

Manufacturability-Driven Topology Optimization Design Method for Structural Continuous Yarn Polymer Composite Parts

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Abstract

Polymer-matrix composites (PMC) are widely used in dynamic structural applications because of their outstanding specific mechanical properties. PMC manufacturing is inherently additive. However, topology optimization (TO) of PMC parts is limited by challenges in manufacturing irregular lattice geometry parts with continuous yarns extending along 3D paths aligned with local principal stress directions. This work introduces a design method for PMC parts featuring 3D TO, focused on manufacturability using continuous yarns. The method was developed from observation of winding and interlacing collimated yarns into 3D TO PMC lattices performed by operators. The outcome of the design method consists in continuous 3D paths for flexible arrays of tensioned collimated yarns wound around curved surface guides located on the periphery of the part envelope, and interlaced with each other within the part envelope. The irregular lattice constitutes the reinforcement of 3D TO PMC parts. The design method features 4 sequential steps: static linear elastic analysis and determination of the principal stress vector field, generation and selection of stress paths, k -neighbours triangulation for lattice generation, and yarn path generation. Collectively, a limited number of yarn paths cover all lattice segments with minimal overlap, entering and exiting the part envelope at points selected to facilitate manufacturing. The algorithms for stress path generation and selection, k -neighbours triangulation and generation of interlacing yarn paths are described. The part designer selects parameters at different points of the design process. The design method is illustrated using an asymmetric torsion link demonstrator subjected to tension.

Keywords

Polymer composites, continuous yarn, topology optimization, design, manufacturing

Symbols

C_i	Clustering term, stress paths
c_1, c_2, c_3	Weighing factors, stress paths
$\mathbf{cp}_i$	Centre point of i^{th} candidate stress path
D	Dijkstra's distance function
d	Euclidean distance, Dijkstra's algorithm
dn	Number of times a graph edge was previously featured in yarn paths
dp	Penalty factor, Dijkstra's algorithm
$\mathbf{F}$	External force vector
$\mathbf{gm}$	Position of centre points geometric median, stress paths
i	Counter, candidate stress paths
$\hat{\mathbf{i}}$	Unit vector along axis $\mathbf{x}$
j	Counter
$\hat{\mathbf{j}}$	Unit vector along axis $\mathbf{y}$
$\mathbf{K}(\rho)$	Stiffness matrix
$\mathbf{K}(\rho, \theta)$	Orthotropic stiffness matrix
k	Number of nearest neighbours to a lattice point by Euclidean distance, number of segments extending from lattice point
$\hat{\mathbf{k}}$	Unit vector along axis $\mathbf{z}$
$\bar{L}$	Average graph edge length
$l(\mathbf{x})$	Direction cosine, axis $\mathbf{x}$
M_i	Clustering term, stress paths

$m(\mathbf{x})$	Direction cosine, axis y
$n(\mathbf{x})$	Direction cosine, axis z
n	Number of equidistant points sampled along a stress path
n_f	Number of segments defining surfaces where loads and boundary conditions are applied
P_i	Prioritization index, stress paths
$\mathbf{p}_j$	Coordinates of j^{th} point, stress paths
$\mathbf{p}_{j+1}$	Coordinates of j^{th} next point, stress paths
q	Number of selected stress paths
$S(\mathbf{x})$	Stress paths
S_i	Stress-length product term, stress paths
s	Number of segments in lattice, k -neighbours triangulation
$U(\rho)$	Total strain energy stored in isotropic part
$U(\rho, \boldsymbol{\theta})$	Total strain energy stored in orthotropic part
u	Candidate edge midpoint, Dijkstra's algorithm
$\mathbf{u}$	Displacement field
V_f	Fibre volume fraction
V_{max}	Maximal material volume fraction
v	Target envelope vertex, Dijkstra's algorithm
$\mathbf{x}$	Position vector
$*$	Normalization, stress paths
$\boldsymbol{\varepsilon}(\rho)$	Strain field
$\boldsymbol{\theta}(\mathbf{x})$	Fibre orientation field
$\rho(\mathbf{x})$	Material density field, $\rho = 1$ solid, $\rho = 0$ void
$\boldsymbol{\sigma}(\rho)$	Stress field
σ_j	Magnitude of stress along segment j , stress paths
Ω	TO domain

1. Introduction

Polymer-matrix composites (PMC) are widely used in dynamic structural applications because of their outstanding specific mechanical properties. In parallel with advances in PMC technology, additive manufacturing (AM) processes for metal parts have evolved rapidly in recent decades. Topology optimization (TO) is central to metal AM, as it enables the positioning of metal along high stress paths only for set load cases. Together, AM and TO enable the manufacturing of metal parts featuring irregular lattice geometry that are highly efficient on a mass basis [1].

PMC manufacturing is inherently additive. However, few cases of structural PMC parts featuring TO were reported [2]. Manufacturing PMC parts where continuous yarns extend along paths aligned with high stress directions in 3D is challenging. Current PMC manufacturing processes are designed for producing beams and shells featuring aligned fibres, as opposed to irregular lattice geometries typical of TO. PMC trusses featuring a repeated structural unit and some level of post-cure assembly were presented [3, 4], but few examples of non-repeating irregular lattice PMC parts were reported [5, 6]. Simpler PMC parts featuring TO were reported, either with continuous yarns extending within 2D layers only where this can constitute an optimal solution [3,7] or featuring short fibres limited to semi-structural applications. As a result, AM metal parts may outperform PMC parts despite the superior specific properties of PMC materials, allowing metals to challenge PMCs over their main advantage: specific structural performance [1, 2].

This work presents a design method for structural continuous yarn PMC parts featuring 3D TO. The method focuses on manufacturability using continuous yarns. It defines continuous 3D paths for flexible arrays of collimated yarns, either dry or pre-impregnated with an uncured polymer. Tensioned collimated yarns wind around curved surface guides located on the periphery of the part envelope, and interlace with each other within the part envelope, forming an irregular lattice. The polymer matrix is consolidated and cured subsequently to winding and interlacing, leading to a PMC part where yarns follow prescribed paths in 3D, hence featuring 3D TO and offering increased specific structural performance. The

outcome of the design method presented in this work is the 3D paths followed by the collimated arrays of yarns within 3D TO PMC parts.

