \in (\Pi^+_{\min}\zeta, \Pi^+_{\max}\zeta) \subseteq (a, b) \subsetneq (0, 1)$. *Define*

$$\Delta_i = \frac{1}{L(1+r)} LnA(e^{\alpha_\star} - 1) \tag{34}$$

$$\mu_0 := \frac{L(r+1)}{\acute{r}\zeta} \tag{35}$$

$$\mu_1 := \frac{L(r+1)}{\pi}\frac{1}{2\acute{r}}\alpha_2^{-1}\big(\frac{q_1\big[\sigma_1(|u|)+\sigma_2(|y|)\big]}{-\Pi}\big) \tag{36}$$

$$\acute{\mu_0} := \big(\pi\lambda_0 - lp\big)\frac{L(r+1)}{\acute{r}\pi\zeta} \tag{37}$$

where $A > 0$ is a positive constant and the last relation is defined for a fixed $t \in [t_0, t_f]$, and $\max\{\frac{a}{\Pi^+_{\min}}, \frac{a}{\Pi^+_{\max}}\} < \zeta < \max\{\frac{b}{\Pi^+_{\min}}, \frac{b}{\Pi^+_{\max}}\}$, $\lambda_0 \neq \frac{lp}{\pi}$,$1 < \lambda_0 < \frac{b}{\pi}$. We also choose μ, $\alpha_\star$, respectively in the following ranges

$$\max\{\mu_0, \acute{\mu_0}\} < \mu < \min\{\mu_1\} \tag{38}$$

$$Ln(1+\frac{1}{A}) < \alpha_\star < Ln(1+\frac{\pi}{\mu plLA(r+1)}) \tag{39}$$

Also consider

$$\acute{\Delta}_i = \frac{1}{L(r+1)}Ln\Big[\frac{\frac{1}{\acute{r}}\big(\alpha_2^{-1}\big(\frac{\sigma_1(|u|)+\sigma_2(|y|)}{-\Pi}\big)}{p^\star}\Big] \tag{40}$$

$$\acute{\pi} \in (\pi, \frac{b}{\zeta}) \tag{41}$$

here

$$p^\star = \Theta p + \Lambda \tag{42}$$

in which

$$\frac{\pi\mu\zeta}{L(r+1)} \leq \Lambda \leq \frac{q_1\big[\sigma_1|u| + \sigma_2|y|\big]}{-2\acute{r}\Pi} \tag{43}$$

$$l \leq \Theta \leq \frac{q_1\big[\sigma_1|u| + \sigma_2|y|\big]}{-2\acute{r}\Pi|p|_0} \tag{44}$$

Then, $t_{i+1} - t_i - \delta_i > 0$ if $\delta_i \leq \Delta_i$.

2. *The error in the plant $(e_\mathfrak{p})(t) = x(t_i) - x(t)|_{t\in[t_{i+1}, t_{i+1}+\delta_{i+1})}$ remains bounded by the following inequality under the previous assumptions and stability of the closed loop system.*

$$e_\mathfrak{p}(t) \leq \big[p + \frac{\acute{\pi}\mu\zeta}{L(r+1)}\big]\big[\frac{\pi\mu\zeta}{L(r+1)}\big] \tag{45}$$

Proof. 1. The derivative of the error system (28) is

$$\begin{aligned}|\dot{e}(t)| &= |\dot{x}| \leq L|x| + L|x_\tau| + L|y| + L|u| \\ &\leq L|e + x(t_i)| + L|r(e + x(t_i)))| + L|u|_\infty + L|y|_\infty \\ &\leq |e|(L(1+r)) + (L|x(t_i)| + Lr|x(t_i)| + L|u|_\infty + L|y|_\infty)\end{aligned} \tag{46}$$

