

Blade Count Versus Chord at Fixed Solidity in a Small Ducted Fan: RANS Performance Trends and the Reynolds-Number Effect They Miss

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Abstract

A small ducted fan can reach a given rotor solidity with more blades or with wider ones, and at chord Reynolds numbers near 10^5 the two routes need not be equivalent. Seven rotors for a 150 mm ducted fan in hover at 6000 rpm, with four or six untwisted S1223 blades of 20 mm to 45 mm chord ($\sigma = 0.34\text{--}1.02$), were simulated with steady RANS and the $k\text{--}\omega$ SST model on single periodic passages. In the simulations, figure of merit and thrust-to-power ratio fall monotonically with solidity for both blade counts (figure of merit from 0.32 to 0.17) because shaft power rises almost linearly with solidity while thrust levels off near 2.2 N. The simulations do not resolve an effect of blade count at fixed solidity, as two matched-solidity pairs differ by amounts comparable to numerical uncertainty, with one pair changing sign under a change of discretisation. Sectional validation shows that at $\text{Re}_c \approx 10^5$ the fully turbulent closure over-predicts S1223 drag by 46 % and captures less than half of its measured fall with Reynolds number, whereas XFOIL reproduces the drag trend for the blade section. A blade-element estimate built on XFOIL polars indicates that this missing Reynolds-number effect favours wider chords: it shifts the matched-pair figure-of-merit differences by 3 to 21 percentage points towards the four-blade rotors for turbulent inflow and removes the simulation advantage of the least solid, narrowest-chord rotor, leaving the optimum unresolved. In this regime efficiency depends on chord Reynolds number as well as solidity, and a fully turbulent model under-represents that effect.

Keywords: ducted fan; blade solidity; blade count; low Reynolds number; laminar separation; RANS; XFOIL; uncertainty quantification

Nomenclature

A_r	rotor disc area, m^2	P	shaft power, W
c	blade chord, m	Q	rotor torque, N m
C_d, C_l	section drag and lift coefficients	R	rotor tip radius, m
C_P	power coefficient, $P/\rho n^3 D^5$	Re_c	chord Reynolds number
C_T	thrust coefficient, $T/\rho n^2 D^4$	T	total system thrust, N
D	rotor diameter, $2R$, m	y^+	wall coordinate
FM	figure of merit	α	section angle of attack
N_b	number of blades	σ	rotor solidity
N_{crit}	e^N transition amplification factor	σ_d	duct area ratio, A_e/A_r
n	rotational speed, 1/s	ω	angular velocity, rad/s

1 Introduction

A ducted fan encloses a rotor within an aerodynamic shroud. The arrangement suits small unmanned aircraft because the shroud weakens the tip vortex and limits the wake contraction of an open rotor, raising static thrust at a given shaft power, and because it shields the rotor. Momentum theory attributes the thrust benefit to the duct’s control of the exit area [1]: for a duct with exit-to-disc area ratio $\sigma_d = A_e/A_r$, the ideal hover power is

$$P_{ideal} = \frac{T^{3/2}}{2\sqrt{\sigma_d \rho A_r}}, \quad (1)$$

which recovers the open-rotor result at $\sigma_d = 1/2$.

How closely a real fan approaches this limit depends on the rotor, and among the rotor parameters the blade solidity

$$\sigma = \frac{N_b c}{2\pi r_{ref}} \quad (2)$$

is one of the most influential. Equation (2) depends only on the product $N_b c$, so the same solidity can be reached with a few wide blades or many narrow ones. The two are not physically equivalent. At fixed σ , more and narrower blades lower the chord Reynolds number in proportion, raise the blade aspect ratio and narrow the passages between blades. At the chord Reynolds numbers of small fans, here 4.8×10^4 to 1.1×10^5 , the sectional lift-to-drag ratio is a steep function of Reynolds number, because laminar boundary layers separate and, if they reattach at all, do so through a laminar separation bubble whose size and losses change rapidly with Re_c [2–4].

The importance of this regime for small rotors is well documented. Profile drag accounts for a larger share of the losses of micro rotors than of full-scale rotors [5], and small propellers lose performance as their Reynolds number falls [6, 7]. For micro rotors at chord Reynolds numbers below 30,000, Benedict et al. found that increasing the chord of a two-bladed rotor raised its figure of merit while adding blades at constant chord had little effect [8]. Dayhoum et al. tested rotors of two to five blades at equal solidity, with S1223-based outer sections, and found little effect of blade count for open rotors but a loss of performance with added blades in a shroud, which they attributed mainly to the low-Reynolds-number behaviour of the narrower chords [9]. At higher Reynolds numbers, by contrast, solidity has been found to be the main geometric parameter of hover performance, with blade aspect ratio secondary [10]. Shrouded-rotor work has otherwise concentrated on the duct and the tip clearance, experimentally [11, 12] and with Reynolds-averaged simulations in a rotating frame [13–15].

Such simulations are the natural tool for design studies of this kind, but the turbulence closures they normally use are fully turbulent and cannot represent a laminar separation bubble. Fully turbulent and transitional closures can agree on the integrated loads of a small rotor while predicting quite different boundary layers [16], which raises a specific question for the comparison above: can a fully turbulent simulation detect a difference whose physical origin is the Reynolds-number dependence of a laminar separation bubble?

This paper answers that question for a 150 mm ducted fan in hover. It makes three contributions. First, seven rotor configurations are simulated in which blade count and chord are varied independently, including two pairs matched in solidity at different levels of loading; the discretisation sensitivity of every configuration is measured, and the grid uncertainty is estimated from a three-grid study of one of them. Second, the simulations show that solidity governs efficiency over $\sigma = 0.34$ – 1.02 , and that they do not resolve an effect of blade count at fixed solidity. Third, a sectional analysis with XFOIL [17], validated against the same wind-tunnel data as the simulations, shows that this null result, and the simulated ranking at the narrow-chord end of the sweep, are at least partly a consequence of the closure, and estimates the size and direction of the effect it misses.

Table 1: Case matrix. Pitch (20°), span (62 mm), tip clearance (4.00 mm), duct and rotational speed (6000 rpm) are identical in all cases. Cases 2 and 4 (pair A) and cases 7 and 5 (pair B) are matched in solidity. Reynolds numbers use $\rho = 1.225 \text{ kg/m}^3$ and $\mu = 1.7894 \times 10^{-5} \text{ Pa}\cdot\text{s}$.

Case	N_b	c (mm)	σ	A_b/A_r	Re_c at $0.75R$	Blade aspect ratio
1	4	20	0.340	0.281	48,400	3.10
2	4	30	0.509	0.421	72,600	2.07
3	4	40	0.679	0.561	96,800	1.55
4	6	20	0.509	0.421	48,400	3.10
5	6	30	0.764	0.632	72,600	2.07
6	6	40	1.019	0.842	96,800	1.55
7	4	45	0.764	0.632	108,900	1.38

2 Configuration

2.1 Rotor and duct

The rotor has a cylindrical hub of radius 13 mm and length 25 mm, from which untwisted blades extend to a tip radius of 75 mm, a span of 62 mm. The blade section is the S1223, a high-lift section designed for low Reynolds numbers [18], with coordinates from the UIUC database [19]. It was chosen because its coordinates and its wind-tunnel characteristics at the relevant Reynolds numbers are published, which allows the sectional behaviour of every method in this paper to be validated directly. The section is truncated at $0.97c$ to give a blunt trailing edge of thickness $0.0131c$ (Appendix C).

Blades are set at a constant geometric pitch of 20° . No twist is applied. This is a deliberate simplification that removes the twist schedule as a variable, so that solidity and its decomposition are the only differences between configurations; the cost is a non-optimal spanwise loading, discussed in Section 6. Blade tips are trimmed to a cylinder of radius 75 mm, which holds the tip clearance at 4.00 mm (5.33% of R) in every configuration.

The duct has a throat radius of 79 mm over $-25 \leq y \leq 25$ mm, an inlet lip flaring to 91 mm at $y = -40$ mm, and a diffuser expanding to 86 mm at $y = 60$ mm, giving $\sigma_d = 1.315$. The wall is 1.5 mm thick and is modelled as a solid.

2.2 Case matrix

Seven configurations were simulated (Table 1, Figure 1). Six form a full factorial of blade count, $N_b = 4$ and 6, and chord, $c = 20, 30$ and 40 mm. The seventh, case 7, has four blades of 45 mm chord, so that $N_b c$ matches case 5. The configurations therefore contain two matched-solidity pairs: cases 2 and 4 (pair A, $\sigma = 0.509$) and cases 7 and 5 (pair B, $\sigma = 0.764$). Within each pair the six-blade rotor has two-thirds of the chord, and therefore two-thirds of the chord Reynolds number, of the four-blade rotor.

Solidity is evaluated from Eq. (2) at $r_{ref} = R/2 = 37.5$ mm. Table 1 also gives the blade area ratio $A_b/A_r = N_b c (R - R_{hub}) / \pi R^2$, which needs no reference radius; both pairs are matched under either definition. Chord Reynolds number is quoted at $0.75R$, using the local blade speed $\omega \cdot 0.75R = 35.34 \text{ m/s}$.