The 3D TO PMC design method presented in this work features 4 sequential steps. In step 1, static linear elastic structural finite element analysis (FEA) is performed on isotropic material bound by the part envelope, for set loads and boundary conditions. The principal stress vector field is extracted. In step 2, stress paths locally tangent to the principal stress vector field are generated from seed points located at domain boundaries and throughout the part volume. Selected stress paths are retained based on stress levels, path length and distribution within the part envelope. In step 3, k -neighbours triangulation is used for generating a continuous triangulated lattice from equidistant points along the selected stress paths and points of load and boundary condition application [8]. In step 4, continuous interlacing yarn paths are generated using graph theory, Dijkstra's algorithm [9] and a distance prioritization scheme. Collectively, a limited number of yarn paths cover all lattice segments with minimal overlap, entering and exiting the part volume at selected points. Yarns are thickened for visualization purposes by attributing them circular cross-sections.

The different algorithms for stress path generation and selection, k -neighbours triangulation and generation of interlacing yarn paths are described. Opportunities for choices and intervention by the 3D TO PMC part designer are discussed.

2. Literature

TO is a mathematical method used for determining optimal material distribution within a given part envelope subject to objectives such as minimizing specific part compliance. Approaches such as the solid isotropic material with penalization (SIMP) method determine which points of the discretized domain are material and which are void [1]. The problem is formulated as:

$$\min_{\rho} U(\rho) = \int_{\Omega} \boldsymbol{\sigma}(\rho) : \boldsymbol{\varepsilon}(\rho) d\Omega \quad (1)$$

subject to:

$$\mathbf{K}(\rho) \mathbf{u} = \mathbf{F} \quad (1.1)$$

$$\int_{\Omega} \rho(\mathbf{x}) d\Omega \leq V_{max} \quad (1.2)$$

where $\mathbf{x}$ is the position vector; $\rho(\mathbf{x})$ is the material density field with $\rho = 1$ for solid and $\rho = 0$ for void; $U(\rho)$ is the total strain energy stored in the structure; $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the stress and strain fields; $\mathbf{K}(\rho)$ is the global stiffness matrix; $\mathbf{u}$ is the displacement field; $\mathbf{F}$ is the external force vector; V_{max} is the maximal material volume fraction; and Ω is the domain where TO is performed. Several methods were developed for solving TO problems including homogenization [10], density-based approaches [11], level set methods [12], moving morphable component optimization [13] and bi-directional evolutionary structural optimization [14].

TO of orthotropic materials such as PMCs involves additional design variables as both material distribution and fibre orientations are optimized [3, 4, 7, 15]:

$$\min_{\rho, \boldsymbol{\theta}} U(\rho, \boldsymbol{\theta}) = \int_{\Omega} \boldsymbol{\sigma}(\rho, \boldsymbol{\theta}) : \boldsymbol{\varepsilon}(\rho, \boldsymbol{\theta}) d\Omega \quad (2)$$

subject to:

$$\mathbf{K}(\rho, \boldsymbol{\theta}) \mathbf{u} = \mathbf{F} \quad (2.1)$$

$$\int_{\Omega} \rho(\mathbf{x}) d\Omega \leq V_{max} \quad (2.2)$$

where $\boldsymbol{\theta}(\mathbf{x})$ is the fibre orientation at each point and $\mathbf{K}(\rho, \boldsymbol{\theta})$ is the orthotropic stiffness matrix. TO methods for orthotropic materials use sequential or concurrent optimization of material distribution, or density, and fibre orientations [16].

Sequential optimization decouples the optimization of density and orientations, reducing computational load and numerical instabilities. Lee et al. [17] proposed a three-step sequential compliance reduction method where step 1 determines material distribution via isotropic density-based TO, and steps

2 and 3 optimize a continuous fibre orientation field and discrete fibre paths. Three-step approaches were also proposed by Nikoleta [18] and Peeters et al. [19] that leverage lamination parameters to produce optimized laminates. Principal stress fields were used for fibre path generation in sequential TO. Zhang et al. [6] and Li et al. [20] proposed two-step sequential processes where step 1 determines material distribution via density-based isotropic TO and step 2 uses a streamline approach for generating fibre paths: the principal stress field obtained in step 1 is used for generating fibre paths tangent to maximum principal stress orientations. Zhou et al. [21], Setoodeh et al. [22] and Xu et al. [23] refined topology and fibre orientations iteratively. Pedersen [24] showed that maximizing stiffness in PMCs requires aligning fibres with maximum principal stress directions. Similarly, Yang et al. [25] increased part stiffness by aligning continuous fibre orientations with principal stress directions obtained from FEA.

Concurrent TO optimizes material distribution and fibre orientations simultaneously; stress paths tangent to optimized fibre orientations lead to increased stiffness and continuous yarn paths [26]. The concurrent problem is more complex, requiring multi-field optimization often utilizing gradient-based methods [3, 27], level-set methods with fibre orientations tracking [28] or genetic algorithms [29] to solve coupled design variables. However, it may improve structural performance in certain cases [16]. Kim et al. [30] proposed a hybrid approach where a density-based method determines the presence of material in a given unit cell and homogenization sets orientation in that unit cell. Size and orientation parameters of unit cells are the optimized design variables.

The above methods originate from a computational perspective and typically do not feature explicit provision for manufacturability. Most work on TO for PMCs addresses fibre path generation for 2D or 2.5D designs only [16, 31]. TO methods proposed by Schmidt et al. [32], Jung et al. [33], Luo et al. [34] and Ye et al. [35] are among rare works that derive 3D continuous fibre trajectories for TO. Schmidt et al. [32] hybridized a gradient-based algorithm with simulated annealing to tackle highly non-convex concurrent TO. The approach was validated in different design examples. Similarly to Kim et al. [30], Jung et al. [33] used a multiscale approach combined with homogenization. The authors additionally used a de-homogenization scheme to create continuous 3D fibre paths from the optimized fibre orientation field. Luo

et al. [34] used density-based methods to optimize initial part topology, while continuous fibre paths were generated using a method inspired by field-based toolpath generation methods combined with a continuity constraint.