By using Lemma 1 for $t \in [t_i, t_i + \delta_i)$ we have

$$|e(t)| \leq pl.exp((L(1+r))(t - t_i)) \tag{47}$$

Where $p = L|x(t_i)| + Lr|x(t_i)| + L|u|_\infty + L|y|_\infty + 1$. and easily we lead to

$$|e(t)| \leq pl.exp((L(1+r))(\Delta_i)) \tag{48}$$

Considering the tolerance Δ_i as in (35)

$$|e(t)| \leq \frac{\pi\mu\zeta}{L(r+1)} \tag{49}$$

Again by using the same approach for $t \in [t_i + \delta_i, t_{i+1} + \delta_{i+1})$ we will obtain

$$|e(t)| \leq \acute{p}.\exp[L(r+1)(t - t_i - \delta_i)] \tag{50}$$

Here $\acute{p} = \frac{\pi\mu\zeta}{L(r+1)} + e(t_i + \delta_i) + pl$. By considering $t = t_{i+1}$ and applying condition (33) in (50) we get to

$$\frac{1}{\acute{r}}\alpha_2^{-1}\Big(\frac{q_1\big[\sigma_1(|u|) + \sigma_2(|y|)\big]}{-\Pi}\Big) \leq \acute{p}.\exp[L(r+1)(t_{i+1} - t_i - \delta_i)] \tag{51}$$

By computing $t_{i+1} - t_i - \delta_i$ from the previous relation we get to

$$t_{i+1} - t_i - \delta_i \geq \frac{1}{L(r+1)} Ln\Big[\frac{r\big(\alpha_2^{-1}\frac{\big(\sigma_1(|u|)+\sigma_2(|y|)\big)}{-\Pi}\big)}{p^\star}\Big] \tag{52}$$

In which $p^\star$ is defined as (43), Θ and Λ are repectively given by (44),(45).Without loss of generality we suppose that $p^\star = \Theta p + \Lambda$ so that $p \leq p^\star$, and the coefficients are determined so that the $p^\star$ and also $s \leq \alpha_2(s)$. and this will lead us to $t_{i+1} - t_i - \delta_i > 0$.

2. Here the changes of the error of the system while $t \in [t_i + \delta_i, t_{i+1} + \delta_{i+1})$ is studied, for nonenegative integer i. Since in $[t_i + \delta_i, t_{i+1} + \delta_{i+1})$, the control signal is not updated, the error in the plant grows and becomes maximum during the $[t_i + \delta_i, t_{i+1} + \delta_{i+1})$, the plant error increases and gets the maximum value during this interval.The error in the plant during $[t_i + \delta_i, t_{i+1} + \delta_{i+1})$ is $e_{\mathfrak{p}}(t)$ and denoted by $e_{\mathfrak{p}}(t) = e(t) - e(t_i)$. For $t \in [t_i, t_i + \delta_i)$, the error $|e|$ rises from zero to $\frac{\pi\mu\zeta}{L(r+1)}$ and overflow this quantity before the input signal is enetered. For about the interval $[t_{i+1}, t_{i+1} + \delta_{i+1}]$ when $\delta_i \leq \Delta_i$ the error satisfies $\frac{\acute{\pi}\mu\zeta}{L(r+1)}$ By applying Lemma 1 by the initial value $\frac{\acute{\pi}\mu\zeta}{L(r+1)}$ for the proposed interval we get to the following inequality

$$\begin{aligned} |e_{\mathfrak{p}}| &\leq (p + \frac{\acute{\pi}\zeta\mu}{L(r+1)}) exp[L(r+1)(t - t_{i+1})] \\ &\leq (p + \frac{\acute{\pi}\zeta\mu}{L(r+1)}) exp(L(r+1)\Delta_{i+1}) \\ &\leq \big[p + \frac{\acute{\pi}\zeta\mu}{L(r+1)}\big]\big[\frac{\pi\mu}{L(r+1)}\big] \end{aligned} \tag{53}$$

Where p is as the previous section.