3 Numerical Method

3.1 Domain, boundary conditions and periodic reduction

The fluid domain is a cylinder of radius 750 mm ($5D$) extending 635 mm ($4.2D$) upstream and 1405 mm ($9.4D$) downstream of the rotor plane (Figure 2). The rotor is enclosed in a rotating cell zone of radius 79 mm, coincident with the duct throat, spanning $-20 \leq y \leq 20$ mm, so the

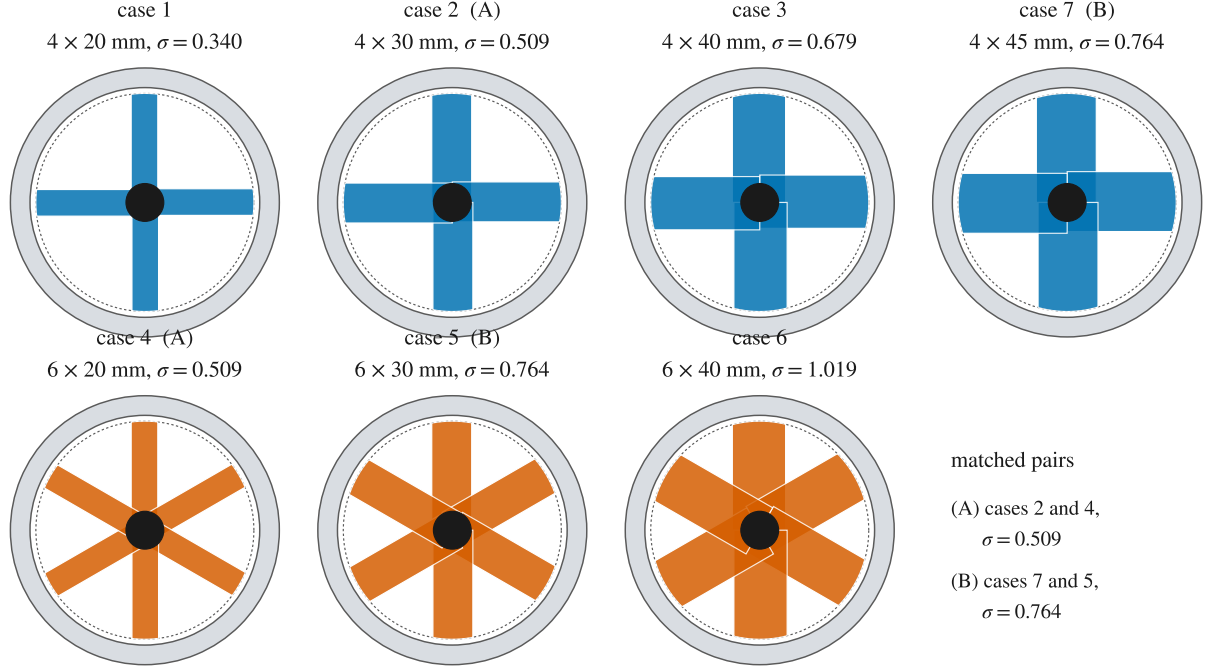


Figure 1: The seven rotor configurations viewed along the axis of rotation. Each blade appears as its axial projection, a strip whose width is the extent of the pitched section seen along the axis, trimmed at the 75 mm tip radius (dotted). The grey annulus is the shroud between the throat radius, 79 mm, and the outer lip, 92.5 mm. (A) and (B) mark the matched-solidity pairs.

tip gap lies inside the rotating zone. All three outer boundaries are pressure outlets at zero gauge pressure, with backflow normal to the boundary, backflow turbulence intensity 1 % and viscosity ratio 10, so that no boundary imposes a flow direction. Blades, hub and duct are no-slip walls; the part of the duct wall inside the rotating zone is stationary in the absolute frame. At 6000 rpm the tip speed is 47.1 m/s, a tip Mach number of about 0.14, and the flow is treated as incompressible with $\rho = 1.225 \text{ kg/m}^3$ and $\mu = 1.7894 \times 10^{-5} \text{ Pa s}$.

Each case is reduced to a single blade passage bounded by rotationally periodic planes, 90° for four blades and 60° for six. The configuration has no stator or other circumferential asymmetry and is assessed in hover, so one passage carries the whole solution; forces and torques are the sector values multiplied by N_b . The reduction is what makes the study possible within the 1,048,576-cell limit of the ANSYS Student licence: a full 360° domain would allow cells no finer than 3 mm in the rotating zone, whereas a single passage admits 1.5 mm, about 2.7 cells across the tip gap. The two periodic faces of every sector were checked to match in area, and a cyclic match control enforced node-for-node conformity.

3.2 Mesh

Meshing used an unstructured tetrahedral scheme with a body sizing of 1.5 mm in the rotating zone. The production meshes occupy 47 % to 80 % of the licence limit, from 497,234 cells for case 5 to 843,262 for case 2; case 7 has 721,988. Prism inflation layers were attempted at first-layer heights of 0.0075 mm to 0.02 mm and 4 to 10 layers, but at every setting the prism stack degenerated where it met the blunt trailing edge and the tip gap, producing up to 671 negative-volume cells. The meshes are therefore built without inflation layers; the final meshes have no negative volumes and a minimum cell volume of $5.37 \times 10^{-12} \text{ m}^3$. The mean y^+ on the rotor surfaces of case 7 is 30.5, in the range in which the wall treatment described next applies wall functions.

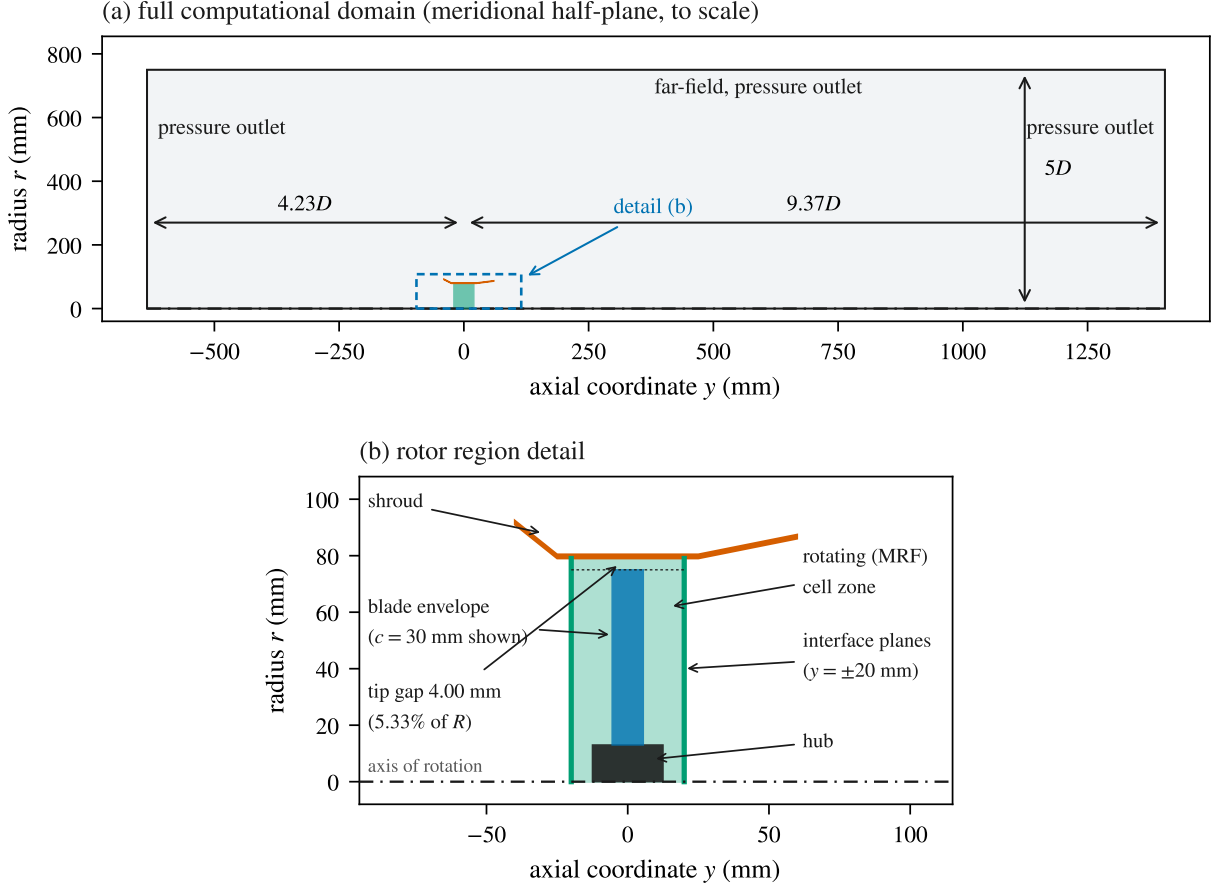


Figure 2: Computational domain in the meridional half-plane. (a) Full extent, to scale. (b) Rotor region, showing the rotating cell zone, the shroud and the interface planes at $y = \pm 20$ mm.