TO typically results in parts with intricate irregular lattice geometry. In AM a model of the lattice is typically sliced along a single direction, and flat layers are deposited sequentially until the part is created. Conventional slicing and layered printing cannot be applied to most structural PMC parts due to constraints brought by yarn continuity and orientations, for 3D yarn paths featuring complex geometry and morphology [3, 16]. AM of continuous yarns PMC parts is challenged by limitations in laying structural yarns along preferred load paths beyond trivial cases where yarns only extend in-plane [25, 26, 36-39]. Further work on AM approaches is conducted, focusing on both software [40] and hardware aspects [41-43].

Fused Deposition Modelling (FDM) and Fused Filament Fabrication (FFF) using a filament combining a thermoplastic polymer and continuous reinforcing fibres enable the production of continuous yarn 3D TO PMC parts with generally constant fibre-volume ratios and some control over yarn orientations [44]. Still, layered printing severely limits continuous yarn orientations in 3D domains and parts.

The free-hanging 3D printing method proposed Liu et al. [45] can produce overhanging and undercut structures by guiding nozzle motion in 3D, leading to continuous fibres better aligned with 3D load paths in parts. However, avoiding fibre breakage, debonding and structural deformations while printing remain key challenges. Liao [46] produced non-periodic PMC lattices by interlacing continuous dry yarns subjected to individually controlled tensions, bringing yarns to optimal 3D geometric configuration and avoiding limitations inherent to conventional 3D printing techniques. In Liao's work, interlacing patterns and initial structure geometry were generated arbitrarily. Additive Fiber Tethering (AFT) was proposed for manufacturing continuous yarn PMCs featuring topology optimization, inspired by truss-based lattices [47, 48]. The technologies illustrate the possibility of manufacturing topology optimized PMC parts where yarns are wound around guides and interlace directly within a delimited 3D envelope, without other support, as envisaged in the present work.

In this work, stress paths constitute the foundational elements of triangulated 3D lattice parts made from flexible yarns wound and interlaced under tension, then consolidated and cured subsequently. The cured lattice consists in a number of interconnected straight segments, leading to a 3D TO PMC part where interconnected collimated continuous yarns extend along high stress orientations within the part envelope. Continuous collimated yarns follow paths along a number of consecutive straight segments, with direction changes along a path happening at winding and interlacing points. Continuous yarn lattice structures are designed in 4 steps described in the following sections.

3. Static linear elastic structural analysis and principal stress vector field

In step 1, a static linear elastic structural finite element analysis (FEA) is performed on continuous isotropic material contained within the part envelope for set loads and boundary conditions, using commercial software. Principal stress vectors are determined for each element by calculating eigenvectors of the Cauchy stress tensor [49]. A 3D vector field is produced by selecting the direction vectors of highest principal stress amplitudes. In this work, commercial software Abaqus/CAE 2022 was used. Figure 1 shows an asymmetric torsion link demonstrator subjected to resulting tension along axis z applied at one end and surfaces with 6 locked degrees of freedom at the other end. Converging simulations resulted from a mesh featuring 4713 tetrahedral elements and material with density 2.70 g/cm^3 , isotropic Young's modulus 70.0 GPa and Poisson's ratio 0.33 . The principal stress vector field appears in Figure 2.

4. Generation and selection of stress paths

In step 2, a large number of stress paths $S(\mathbf{x})$ tangent to the principal stress field are generated from seed points positioned as detailed below. The paths are defined as:

$$\nabla S(\mathbf{x}) = l(\mathbf{x}) \hat{\mathbf{i}} + m(\mathbf{x}) \hat{\mathbf{j}} + n(\mathbf{x}) \hat{\mathbf{k}} \quad (3)$$

where $l(\mathbf{x})$, $m(\mathbf{x})$ and $n(\mathbf{x})$ are direction cosines relative to unit directional vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. Bidirectional stress paths grow from seed points through successive elements until the part envelope is reached or a change in direction exceeds a set threshold. Seed points are positioned along a uniform rectilinear grid with feature-based spacing, ensuring that stress paths reach every zone of the part. Additional seed points are positioned in an adjustable fraction of the most loaded elements, ensuring that stress paths are present in high stress zones. Finally, seed points are positioned on envelope edges where loads and boundary conditions are applied, ensuring that stress paths are present at these locations. Seed points and initial stress paths for the asymmetric torsion link appear in Figure 3.

A limited number q of stress paths are selected. Stress paths generated from points of application of loads and boundary conditions are imposed in the selection. A prioritization index P_i based on stress levels along the path, path length and distance is calculated for all other initial stress paths i :

$$P_i = c_1 S_i^* + c_2 C_i^* \quad (4)$$

where S_i and C_i are the stress-length product and clustering terms; superscript* denotes normalization; c_1 and c_2 are weighing factors; and:

$$S_i = \sum_{j=1}^{n-1} \sigma_j \|\mathbf{p}_j - \mathbf{p}_{j+1}\| \quad (4.1)$$

$$C_i = \|\mathbf{cp}_i - \mathbf{gm}\| \quad (4.2)$$

S_i promotes paths that carry higher stresses and extend longer in the part envelope; $\mathbf{p}_j$ and $\mathbf{p}_{j+1}$ are the coordinates of the j^{th} and next point delimiting a straight segment along a stress path; σ_j is the magnitude of principal stress at point j ; n is the number of equidistant points sampled along the stress path. C_i prevents local clustering of selected stress paths; $\mathbf{cp}_i$ is the centre point of the i^{th} candidate stress path and $\mathbf{gm}$ is the position vector of the geometric median of centre points. Stress path selection is iterative, with prioritization

indices calculated for every stress path remaining at each iteration. Additional utility functions enable the designer to copy symmetric paths in symmetric parts loaded symmetrically, and to displace or merge overlapping paths. Selected stress paths for the asymmetric torsion link appear in Figure 4.

5. *k*-neighbours triangulation for lattice generation

In step 3, a *k*-neighbours triangulation algorithm is used for generating a continuous, interlaced lattice of flexible yarns from points along the selected stress paths and points where loads and boundary conditions are applied. Physically, points located on stress paths within the part envelope paths are locations where flexible yarns interlace under tension, prior to consolidation and cure. Straight segments extend between these points.