$\square$

5 Normed Observer Stablization

Now we are going to represent our main theorem in this paper which is studying normed observer sablizability of the system (1)-(2).

Theorem 4. *Consider the system (1)-(2) under the control laws (55) and the update laws (56) , then the system is normed observer stablized.*

$$\begin{aligned} u = \omega_0 \Big[& Kx(t_i) + e + \epsilon + \frac{P^{-1} e \epsilon^T Q \epsilon}{|e|^2} + [x^T K^T P e + e^T P K x] + G\Big[(f^T P e + e^T P f + (Az - \sigma^\star)^T Q \epsilon\Big] \\ & + G\Big[\epsilon^T Q(A + rB|z| - \sigma^\star + r|z|^T B^T Q(e - x))\Big] - \frac{P^{-1} e \mathbf{W}}{2|e|^2}\Big] \end{aligned} \tag{54}$$

$$
\begin{aligned}
\dot{\hat{G}}(t) &= \big[\exp(-t)\Big(\big(f^T Pe + e^T Pf + (Az-\sigma^\star)^T Q\epsilon\big)^T Pe + e^T P\big(f^T Pe + e^T Pf + (Az-\sigma^\star)^T Q\epsilon)\big)\Big)\big]\\
&+ \big[\exp(-t)\Big(\big(\epsilon^T Q(A + rB|z| - \sigma^\star + r|z|^T B^T Q(e-x))\big)^T Pe + e^T P\big(\epsilon^T Q(A + rB|z| - \sigma^\star)\\
&+ r|z|^T B^T Q(e-x)\big)\Big) - \exp(-(t-\tau))\epsilon(t-\tau)^T S_1\epsilon(t-\tau) + \exp(-t)\epsilon^T S_1\epsilon\\
&- \exp(-(t-\tau))\epsilon(t-\tau)^T S_1\epsilon(t-\tau) - \exp(-(t-\tau))\acute{\epsilon}(t-\tau)^T S_2\acute{\epsilon}(t-\tau)
\end{aligned}
\tag{55}
$$

, and

$$
\omega_0 = \frac{1}{|g_0|+1}. \tag{56}
$$

Where $\hat{G}_1(0) = G_{10}, \hat{G}_2(0) = G_{20}$ *are the initial value of the parameters* $\hat{G}_1, \hat{G}_2$.

Proof. The general format of the observer is considered as

$$
\dot{z} = -Az - Bz_\tau + \sigma_1(|u|) + \sigma_2(|y|) \tag{57}
$$

Consider the Lyapunov-Krasovskii functional as

$$
\begin{aligned}
V(e) = \exp(-t)\big[e^T Pe + \epsilon^T Q\epsilon\big] + \int_{t-\tau}^{t} &(e^T S_0 e\\
+ \exp(-s)[\epsilon^T S_1\epsilon + \acute{\epsilon}^T S_2\acute{\epsilon}])ds &+ \frac{1}{2}\tilde{G}^T\tilde{G}
\end{aligned}
\tag{58}
$$

Here $e(t)$ is as (34), τ is the delay of the system (1)-(2), ϵ, $\acute{\epsilon}$, ξ is the error of the observation which is

$$
\begin{aligned}
\epsilon(t) &= x(t_i) - z(t),\\
\acute{\epsilon}(t) &= z(t_i) - z(t),
\end{aligned}
\tag{59}
$$

and $\hat{G}$ is the parameter estimation, also $\hat{G} = \big[G_1, G_2, G_3\big]$ is the estimation of the unknown parameter G and $\tilde{G} = G - \hat{G}$. Moreovere P, Q, S_0, S_1 are symmetric matrices of approporiate dimensions so that $e^T S_0 e + [\epsilon^T Pe + e^T P\epsilon] + \epsilon^T Q\epsilon = \mathbf{W}(t)$. Taking time derivative of (58) along the error trajectories (34) we get to