3.3 Turbulence closure

The steady Reynolds-averaged Navier–Stokes equations are solved with ANSYS Fluent [20] in a moving reference frame: the rotating zone is solved in a frame rotating at ω , the stationary zone in the inertial frame. Turbulence is closed with the k – ω SST model [21] with automatic wall treatment. A transition-sensitive closure such as the γ – Re_θ model [22, 23] would suit this Reynolds-number range better, but it requires $y^+ \approx 1$ and hence the inflation layers that this geometry would not accept within the cell budget. The consequence of a fully turbulent closure is measured in Section 4.4 and its effect on the comparison between configurations is estimated in Section 5.3.

3.4 Solver and convergence

Each case is solved in two stages. Stage A uses the SIMPLE scheme with first-order upwind momentum and turbulence equations and second-order pressure, with the rotational speed ramped through 157.08, 314.16 and 628.32 rad/s, from an initial field with $k = 0.1 \text{ m}^2/\text{s}^2$ and $\omega = 1000/\text{s}$. Stage B, which produces the reported results, switches to the pressure-based coupled solver with second-order upwind momentum and the pseudo-transient formulation (time scale factor 0.5); the turbulence equations stay first order. The steady formulation is iteratively unstable at second order without the pseudo-transient term, for reasons given in Appendix A.

A solution is accepted when total thrust and rotor torque each change by less than 0.5% over each of two consecutive blocks of 1000 iterations, with the scaled residuals falling. At that point the rotor and duct contributions can still exchange load at up to about $\pm 0.7\%$ per

Table 2: Grid study on case 5. The fine grid is the production mesh. Neither thrust nor power is in the asymptotic range, so no grid convergence index is formed; each quantity is bounded by the spread of the three solutions, relative to the fine-grid value.

Mesh	Cells (10^6)	C_T	C_P	FM
Coarse (2.54 mm)	0.150	0.3410	0.4168	0.235
Medium (1.95 mm)	0.267	0.3468	0.4430	0.227
Fine (1.50 mm, used)	0.497	0.3590	0.4372	0.242
Refinement ratios r_{21} / r_{32}		1.230 / 1.213		
Spread: $T / P / T/P / FM$		5.0 / 6.0 / 4.6 / 6.3 %		

1000 iterations while their sum changes by less than 0.4 %, so the rotor–duct split carries an uncertainty of ± 0.7 %. Six cases reached a fixed point; case 1 settled into a bounded limit cycle of period about 250 iterations and amplitude ± 2.9 % in rotor thrust, and its values are averages over 2000 iterations. As a physical check, the loads should scale with ω^2 between 157.08 and 314.16 rad/s, a factor of 4.00: in cases 2–7 the factors were 4.05 to 4.15 for rotor thrust, 4.08 to 4.21 for duct thrust and 3.94 to 3.97 for torque.

3.5 Performance metrics

Total thrust T is the sum of the axial forces on the rotor and the duct, and shaft power is $P = Q\omega$. Results are also given as $C_T = T/\rho n^2 D^4$ and $C_P = P/\rho n^3 D^5$, for which $\rho n^2 D^4 = 6.202$ N and $\rho n^3 D^5 = 93.02$ W. Efficiency is reported in two forms: the thrust-to-power ratio T/P , and the figure of merit referenced to the ideal power of Eq. (1),

$$FM = \frac{T^{3/2}}{2P\sqrt{\sigma_d \rho A_r}}, \quad (3)$$

with $\sigma_d = 1.315$ for this duct; values are lower by $\sqrt{1.315} = 1.147$ than they would be under a straight-duct convention.

The figure of merit is the primary measure in this paper. All configurations run at the same rotational speed, so they produce different thrusts, here differing by up to 17.5 %, and T/P falls with thrust even for an ideal rotor. If $T \propto \omega^2$ and $P \propto \omega^3$, the figure of merit does not depend on rotational speed, so it compares configurations as if each had been run at the speed that gives a common thrust. The scaling exponents implied by the check above ($T \propto \omega^{2.03-2.06}$, $Q \propto \omega^{1.98-1.99}$) change the figure of merit by less than 1 % over the 8.4 % change in speed needed to equalise thrust here.

4 Numerical Uncertainty and Sectional Validation

4.1 Grid

Grid uncertainty was assessed on case 5 with three systematically coarsened meshes, following Celik et al. [24] and Roache [25]. Case 5 was chosen because it is one of the two least well resolved configurations: the smallest distance between adjacent blade surfaces, computed from the geometry, is 1.09 mm near $r = 15$ mm, less than one nominal cell. Only case 6 is tighter (0.17 mm); the other cases range from 2.08 to 5.38 mm (Table 9). Because the production meshes already use up to 80 % of the cell limit, the production mesh is the finest of the three, and the coarser meshes use rotating-zone cell sizes of 1.95 and 2.54 mm; the refinement ratios computed from cell counts and volumes are 1.230 and 1.213 (Table 2).

Thrust rises at each refinement but by a growing increment (C_T changes by 0.0058 and then 0.0122, a convergence ratio of 2.10), and power oscillates, with rotor torque 0.0617, 0.0656 and

Table 3: Sensitivity to momentum discretisation: change from the first-order to the second-order solution on identical meshes. The second-order values of case 1 are averages over its limit cycle.

Case (σ)	ΔT	ΔP	$\Delta(T/P)$	ΔFM	Δ duct share
1 (0.340)	−16.1 %	−14.1 %	−2.4 %	−10.6 %	−7.3 pts
2 (0.509)	−17.3 %	−15.9 %	−1.7 %	−10.5 %	−3.3 pts
3 (0.679)	−17.7 %	−13.1 %	−5.3 %	−14.1 %	−0.3 pts
4 (0.509)	−13.6 %	−14.7 %	+1.3 %	−5.8 %	−8.3 pts
5 (0.764)	−16.3 %	−16.8 %	+0.7 %	−7.9 %	−7.0 pts
6 (1.019)	−24.4 %	−16.4 %	−9.6 %	−21.4 %	−2.2 pts
7 (0.764)	−18.0 %	−9.2 %	−9.6 %	−18.2 %	+2.0 pts

Table 4: Change in T/P and figure of merit between configurations, evaluated separately at each discretisation. Upper block: adjacent solidity steps within a blade count. Lower block: the matched pairs, six blades relative to four.

Comparison	T/P		FM	
	1st order	2nd order	1st order	2nd order
Case 1 $\rightarrow$ 2 (σ 0.340 $\rightarrow$ 0.509)	−10.4 %	−9.8 %	−5.1 %	−5.0 %
Case 2 $\rightarrow$ 3 (σ 0.509 $\rightarrow$ 0.679)	−14.2 %	−17.4 %	−13.3 %	−16.8 %
Case 3 $\rightarrow$ 7 (σ 0.679 $\rightarrow$ 0.764)	−7.1 %	−11.3 %	−7.9 %	−12.3 %
Case 4 $\rightarrow$ 5 (σ 0.509 $\rightarrow$ 0.764)	−18.8 %	−19.3 %	−17.6 %	−19.4 %
Case 5 $\rightarrow$ 6 (σ 0.764 $\rightarrow$ 1.019)	−17.3 %	−25.8 %	−19.7 %	−31.4 %
Pair A: case 2 $\rightarrow$ 4 (σ = 0.509)	−6.4 %	−3.6 %	−6.2 %	−1.3 %
Pair B: case 7 $\rightarrow$ 5 (σ = 0.764)	−4.7 %	+6.2 %	−3.2 %	+8.9 %

0.0647 N m on the coarse, medium and fine grids. Neither is in the asymptotic range, so the uncertainty is bounded by the spread of the three solutions: 5.0 % on thrust, 6.0 % on power, 4.6 % on T/P and 6.3 % on the figure of merit. The probable limiting feature is the tip clearance, which spans 2.7, 2.1 and 1.6 cells on the three grids. The band was measured on one case; the others were meshed by the same rules and share the same tip-gap resolution, but that their error is similar is not independently verified.

4.2 Discretisation

The first-order Stage A solution is available on the same mesh for every case, so the sensitivity to momentum discretisation can be measured directly (Table 3). Thrust changes by 14 % to 24 % and power by 9 % to 17 % between the two schemes, while T/P changes by only 0.7 % to 9.6 %, probably because the artificial diffusion of first-order upwind alters lift and drag in the same sense. The figure of merit, which weights thrust by 3/2, benefits less from this cancellation (5.8 % to 21.4 %).

What matters for this study is whether the *differences* between configurations survive the change of scheme (Table 4). Every adjacent solidity step reduces both T/P and the figure of merit at both orders; the magnitudes differ by 0.5 to 8.5 percentage points for T/P and by up to about 12 for the figure of merit, the largest shift being the step from case 5 to case 6, the configuration with the narrowest inboard passages. The matched pairs behave differently: at $\sigma = 0.764$ the six-blade rotor is worse at first order and better at second order, in both metrics.