Straight lattice segments are formed between a given point and its *k* nearest neighbours, where *k* is defined by the part designer. A first set of segments are constructed for defining surfaces where loads and boundary conditions are applied, and a second set of segments are constructed between *n* sequential points sampled along each selected stress path. Then, for every point in the lattice, *k* straight segments extending from the point to its *k* closest neighbours by Euclidean distance are created. The number *s* of segments in a structure generated by *k*-neighbours triangulation is:

$$s(k) = n_f(k + 1) + q(n(k + 1) - 1) \quad (5)$$

where *n_f* is the number of segments defining surfaces where loads and boundary conditions are applied; *k* is the number of neighbours; *q* is the number of selected stress paths; and *n* is the number of points sampled per stress path. The resulting structure is defined by values of *k* and *n* ; different values may be probed by the part designer. Structure complexity increases with *k* for set numbers of main stress paths *q* and points sampled per path *n* . Figure 5 shows a triangulated structure generated for the asymmetric torsion link.

6. Yarn path generation

Step 4 consists in defining the different paths along which collimated flexible yarns extend within the 3DTO PMC part envelope under tension, wind around surface guides and interlace among themselves. Each yarn path follows a different sequence of lattice segments. Collectively, yarns laid along these paths constitute the complete reinforcement of the 3D TO irregular lattice.

The paths are generated using graph theory, a distance prioritization scheme along with Dijkstra's algorithm [9]. Physical segments in the lattice are represented by edges in the graph; physical distances are preserved. The foremost challenge in generating a set of paths that cover all segments making the irregular lattice is the very large number of possible combinations, which increases rapidly with the number of edges in the graph. Requirements were set for guiding yarn path generation using Dijkstra's algorithm. Firstly, paths must originate and end on the part envelope, enabling adequate securing and tensioning of the yarns in physical manufacturing. Secondly, all paths must cross the envelope through the same two entry/exit points, further facilitating manufacturing. Thirdly, these entry/exit points are positioned at opposite ends along the longest dimension, promoting a smaller number of longer paths covering all the lattice, again facilitating manufacturing. Lastly, different paths may overlap over any given segment in the final lattice part. However, corresponding graph edges reused upon generating a new path in the graph are penalized, reducing the number of such overlaps and balancing the number of paths, hence yarns, present along any given segment in the final lattice part. As such, segments with generally similar cross-section areas are generated upon manufacturing.

The algorithm proceeds as follows. Two envelope vertices furthest from each other along the longest dimension of the envelope are identified. Then, graph edges are sorted in decreasing order of shortest distance measured from the edge endpoint to the closest selected envelope vertex. Dijkstra's algorithm [9] is executed with the selected envelope vertex to which it is closest, determining the shortest path from the edge endpoint to reach the selected envelope vertex. At this stage, the generation of yarn paths begins. Each path is progressively formed by merging the original edge with the shortest paths reaching the envelope at the selected vertices, on either side of the edge, creating an allowable yarn path. The following distance function is used for Dijkstra's algorithm:

$$D(u, v, n) = d(u, v) + dp \cdot dn \cdot \bar{L} \quad (6)$$

where d is the Euclidean distance; u is the candidate edge midpoint; v is the target envelope vertex; dp is a penalty factor; dn is the number of times the candidate edge is already featured in the set of constructed yarn paths; and $\bar{L}$ is the average graph edge length. The Euclidean distance term minimizes the distance between the path being formed and the envelope vertex targeted, while the average edge length term serves as an edge overlap penalty. The edge overlap penalty scales linearly with multiple edge use, ensuring that repeatedly used edges are penalized relatively to typical edge lengths in the physical lattice, thereby discouraging overlap.

Figure 6 shows four examples of yarn paths for the simplified asymmetric torsion link; the complete irregular lattice required 24 paths. Figure 7 shows the numbers of yarn paths present over each segment. The figure shows that the numbers are generally contained. An additional utility function enables the designer to modify the paths manually. Figure 8 shows a representation of the 3D TO PMC part where segments were attributed arbitrary circular cross-sections for visualization purposes.

7. Discussion

The design method presented above originates from a pragmatic manufacturing rationale for 3D TO PMC lattice parts featuring continuous yarns, where collimated flexible yarns under tension are progressively wound around guide surfaces and interlaced within a part envelope. Manufacturing uses two robotic manipulators exchanging a yarn carrier and directing tensioned collimated yarns around guide surfaces, their support structure and other yarns. The manufacturing apparatus and operations that it performs were inspired by manual winding and interlacing of collimated dry yarns into 3D TO PMC lattice parts performed by operators [46]. The design method incorporates elements of cited work such as the usage of continuous paths tangent to local principal stress vectors for guiding yarn orientations, and the interlacing of yarns

within the part envelope to change their directions without systematic usage of physical guides. It achieves its objective of defining final paths extending in 3D for collimated yarns constituting the reinforcement of 3D TO PMC lattice parts. Practical manufacturability was a foremost concern.

Routine manufacturing involves other considerations. Yarn paths defined by the method presented above correspond to the final lattice reinforcement configuration in 3D TO PMC parts. Paths followed by robotic manipulators are different due to obstacle avoidance and the progressive nature of yarn positioning, particularly when interlacing within the part envelope without using guide surfaces. Yarns are positioned sequentially and progressively, hence intermediate paths are used for positioning a given yarn prior to it being interlaced with another yarn at a latter stage of manufacturing. Final equilibrium positions between interlacing yarns depend on individual yarn tensions, inter-yarn friction, and their variability [46].

Manufacturing positions a number of collimated flexible yarns along each path. Total fibre cross-sectional area is constant along the segments forming a given path; it may vary for different paths. Segment cross-sectional area and fibre volume fraction v_f may vary; more significantly, segment cross-sectional shape is determined by surface guide geometry and the behaviour of interlacing collimated yarns [50]. Larger segment cross sections can warrant the usage of vertex contraction, a graph theory operation that merges adjacent vertices, effectively removing an edge from the graph and the corresponding segment from the lattice, reassigning incident edges to the new vertex. The transformation simplifies the graph, and lattice, while preserving its connectivity properties [51].

Lattice parts are finalized by introducing and consolidating the matrix using curable thermoplastic or thermosetting liquid polymers or prepreg yarns, in partial mould-less processing or temporary moulds. All forms proceed differently and present specific challenges related to consolidation and external surfaces.

All above technical aspects: robotic path planning for intermediate yarn paths and obstacle avoidance; segment cross-sections for guided and interlacing collimated yarns with interference correction using vertex contraction; processes for introducing and consolidating the polymer matrix, warrant standalone documentation included in parallel publications.