$$
\begin{aligned}
\dot{V}(e) &= \exp(-t)\Big[\dot{e}^T Pe + e^T P\dot{e} + \dot{e}^T Pe + \dot{\epsilon}^T Q\epsilon + \epsilon^T Q\dot{\epsilon}\Big] - \exp(-t)\Big[e^T Pe + \epsilon^T Q\epsilon\Big]\\
&+ e^T(t)S_0 e(t) - e^T(t-\tau)S_0 e(t-\tau) + \exp(-t)\epsilon^T S_1\epsilon - \exp(-(t-\tau))\epsilon(t-\tau)^T S_1\epsilon(t-\tau)\\
&+ \exp(-t)\acute{\epsilon}^T S_2\acute{\epsilon} - \exp(-(t-\tau))\acute{\epsilon}(t-\tau)^T S_2\acute{\epsilon}(t-\tau) + (\hat{G}(t) - G)^T\dot{\hat{G}}
\end{aligned}
\tag{60}
$$

After that considering the error dynamics (34), (59) and applying $\aleph_r(x_\tau)$-property for the system (1)-(2) together with Assumption (4) we have

$$
\begin{aligned}
\dot{V}(e) \leq \exp(-t)\Big[&(f_0+g_0u)^T Pe + e^T P(f_0+g_0u) + \epsilon^T Q(Az+Bz_\tau - \sigma_1(|u|) - \sigma_2(|y|)) + (Az+Bz_\tau \\
&- \sigma_1(|u|) - \sigma_2(|y|) + (Az+Bz_\tau - \sigma_1(|u|) - \sigma_2(|y|)^T Q\epsilon)\Big] + (1-r^2)e^T S_0 e + \exp(-t)\epsilon^T S_1 \epsilon \\
&- \exp(-(t-\tau))\epsilon(t-\tau)^T S_1 \epsilon(t-\tau) + \exp(-t)\acute{\epsilon}^T S_2 \acute{\epsilon} - \exp(-(t-\tau))\acute{\epsilon}(t-\tau)^T S_2 \acute{\epsilon}(t-\tau) \\
&+ (\hat{G}(t) - G)^T \dot{\hat{G}}
\end{aligned}
\tag{61}
$$

By considering $-\sigma_1(|u|_0) - \sigma_2(|y|_0) = -\sigma^\star$ where $y_0 = \min|y|$, $u_0 = \min|u|$, then by employing the following relation

$$
\begin{aligned}
-G\dot{\hat{G}} = &\Big[(f^T Pe + e^T Pf + \epsilon^T Q(A + rB|z| - \sigma^\star)\Big] + \Big[(Az - \sigma^\star)^T Q\epsilon + r|z|^T B^T Q(e-x)\Big] \\
&- \exp(-(t-\tau))\epsilon(t-\tau)^T S_1 \epsilon(t-\tau) + \exp(-t)\epsilon^T S_1 \epsilon - \exp(-(t-\tau))\epsilon(t-\tau)^T S_1 \epsilon(t-\tau) \\
&- \exp(-(t-\tau))\acute{\epsilon}(t-\tau)^T S_2 \acute{\epsilon}(t-\tau)
\end{aligned}
\tag{62}
$$

We obtain the next inequality

$$
\dot{V}(e) \leq \exp(-t)\Big[u^T Pe + e^T Pu\Big] + r|z_\tau|^T B^T Q|z_\tau| + |\acute{\epsilon}|^T S_2 |\acute{\epsilon}| + (1-r^2)e^T S_0 e + \hat{G}(t)^T \dot{\hat{G}}(t)
\tag{63}
$$