4.3 What the numerical uncertainty resolves

Taken together, the grid band and the discretisation sensitivity separate three kinds of comparison. The overall decline of efficiency with solidity is resolved without ambiguity: from case 1 to

Table 5: Computed performance at 6000 rpm. T_{duct}/T is the duct’s share of total thrust. Case 1 is the average over its limit cycle.

Case	σ	T_{rotor} (N)	T_{duct} (N)	T (N)	Q (N m)	P (W)	C_T	C_P	T/P (N/W)	FM	T_{duct}/T
1	0.340	1.328	0.592	1.920	0.0392	24.6	0.3095	0.2647	0.0780	0.320	30.8 %
2	0.509	1.422	0.709	2.131	0.0482	30.3	0.3436	0.3258	0.0703	0.304	33.3 %
3	0.679	1.397	0.766	2.163	0.0592	37.2	0.3487	0.4001	0.0581	0.253	35.4 %
4	0.509	1.576	0.656	2.232	0.0524	32.9	0.3600	0.3539	0.0678	0.300	29.4 %
5	0.764	1.577	0.649	2.226	0.0647	40.7	0.3590	0.4372	0.0547	0.242	29.2 %
6	1.019	1.276	0.624	1.900	0.0744	46.8	0.3064	0.5028	0.0406	0.166	32.8 %
7	0.764	1.323	0.794	2.117	0.0654	41.1	0.3414	0.4417	0.0515	0.222	37.5 %

case 6 the figure of merit falls by 48 %. Adjacent solidity steps are resolved in T/P , where they are 2.1 to 5.6 times the 4.6 % grid band, and in the figure of merit except for the smallest step, case 1 to case 2 (−5.0 % against a 6.3 % band). The matched-pair differences are not resolved: that of pair A lies inside the grid band, and that of pair B, slightly outside it, changes sign with the scheme.

4.4 Sectional validation

The closure was validated in two dimensions against the UIUC wind-tunnel measurements of the S1223 [18, 19], whose stated uncertainty is 1.5 %. The two-dimensional mesh used the production resolution and wall treatment, with no inflation layers and a mean y^+ of 95.4, and the same solver settings, so the comparison tests the method as applied. Fifteen conditions were computed at $Re_c = 61,000$ (lift only in the reference data), 100,600 and 200,400; the full comparison is in Appendix B.

At $Re_c = 100,600$, the condition closest to the wider-chord configurations, lift is under-predicted by 3.2 % to 9.1 % (5.4 % on average) and drag over-predicted by 41 % to 51 % (46 % on average). The more important result for this study is the Reynolds-number trend. Between $Re_c = 100,600$ and 200,400 the measured drag near zero incidence falls by 21.7 %, as the laminar separation bubble shortens; the computed drag falls by only about 9 %, because a closure that is turbulent from the leading edge contains no bubble whose length can respond. The simulations therefore contain less than half of the Reynolds-number sensitivity of the section, which is exactly the physics that distinguishes the members of a matched pair. Section 5.3 estimates what the missing part does to the comparison.

5 Results

5.1 Efficiency falls with solidity for both blade counts

Table 5 gives the computed performance of all seven configurations and Figure 3 plots it against solidity. The figure of merit and T/P fall at every step in solidity for both blade counts. With four blades the figure of merit is 0.320, 0.304, 0.253 and 0.222 at $\sigma = 0.340, 0.509, 0.679$ and 0.764; with six it is 0.300, 0.242 and 0.166 at $\sigma = 0.509, 0.764$ and 1.019. A least-squares straight line through all seven points, $FM = 0.416 - 0.240 \sigma$, has $R^2 = 0.968$, and no configuration lies more than 4.3 % from it; the corresponding line for T/P , $0.0972 - 0.0566 \sigma$ (in N/W), has $R^2 = 0.987$ and a largest deviation of 4.5 %. Both deviations are about the size of the grid band, so within the resolution of the simulations the two blade counts fall on one line. In the simulations there is no interior maximum, and the least solid configuration is the most efficient; Section 5.3 shows that this ranking at the narrow-chord end is not robust to the Reynolds-number effect.

The decline comes mainly from power. Shaft power rises almost linearly with solidity: the line $P = 14.6 + 33.1 \sigma$ (in watts) fits all seven cases with $R^2 = 0.972$, and every adjacent step in solidity costs 10 % to 24 % more power (Table 6). Thrust does not keep pace. In the four-blade

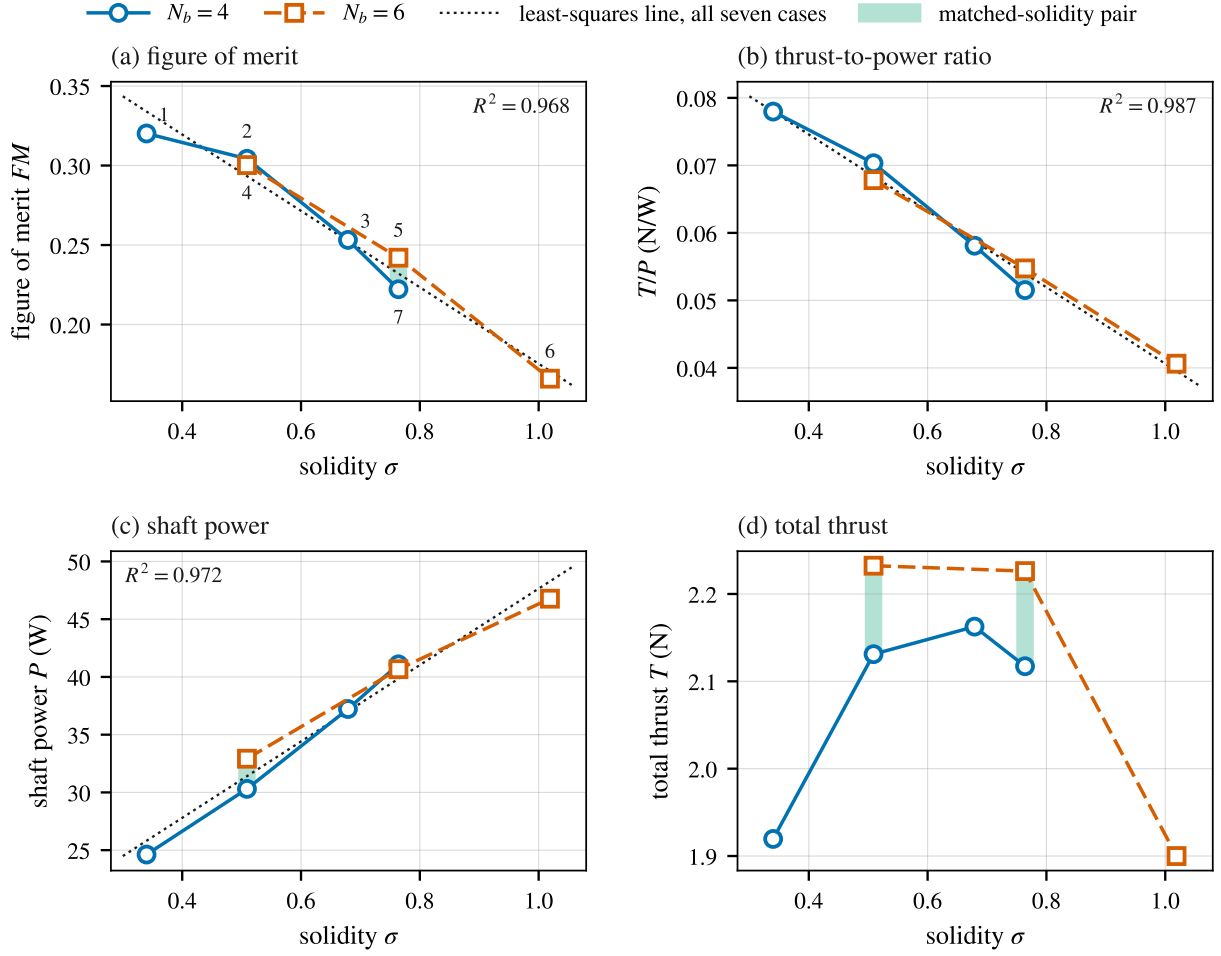


Figure 3: Performance against solidity, with case numbers marked in (a). (a) Figure of merit, (b) thrust-to-power ratio and (c) shaft power, each with the least-squares straight line through all seven cases. (d) Total thrust. The shaded vertical links join the matched pairs, cases 2 and 4 at $\sigma = 0.509$ and cases 7 and 5 at $\sigma = 0.764$.

Table 6: Change in thrust, power, T/P and figure of merit across each adjacent solidity step within a blade count.