8. Conclusion

A design method for structural PMC lattice parts featuring continuous yarns and 3D topology optimization is presented. The method was developed from observation of manual winding and interlacing of collimated yarns into 3D TO PMC lattice parts performed by operators; practical manufacturability was a foremost concern. The method features 4 steps: static linear elastic structural analysis and determination of principal stress vector field; generation and selection of stress paths; k -neighbours triangulation for lattice generation; and yarn path generation. The PMC 3D TO part designer selects parameters and they can intervene at different points of the design process. All steps of the design method are illustrated through the case of an asymmetric torsion link demonstrator subjected to tension. Technical aspects of manufacturing including robotic path planning, segment cross-sections and matrix consolidation are discussed in parallel publications.

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Figure captions

Figure 1. Asymmetric torsion link demonstrator subjected to resulting tension along axis z

Figure 2. Principal stress vector field in axisymmetric torsion link

Figure 3. Seed points and initial stress paths for axisymmetric torsion link

Figure 4. Selected stress paths for axisymmetric torsion link

Figure 5. Triangulated structure generated for axisymmetric torsion link

Figure 6. Four examples of yarn paths out of a total of twenty-four, asymmetric torsion link

Figure 7. Numbers of yarn paths present over each segment, axisymmetric torsion link

Figure 8. Visualization of 3D TO irregular lattice PMC part, axisymmetric torsion link

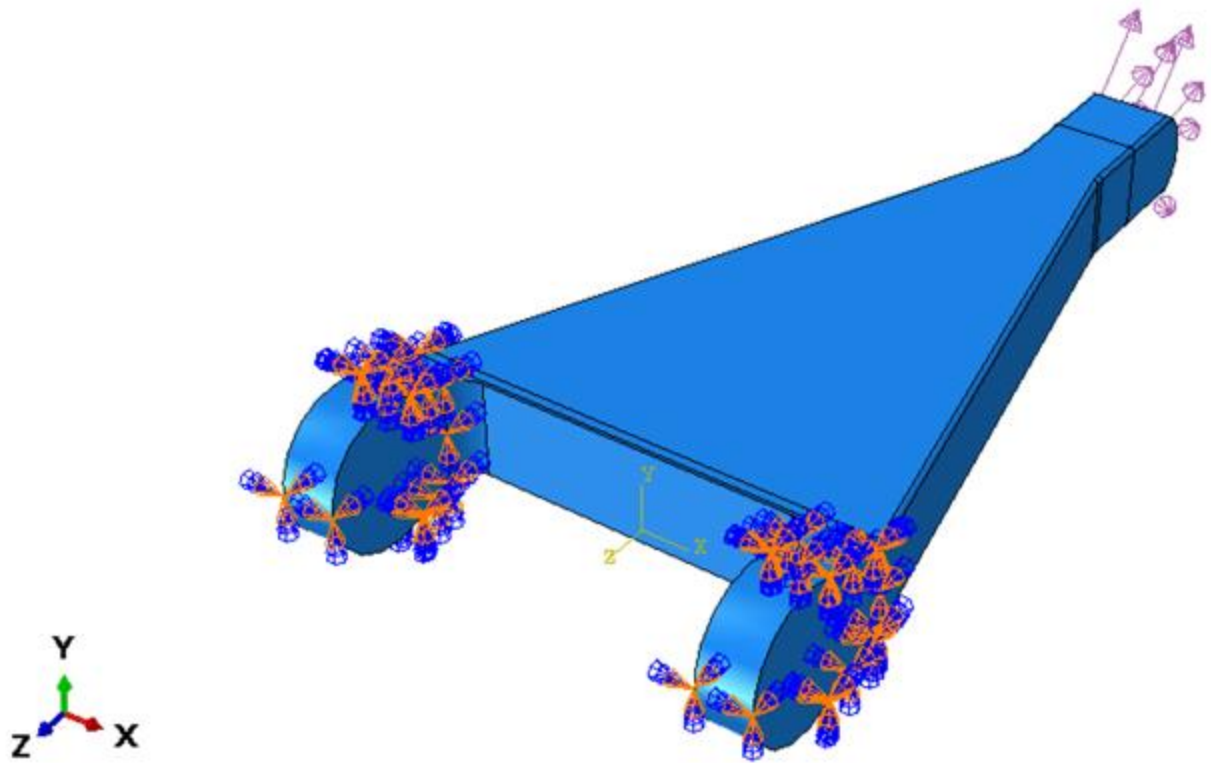


Figure 1. Asymmetric torsion link demonstrator subjected to resulting tension along axis z