At this stage by utilizing the control input (55) we get to

$$
\begin{aligned}
\dot{V}(e) \leq exp(-t)\Big[&\omega_0\Big(Kx(t_i) + e + \epsilon + \frac{P^{-1}e\epsilon^T Q\epsilon}{|e|^2} + [x^T K^T Pe + e^T PKx] + G_1\Big[(f^T Pe + e^T Pf + (Az - \sigma^\star)^T \\
&Q\epsilon\Big] + G\Big[\epsilon^T Q(A + rB|z| - \sigma^\star + r|z|^T B^T Q(e-x))\Big] - \frac{P^{-1}eW}{2|e|^2}\Big)^T Pe + e^T P\omega_0\Big(Kx(t_i) + e + \epsilon + \frac{1}{|e|^2} \\
&+ P^{-1}e\epsilon^T Q\epsilon + [x^T K^T Pe + e^T PKx] + G\Big[(f^T Pe + e^T Pf + (Az - \sigma^\star)^T Q\epsilon\Big] + G_2\Big[\epsilon^T Q(A + rB \\
&|z| - \sigma^\star + r|z|^T B^T Q(e-x))\Big] - \frac{P^{-1}eW}{2|e|^2}\Big)\Big] + r|z_\tau|^T B^T Q|z_\tau| + (1-r^2)e^T S_0 e \\
&+ |z|^T S_1 |z| + \hat{G}(t)^T \dot{\hat{G}}(t)
\end{aligned}
\tag{64}
$$

Using

$$\hat{G}(t)^T\dot{\hat{G}}(t) = G\big[\exp(-t)\Big(\big(f^TPe + e^TPf + (Az-\sigma^\star)^TQ\epsilon\big)^TPe + e^TP\big(f^TPe + e^TPf + (Az-\sigma^\star)^TQ\epsilon)\big)\Big)\big] + G\big[\exp(-t)\Big(\big(\epsilon^TQ(A+rB|z| - \sigma^\star + r|z|^TB^TQ(e-x))\big)^TPe + e^TP\big(\epsilon^TQ(A+rB|z|-\sigma^\star\big) + r|z|^TB^TQ(e-x)\Big) + G(-\exp(-(t-\tau))\epsilon(t-\tau)^TS_1\epsilon(t-\tau) + \exp(-t)\epsilon^TS_1\epsilon - \exp(-(t-\tau))\epsilon(t-\tau)^TS_1\epsilon(t-\tau)) + G(-\exp(-(t-\tau))\acute{\epsilon}(t-\tau)^TS_2\acute{\epsilon}(t-\tau)) \tag{65}$$

From Lemma 2, and if Ω satisfies the following inequality

$$K^TP + PK + P + \Omega < 0 \tag{66}$$

then we lead to the desired result

$$\dot{V}(e) \leq exp(-t)[\lambda_{\min}(\Omega)e^Te + r\lambda_{\min}((B^TQ)|z_\tau|^T|z_\tau|) + \lambda_{\min}(S_2)|\acute{\epsilon}|^T|\acute{\epsilon}| \tag{67}$$

Consequently, it yields

$$|e(t)| \leq \alpha_1^{-1}(2[exp(-t)[\lambda_{\min}(\Omega)e^Te + \lambda_{\min}(B^TQ)|z_\tau|^T|z_\tau|) + \alpha_1^{-1}(2[\lambda_{\min}(S_2)|\acute{\epsilon}|^T|\acute{\epsilon}|]) \tag{68}$$

By taking $\beta := \alpha_1^{-1}(2[.])$, $\theta := \alpha_1^{-1}(2[.])$ normed observer stablizability of the system (1)-(2) is proved.

□

Corollary 4.1. *If $\acute{\epsilon} \to 0$ as $t \to \infty$ then the system (1)-(2) with the contol input (55) and adaptive laws (56) is asymptotically normed observer.*

Proof. The proof is easily achieved by considering the assumptions on (68).

□

Example 1

6 Conclusion

The Lyapunov–Krasovskii functional to investigate the problem of the event-triggered normed estimator of nonlinear delay systems is generalized. In this paper, an event-based observer method for nonlinear time-delayed systems by applying an adaptive controller is introduced which ensures the stability of the event triggering technique.

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