Step	$\Delta\sigma$	T	P	T/P	FM
Four blades, case 1 $\rightarrow$ 2 (20 mm to 30 mm)	0.169	+11.0 %	+23.1 %	−9.8 %	−5.0 %
Four blades, case 2 $\rightarrow$ 3 (30 mm to 40 mm)	0.170	+1.5 %	+22.8 %	−17.4 %	−16.8 %
Four blades, case 3 $\rightarrow$ 7 (40 mm to 45 mm)	0.085	−2.1 %	+10.4 %	−11.3 %	−12.3 %
Six blades, case 4 $\rightarrow$ 5 (20 mm to 30 mm)	0.255	−0.3 %	+23.5 %	−19.3 %	−19.4 %
Six blades, case 5 $\rightarrow$ 6 (30 mm to 40 mm)	0.255	−14.7 %	+15.0 %	−25.8 %	−31.4 %

series it gains 11.0 % across the first step, 1.5 % across the second and loses 2.1 % across the third; in the six-blade series it is unchanged across the first step and falls 14.7 % across the second. Total thrust lies between 2.12 and 2.23 N for all five configurations with $0.509 \leq \sigma \leq 0.764$, against 1.92 N at $\sigma = 0.340$ and 1.90 N at $\sigma = 1.019$. Beyond $\sigma \approx 0.5$ added blade area no longer produces thrust, while the power needed to drive it keeps rising.

The duct carries 29.2 % to 37.5 % of total thrust. In the four-blade series its share rises with solidity, from 30.8 % to 37.5 %, and at matched solidity it is larger for the four-blade rotor, by 3.9 and 8.3 percentage points. The rotor–duct split is, however, the quantity most sensitive to discretisation, moving by up to 8.3 points between schemes (Table 3), and these differences

Table 7: Matched-solidity comparisons. Differences are for the six-blade rotor relative to the four-blade rotor of the same pair.

	Pair A, $\sigma = 0.509$			Pair B, $\sigma = 0.764$		
	Case 2 4×30 mm	Case 4 6×20 mm	Difference	Case 7 4×45 mm	Case 5 6×30 mm	Difference
Re_c at $0.75R$	72,600	48,400	−33 %	108,900	72,600	−33 %
T (N)	2.131	2.232	+4.8 %	2.117	2.226	+5.1 %
P (W)	30.3	32.9	+8.6 %	41.1	40.7	−1.0 %
FM	0.304	0.300	−1.3 %	0.222	0.242	+8.9 %
T/P (N/W)	0.0703	0.0678	−3.6 %	0.0515	0.0547	+6.2 %
FM , first order			−6.2 %			−3.2 %
T/P , first order			−6.4 %			−4.7 %

are reported rather than interpreted. All figures of merit are low against the 0.4 to 0.6 of well-developed small ducted fans, consistent with the untwisted blade, the 5.33 % tip clearance and the absence of a stator.

5.2 The matched pairs: no blade-count effect is resolved

Pairs A and B each reach one solidity with four blades and with six (Table 7). In pair A the six-blade rotor produces 4.8 % more thrust for 8.6 % more power, so its figure of merit is 1.3 % lower and its T/P 3.6 % lower. In pair B it produces 5.1 % more thrust for 1.0 % less power, so its figure of merit is 8.9 % higher. The two pairs disagree in sign. The difference in pair A lies inside the 6.3 % grid band; that in pair B is slightly larger than the band but reverses sign between first- and second-order discretisation. The simulations therefore resolve no effect of blade count at fixed solidity: within their resolution, solidity alone describes efficiency over $\sigma = 0.34$ to 1.02.

That conclusion holds only for the method. The sectional validation showed that the fully turbulent closure contains less than half of the Reynolds-number sensitivity of S1223 drag, and the Reynolds number is precisely what differs within a pair. The next section estimates what the missing sensitivity does to the comparison.

5.3 The Reynolds-number effect missing from the simulations

Sectional data. Lift and drag polars of the S1223 were computed with XFOIL 6.99 [17], which couples a panel method to an integral boundary layer with e^N transition prediction and therefore represents laminar separation bubbles. Polars were computed at twelve Reynolds numbers from 30,000 to 200,400, for α from -6° to 14° , at three values of the critical amplification factor. $N_{crit} = 9$ corresponds to a low-disturbance wind tunnel such as the one in which the reference data were measured; lower values represent the higher freestream turbulence expected in the inflow of a ducted rotor [26], and $N_{crit} = 3-5$ is taken as representative of it. At $N_{crit} = 3$ and 5 the polars are nearly complete and vary smoothly with Reynolds number. At $N_{crit} = 9$ they are not: at 30,000 and 40,000 XFOIL converges at positive incidence only to separated solutions, with lift-to-drag ratios of 12 or less, and at 48,400 the solution depends on where the incidence sweep starts, attached or separated. Sweeps were therefore started at both 0 and 2° at 30,000 to 61,000, and the quiet-inflow results below span the more and the less favourable branch. Some $N_{crit} = 9$ polars at higher Reynolds numbers end before 14° (at 7° for 72,600).

At $N_{crit} = 9$ XFOIL under-predicts the measured drag by 10.1 % on average at $Re_c = 100,600$ and 4.0 % at 200,400, and, crucially, it reproduces the Reynolds-number trend: between these two conditions its drag near zero incidence falls by 22.6 %, against 21.7 % measured and about 9 % with the fully turbulent closure (Figure 4a). It over-predicts lift, by 13 to 16 % on average

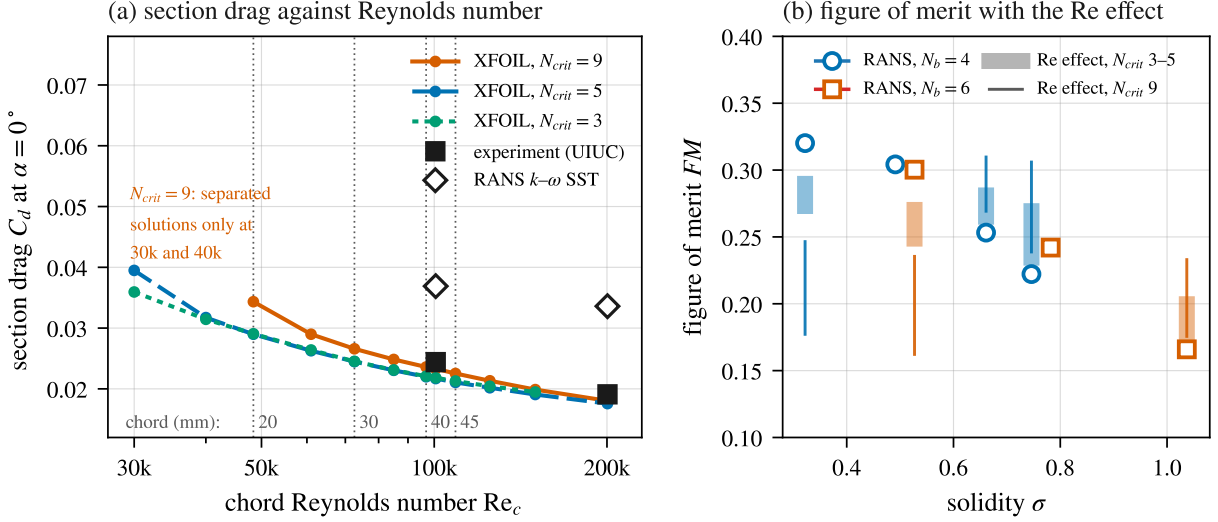


Figure 4: (a) S1223 drag at zero incidence against chord Reynolds number: XFOIL at three transition settings, the UIUC measurements and the fully turbulent RANS closure. Dotted lines mark the Reynolds number at $0.75R$ of each chord. (b) Figure of merit against solidity: RANS (markers) and the blade-element estimate with the Reynolds-number effect included, as the range over three inflow treatments for turbulent (thick bars, N_{crit} 3–5) and quiet (thin bars, N_{crit} 9) inflow. Cases 2 and 5, with 30 mm chords, are the reference and are unchanged. Four- and six-blade points at the same σ are offset slightly.

at these two Reynolds numbers, a familiar limitation for highly cambered sections, so XFOIL is used here only for differences between Reynolds numbers, never for absolute levels. The polars show how steeply the section degrades over the range of this rotor. At $\alpha = 1-3^\circ$ the lift-to-drag ratio at $N_{crit} = 3-5$ is 36–41 at $Re_c = 48,400$, 46–51 at 72,600 and 55–60 at 108,900; at $N_{crit} = 9$ it is 27–31 (on the attached branch), 45–46 and 54–57.