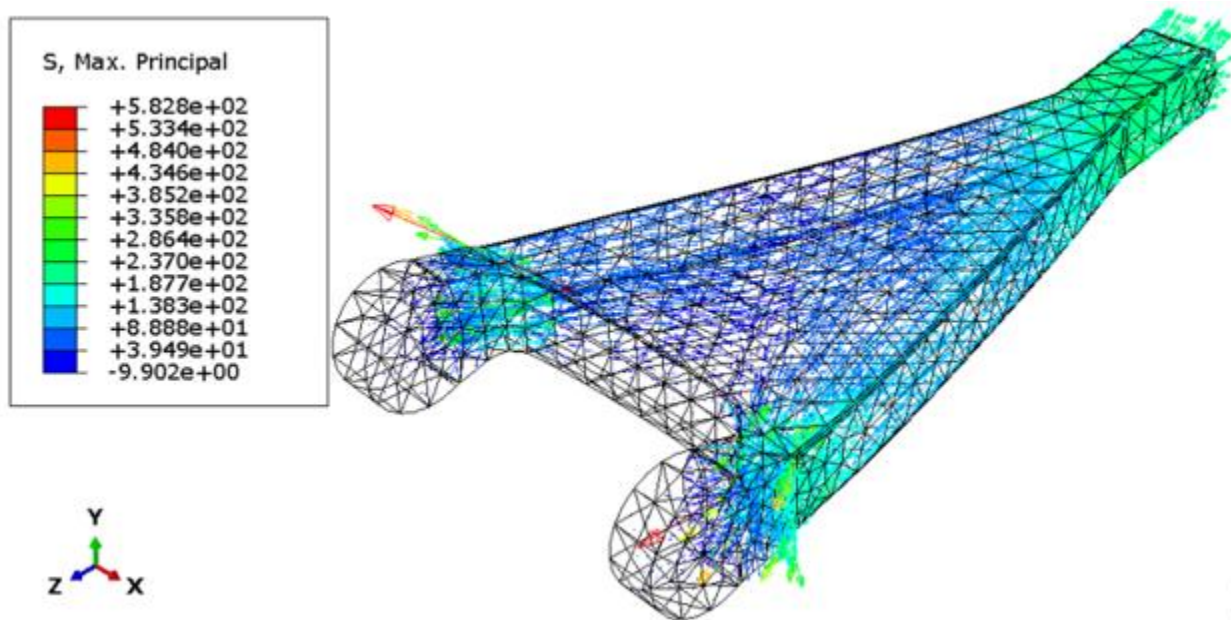


Figure 2. Principal stress vector field in axisymmetric torsion link

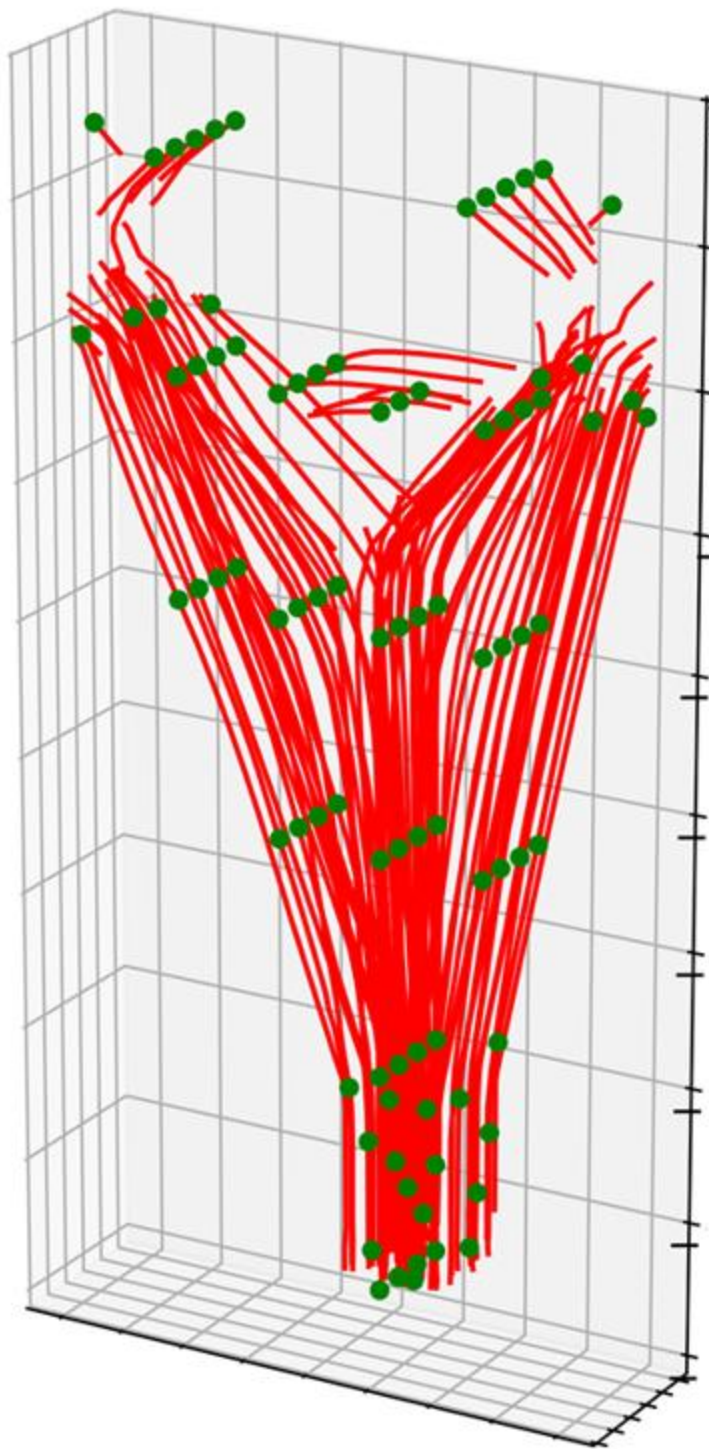


Figure 3. Seed points and initial stress paths for axisymmetric torsion link

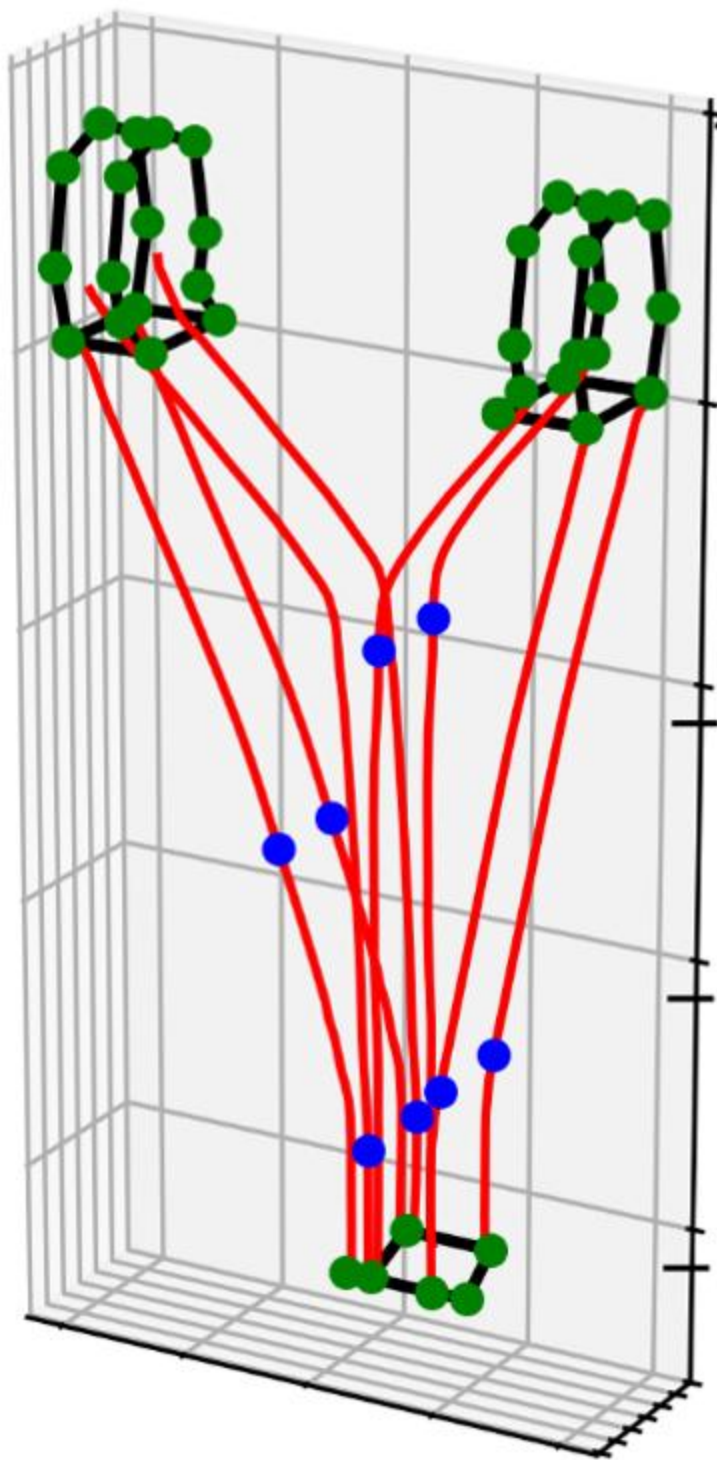


Figure 4. Selected stress paths for axisymmetric torsion link

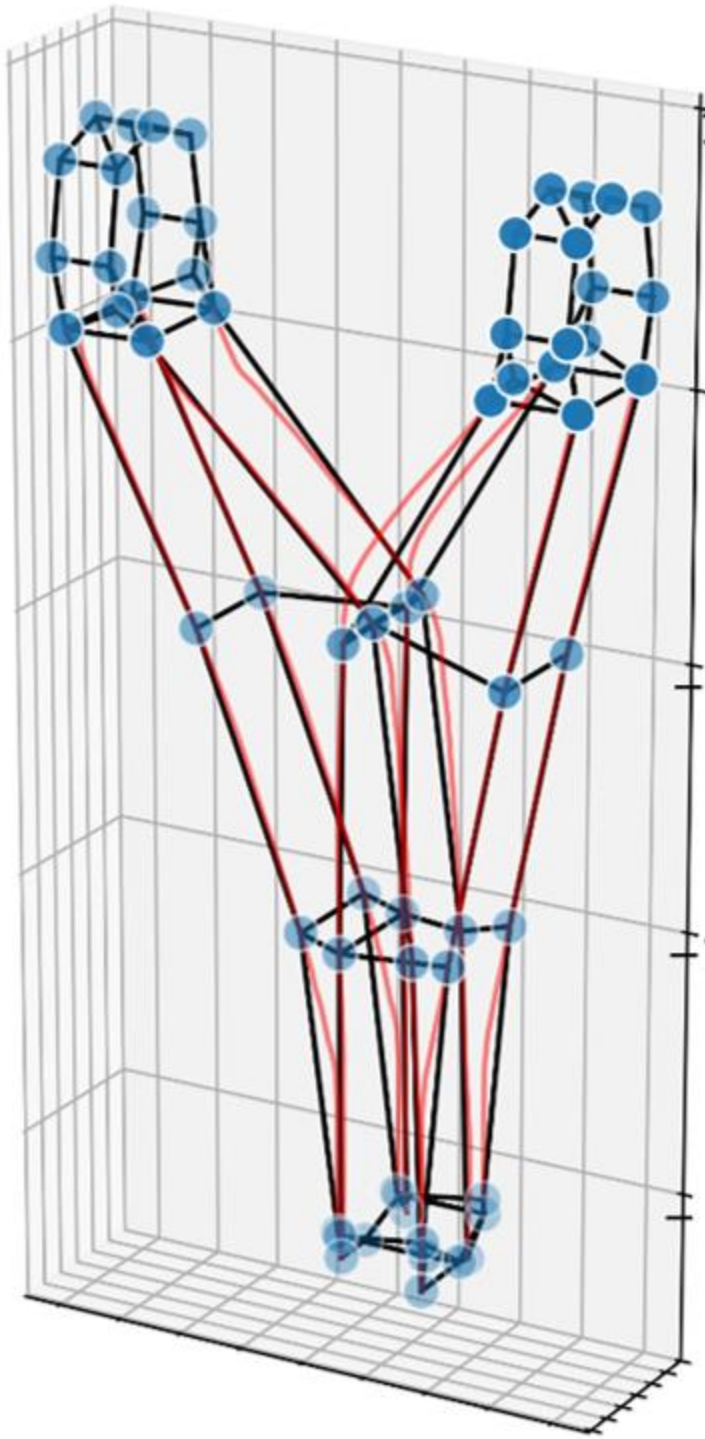


Figure 5. Triangulated structure generated for axisymmetric torsion link