Blade-element estimate. The effect of these polars on each rotor was estimated with a blade-element model. At each radius r the axial velocity $u(r)$ is the passage-averaged value from each rotor’s own RANS solution at the throat entry (Section 5.4), the inflow angle is $\phi = \arctan[u/(\omega r - v_\theta/2)]$, the section incidence is $\alpha = 20^\circ - \phi$, and

$$dT = N_b \frac{1}{2} \rho W^2 c (C_l \cos \phi - C_d \sin \phi) dr, \quad dQ = N_b \frac{1}{2} \rho W^2 c (C_l \sin \phi + C_d \cos \phi) r dr, \quad (4)$$

with W the local relative velocity. Each rotor is evaluated twice: with polars at its own chord Reynolds number, and with polars at the Reynolds number a 30 mm chord would have at the same station. For the matched pairs the second is the Reynolds number of the other member of the pair, and for the sweep it is a common reference, so that cases 2 and 5 are unchanged by construction. The ratio of the two gives the change in rotor thrust and torque due to Reynolds number alone. It is applied to the RANS power, and to the RANS total thrust on the assumption that the duct thrust scales with the rotor thrust. Three treatments of the inflow bracket the modelling uncertainty: axial inflow only ($v_\theta = 0$); swirl v_θ from annular momentum; and swirl plus a uniform incidence offset, 1.2° to 8.6° , chosen so that the blade-element thrust with the reference polars equals the RANS rotor thrust. Without that calibration it is 1.2 to 10.2 times the RANS value, because a two-dimensional section model omits the three-dimensional, cascade and duct effects on these low-aspect-ratio blades; this is why only ratios are used. Stations where the polars give no data are excluded and the ratios are formed over the remaining span. This removes the span inboard of $r = 22-33$ mm in the first two treatments and $r = 34-61$ mm in the calibrated one, and in two thirds of the runs also up to the outermost 6 mm, where the low axial velocity next to the duct wall raises the incidence beyond the polars.

Table 8: Figure of merit with the Reynolds-number effect, from the blade-element model with XFOIL polars, as the range over three inflow treatments, against the RANS value. Sections are referenced to the Reynolds number of a 30 mm chord, so cases 2 and 5 are unchanged. Lower block: six-blade relative to four-blade figure of merit within each matched pair.

Case ($N_b \times c$)	σ	RANS	With Reynolds-number effect	
			$N_{crit} = 3-5$	$N_{crit} = 9$
1 (4×20 mm)	0.340	0.320	0.267 to 0.296	0.177 to 0.246
2 (4×30 mm)	0.509	0.304	0.304	0.304
3 (4×40 mm)	0.679	0.253	0.259 to 0.287	0.269 to 0.310
7 (4×45 mm)	0.764	0.222	0.229 to 0.275	0.239 to 0.306
4 (6×20 mm)	0.509	0.300	0.243 to 0.276	0.162 to 0.235
5 (6×30 mm)	0.764	0.242	0.242	0.242
6 (6×40 mm)	1.019	0.166	0.169 to 0.206	0.176 to 0.233
Pair A, six vs four	0.509	-1.3 %	-20.2 to -9.2 %	-46.7 to -22.6 %
Pair B, six vs four	0.764	+8.9 %	-12.1 to +5.9 %	-20.9 to +1.4 %

Result. The direction of the change follows from the polars, whose lift-to-drag ratio rises with Reynolds number at every transition setting; the blade-element model supplies its size, which depends strongly on the inflow treatment (Table 8, Figure 4b). Relative to the 30 mm chords, the 20 mm chords of cases 1 and 4 lose 8 % to 19 % of their figure of merit with turbulent inflow and 22 % to 46 % with quiet inflow, while the 40 and 45 mm chords gain 2 % to 24 % and 6 % to 40 %. The calibrated treatment, which lowers the outer-span incidence towards zero, where the S1223 is most sensitive to Reynolds number, gives the largest changes. Because the RANS solutions already contain a little under half of the Reynolds-number sensitivity of drag, the part they miss is somewhat smaller than these changes.

Two consequences follow. Within the matched pairs, the change favours the four-blade, wider-chord rotor in every variant. With turbulent inflow the figure-of-merit difference of pair A moves from -1.3 % to between -20.2 and -9.2 %, outside the grid band, and the six-blade advantage of pair B shrinks or reverses, to between -12.1 and +5.9 %; the shifts are 3 to 21 percentage points, and the corresponding T/P differences become -14.8 to -7.2 % and -5.7 to +4.6 %. Along the sweep, the change removes the advantage of the least solid rotor: with the Reynolds-number effect included, case 1 falls to or below case 2 in every variant. For turbulent inflow the margin is 2.8 % to 5.4 % without calibration, inside the grid band, and 10.5 % to 12.2 % with it; if only the part of the effect missing from the RANS solutions is applied, cases 1 and 2 are about level without calibration (-1.1 to +0.4 %) and 4 to 5 % apart with it. Beyond $\sigma \approx 0.5$ the figure of merit still falls, or stays level, with solidity in every turbulent-inflow variant, for both blade counts. For quiet inflow the six-blade series peaks at case 5, and in the calibrated treatment the four-blade series rises slightly from $\sigma = 0.509$ to 0.679. Where the optimum lies is therefore not resolved: the estimate removes the simulated advantage of the narrowest chord without establishing a new optimum.

The absence of a blade-count effect in the simulations, and their ranking of the narrow-chord rotors, are therefore at least partly consequences of the closure. The direction of the estimated effect agrees with experiments in which increasing the chord of a micro rotor raised its figure of merit while adding blades did not [8], and in which shrouded rotors lost performance as blades were added at fixed solidity [9]. It is largest where the Reynolds number is lowest, for the 20 mm chords that run near $Re_c = 48,400$.

5.4 Flow near the hub

Axial velocity was exported on three planes of each converged solution: the meridional half-plane through the mid-chord of one blade; the throat entry, $y = -25$ mm, 15 mm to 20 mm upstream

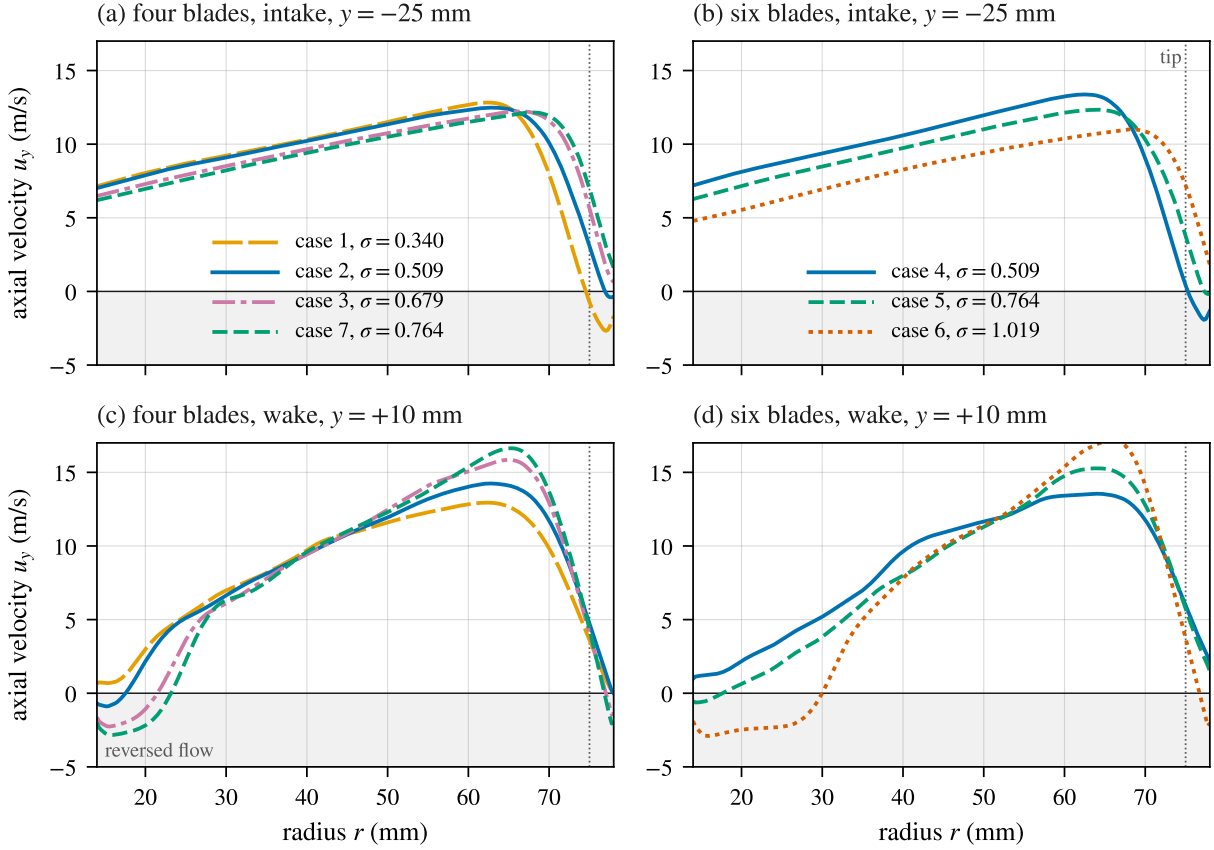


Figure 5: Passage-averaged radial profiles of axial velocity at the intake ($y = -25$ mm) and near-wake ($y = +10$ mm) stations. (a, c) Four-blade cases 1, 2, 3 and 7; (b, d) six-blade cases 4, 5 and 6. Colour and line style denote solidity, so cases 2 and 4, and cases 7 and 5, share a style. The shaded band is reversed flow; the dotted line is the tip radius.

of the blade; and the near wake, $y = +10$ mm, 3.6 mm to 7.2 mm behind it. The two cross-flow planes cover a whole blade passage, and the profiles and quantities below are averages over it at constant radius (Figure 5, Table 9); a negative axial velocity denotes reversed flow. The passage-averaged intake profiles imply that the untwisted blade runs at roughly -19 to -9° incidence at $r = 14$ mm and $+1$ to $+5^\circ$ at $r = 60$ mm to 65 mm, so the inboard span carries little thrust. Figure 6 shows the meridional plane of the two matched pairs and Figure 7 the wake plane of all seven rotors.