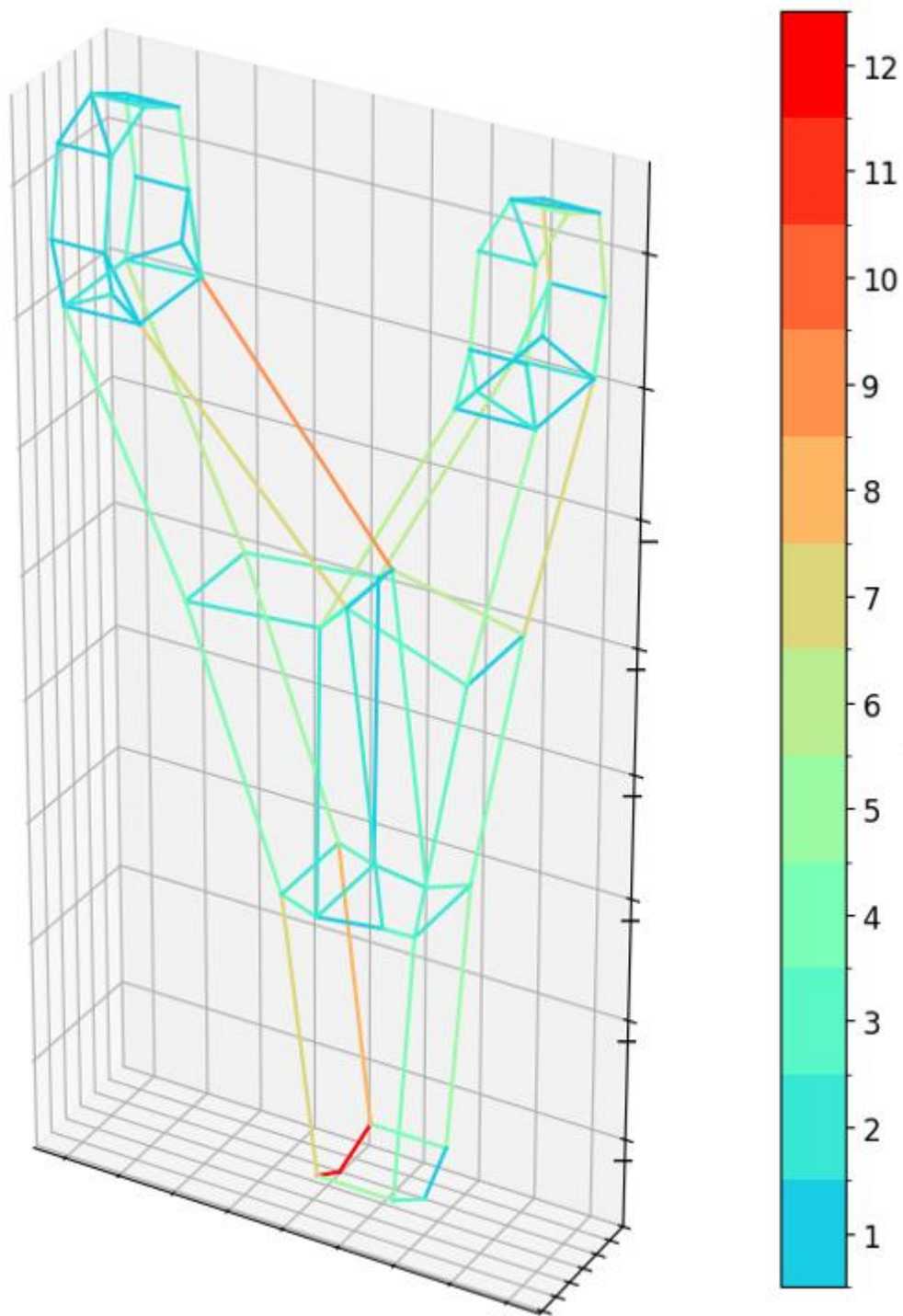


Figure 7. Numbers of yarn paths present over each segment, axisymmetric torsion link

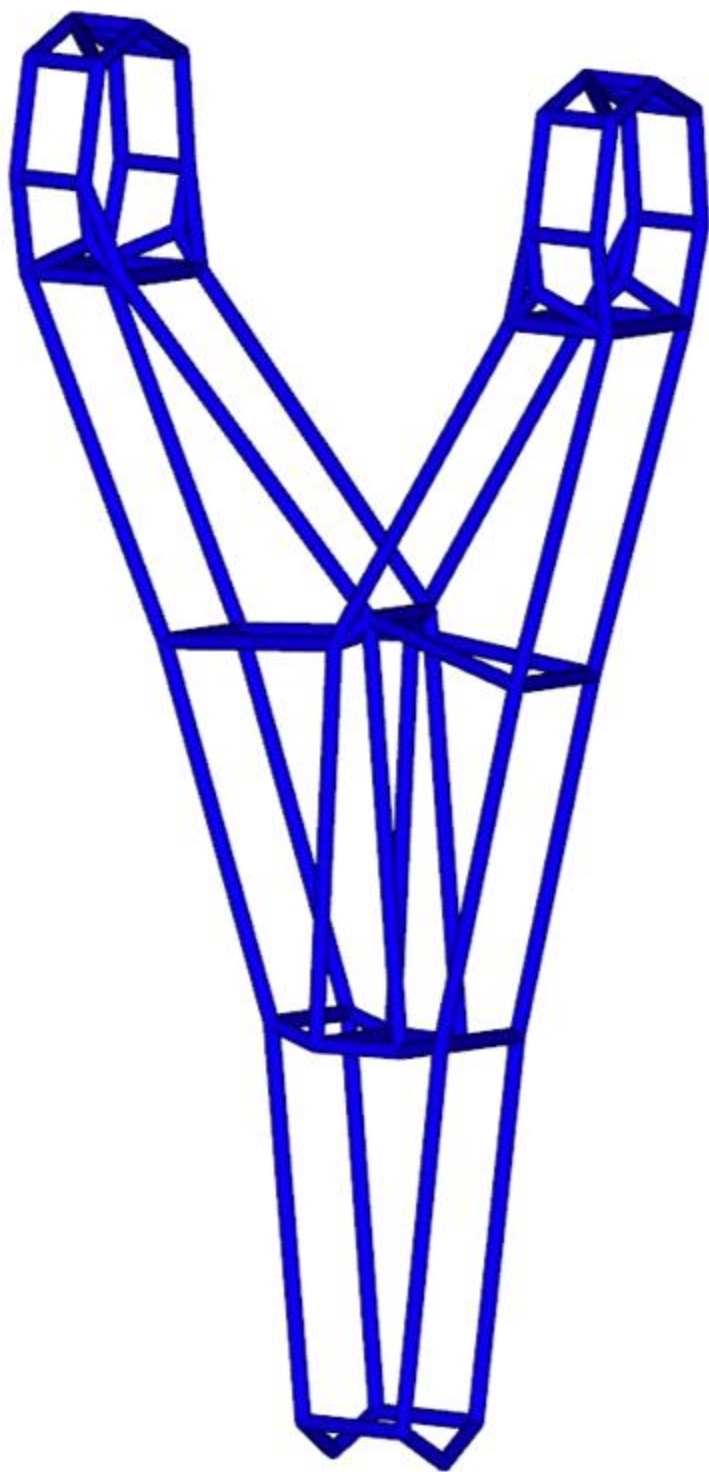


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