Four features stand out. First, the through-flow near the hub falls broadly with solidity: the mean wake velocity over $r = 14$ mm to 35 mm is $+4.67$ m/s at $\sigma = 0.340$, $+3.61$ and $+4.10$ m/s at 0.509, between $+1.91$ and $+2.59$ m/s at 0.679 and 0.764, and -1.04 m/s at 1.019.

Second, a core of reversed flow adjoins the hub in five of the seven rotors, and where it exists it is present at every azimuth of the passage (Figure 7). The meridional plane shows that it is the upstream end of a region of reversed flow behind the blunt downstream face of the hub, which extends along the axis beyond the duct exit (Figure 6). Its size follows the chord more closely than the solidity or the blade count: it is absent in both rotors with 20 mm chords, covers 2.0 and 2.1 % of the sampled annulus with 30 mm chords, 4.6 and 5.8 % with four blades of 40 and 45 mm, and 11.6 % in case 6, whose six 40 mm blades leave a gap of only 0.17 mm at the hub. It is not ordered by the local solidity at the hub: cases 2 and 4 have the same value, 1.47, and the core is present in case 2 and absent in case 4.

Third, the flow in the tip gap, averaged over the passage, is reversed in the three rotors with chords of 40 mm or more, cases 3, 6 and 7, at -1.4 to -2.0 m/s, and forward or about zero in the others. In every case it varies around the passage, by 2.3 m/s to 7.4 m/s between its extremes.

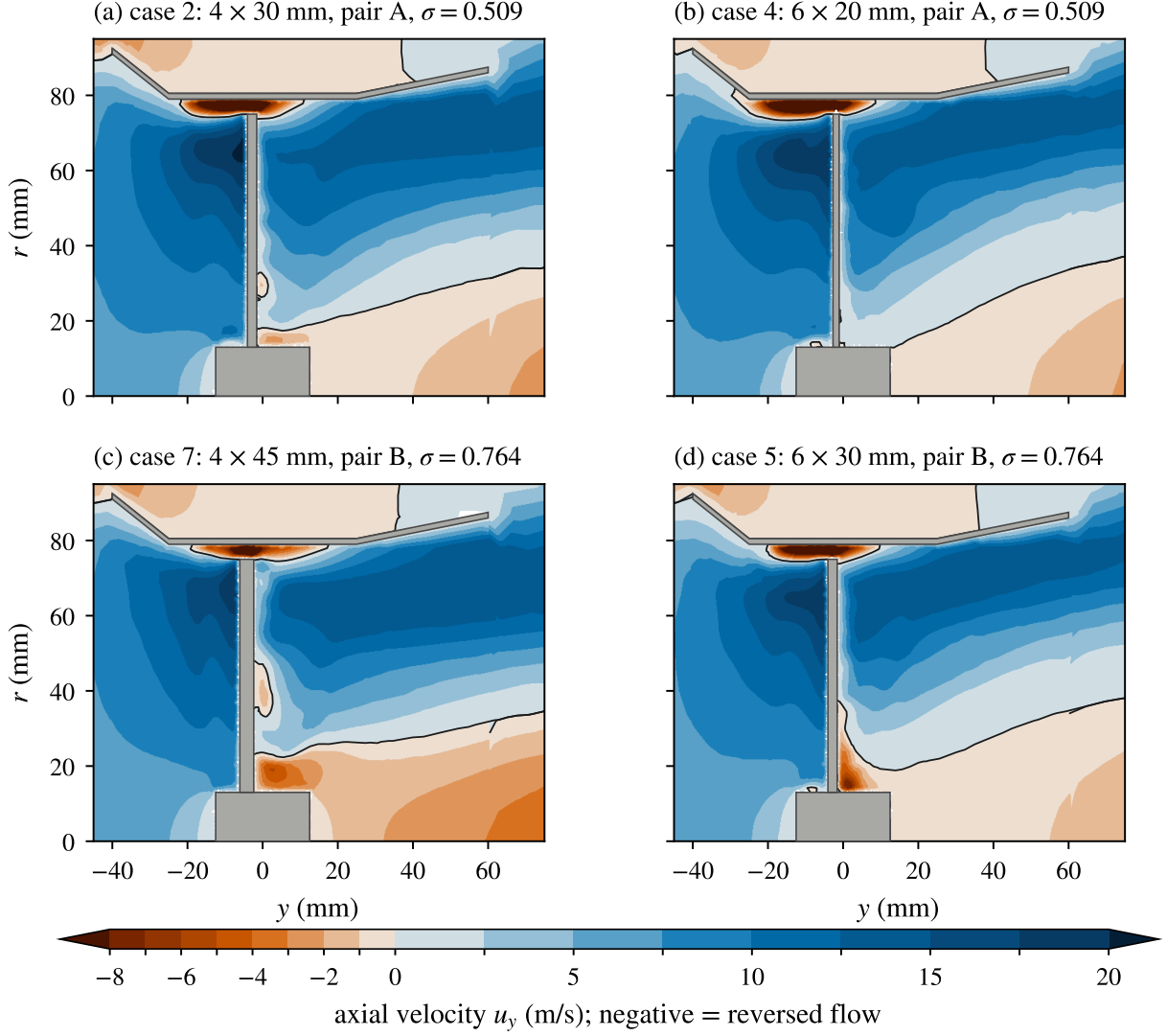


Figure 6: Axial velocity in the meridional plane through the mid-chord of one blade, for the two matched pairs: (a, b) pair A, $\sigma = 0.509$; (c, d) pair B, $\sigma = 0.764$. Flow is from left to right; $y = 0$ is the rotor plane and $r = 0$ the axis. Grey: hub, blade section and duct wall. The black line is $u_y = 0$. The colour scale is not symmetric about zero.

Table 9: Inboard geometry and near-wake flow, in order of chord. The local solidity at the hub is $\sigma_{loc} = N_b c / 2\pi r$ at $r = 13$ mm, and the narrowest gap is the smallest distance between adjacent blade surfaces, computed from the geometry. Wake quantities are from the plane $y = +10$ mm: the passage-averaged axial velocity, as a mean over $r = 14$ mm to 35 mm; the radius at which the passage-averaged reversed flow next to the hub ends; the share of the annulus $r = 14$ mm to 78 mm, over the whole passage, in which the flow inboard of $r = 60$ mm is reversed; and the passage-averaged axial velocity in the tip gap at $r = 78$ mm.

Case	$N_b \times c$ (mm)	σ	σ_{loc} at hub	Narrowest gap (mm)	Inboard mean u_y (m/s)	Core edge (mm)	Core area	Tip-gap u_y (m/s)
1	4×20	0.340	0.98	5.38	+4.67	none	0	+0.04
4	6×20	0.509	1.47	2.08	+3.61	none	0	+2.05
2	4×30	0.509	1.47	4.19	+4.10	17.5	2.0 %	-0.01
5	6×30	0.764	2.20	1.09	+2.15	17.6	2.1 %	+1.51
3	4×40	0.679	1.96	3.41	+2.59	21.5	4.6 %	-1.38
6	6×40	1.019	2.94	0.17	-1.04	30.0	11.6 %	-2.03
7	4×45	0.764	2.20	3.06	+1.91	23.2	5.8 %	-1.98

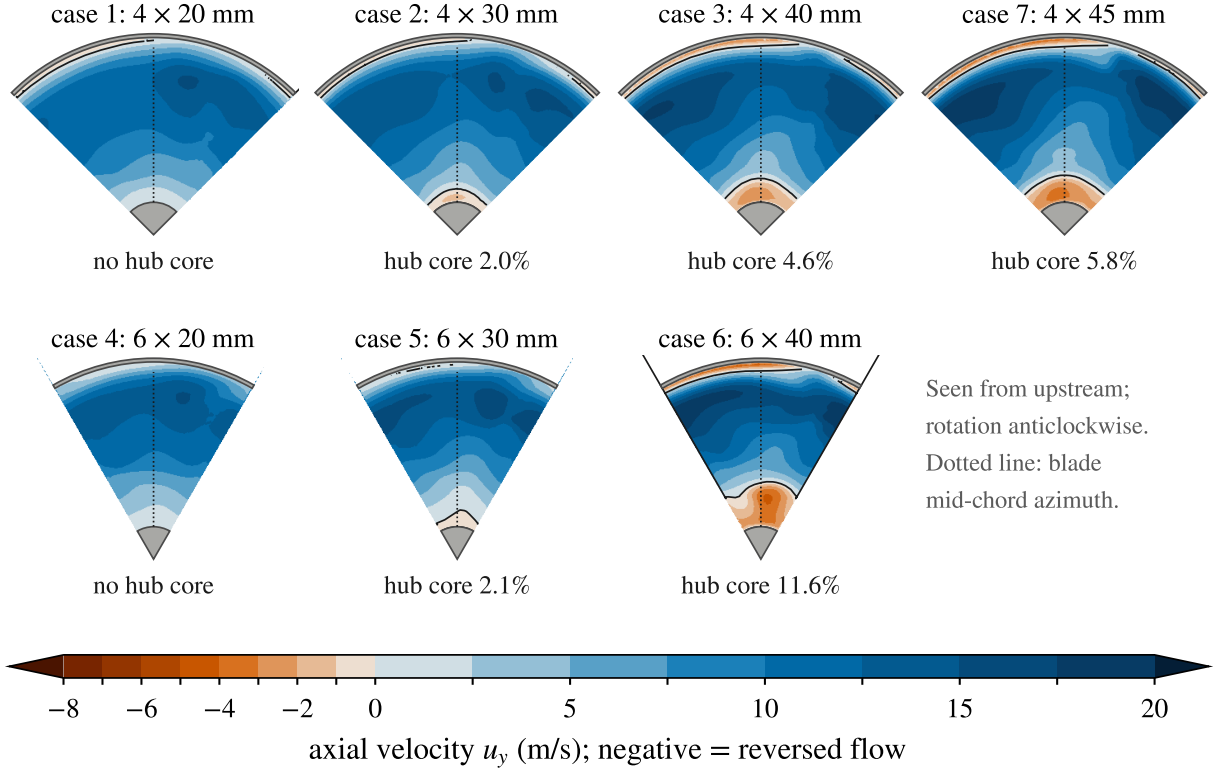


Figure 7: Axial velocity in the near-wake plane, $y = +10$ mm, over one blade passage of each rotor, seen from upstream: columns by chord, rows by blade count. The dotted line is the mid-chord azimuth at which Figure 6 is cut, and the black line is $u_y = 0$. Below each passage is the share of the annulus with reversed flow next to the hub (Table 9).

Fourth, the inboard flow is not set by solidity alone, whereas efficiency, within the resolution of the simulations, is. In each matched pair the two rotors differ near the hub: the core is present in case 2 and absent in case 4, and it covers 5.8 against 2.1 % of the annulus in cases 7 and 5, whose passage-averaged wake profiles differ by 1.63 m/s root-mean-square. Yet the efficiencies within each pair do not differ measurably. The reversed core is therefore reported as an observed feature, not as a mechanism that sets efficiency. On the three grids of the case 5 study, sampled at the blade azimuth, the core is always present, its edge moving from 19.2 to 20.5 to 21.4 mm as the grid is coarsened, so its existence is not a feature of the production mesh, although its extent is not grid-converged. Sampling at the blade azimuth alone would bias these quantities: there the inboard mean velocity is 0.2 m/s to 1.2 m/s lower, and the core 0.4 to 1.0 percentage points larger, than the passage average.

6 Limitations

Blade section and twist. The S1223 is a high-camber wing section rather than a propeller section, and at the Reynolds numbers of this rotor it operates below the range in which it is a high-lift aerofoil (Appendix B). The blades are untwisted, so the inboard span runs at large negative incidence. Both choices were made to isolate solidity, and both make the absolute performance specific to this rotor. Whether the trends hold for a twisted blade with a purpose-designed section has not been tested.

Turbulence closure and wall resolution. The fully turbulent closure over-predicts section drag by 46 % at the closest validated condition and contains less than half of its Reynolds-number sensitivity. No inflation layers could be generated, so the boundary layers are not resolved; the rotor runs at a mean y^+ of 30.5 and the two-dimensional validation at 95.4, so the

sectional errors transfer to the rotor only approximately. These are the reasons for the sectional analysis of Section 5.3.

The blade-element estimate. The estimate of Section 5.3 uses two-dimensional polars, a frozen, passage-averaged axial inflow taken from each RANS solution, a crude incidence calibration and the assumption that duct thrust scales with rotor thrust, and it excludes inboard and, in some runs, tip stations where the polars give no data. Its ranges are correspondingly wide. Its direction follows from the polars; its magnitude is an order of magnitude, not a corrected value. A wall-resolved, transition-sensitive simulation of the matched pairs and of cases 1 and 2, or an experiment, is needed for that.

Steady formulation. The moving-reference-frame solution cannot capture blade-passing unsteadiness, and the iterative instability of Appendix A is evidence that the separated blade flow is genuinely unsteady. The steady solutions are best read as approximations to the time-mean of an unsteady flow, with an uncharacterised error; a time-accurate computation of at least one case would test this.

Resolution of the inboard passages and the tip gap. The narrowest gap between adjacent blades is 1.09 mm in case 5 and 0.17 mm in case 6, less than one nominal cell, so the inboard flow of these cases is resolved only qualitatively. The 4.00 mm tip clearance spans fewer than three cells. At 5.33% of the tip radius it is also large compared with the sub-1 % clearances of high-performance fans, which makes absolute performance pessimistic [11, 14]; it is identical in every case, but the flow through it differs between cases (Section 5.4).

Scope. Each case is a single periodic passage, which excludes non-axisymmetric flow, and is computed at one operating point in hover.

7 Conclusions

Seven rotors for a 150 mm ducted fan in hover at 6000 rpm, with blade count and chord varied independently over $\sigma = 0.34$ to 1.02 and two pairs matched in solidity, were simulated with steady RANS and the $k-\omega$ SST closure, with the discretisation sensitivity measured for every configuration and the grid uncertainty for one, and the Reynolds-number effect that the closure under-represents was estimated with XFOIL.

1. In the simulations, efficiency falls monotonically with solidity for both blade counts: the figure of merit from 0.320 at $\sigma = 0.340$ to 0.166 at $\sigma = 1.019$, and T/P from 0.0780 to 0.0406 N/W. One straight line in σ describes all seven configurations to within 4.5 %. The decline is driven mainly by power, which rises by about 33 W per unit of solidity, while thrust levels off at 2.12 N to 2.23 N and falls at the highest solidity.
2. The simulations do not resolve an effect of blade count at fixed solidity. The matched-pair differences in figure of merit, -1.3% and $+8.9\%$, are comparable to the 6.3 % grid band, and the second changes sign with the discretisation scheme.
3. Both results are at least partly a consequence of the closure. The fully turbulent model reproduces less than half of the measured fall of S1223 drag with Reynolds number, whereas XFOIL reproduces the drag trend for the blade section. A blade-element estimate built on XFOIL polars indicates that the missing effect favours wider chords in every variant. It moves the matched-pair figure-of-merit differences 3 to 21 percentage points towards the four-blade rotors for turbulent inflow, and more for quiet inflow, in line with experiments on micro rotors [8] and shrouded rotors [9]. It also costs the 20 mm chords enough to remove the simulated advantage of the least solid rotor, which falls to or below the 30 mm rotor in every variant, so where the optimum lies is not resolved; above $\sigma \approx 0.5$ efficiency still falls with solidity for turbulent inflow.

4. Comparisons between configurations are more robust to discretisation than the forces themselves. Thrust and power change by 9% to 24% between first- and second-order momentum schemes while T/P changes by 0.7% to 9.6%; every solidity step keeps its sign, but one of the two matched-pair differences does not. The steady formulation is iteratively unstable at second order on this configuration and needs a pseudo-transient formulation to converge.
5. A core of reversed flow adjoins the hub, at every azimuth, in every rotor with a chord of 30 mm or more and in neither 20 mm rotor; it is the upstream end of the reversed flow behind the hub and, for each blade count, grows with chord. It is present on all three grids of the convergence study, but it differs between configurations of equal solidity whose efficiencies do not differ measurably, and it is reported as an observation rather than as a cause of the trend.

In this regime efficiency depends on chord Reynolds number as well as on solidity. A fully turbulent RANS model under-represents the effect of the Reynolds number, so it should not be relied on to compare configurations that differ in chord, whether at equal solidity or along a chord sweep. The natural next steps are a wall-resolved, transition-sensitive simulation or an experiment on the two matched pairs and on cases 1 and 2, which would turn the estimate of Section 5.3 into a measurement; low-solidity rotors with fewer, wider blades, which would show whether the optimum is set by solidity or by chord; and a twisted blade with a purpose-designed section.

Acknowledgements

All computations were performed with the ANSYS 2026 R1 Student licence on personal hardware, within its 1,048,576-cell limit. The experimental data used for validation are from the University of Illinois at Urbana-Champaign *Summary of Low-Speed Airfoil Data*, distributed by the UIUC Applied Aerodynamics Group [19]. XFOIL is distributed by M. Drela under the GNU General Public License.

Availability of data and materials

The data and scripts supporting the conclusions of this article are available in the Zenodo repository, <https://doi.org/10.5281/zenodo.23361018> [27]. They comprise the converged forces and torques of all seven configurations, exports of the three flow-field planes of each solution, the XFOIL polars, and the Python scripts that produce every table and figure. The ANSYS Fluent case and data files are available from the author on request.

Declaration of AI-assisted tools

The author conceived the study, set up and ran all CFD simulations and XFOIL calculations, and exported all data used here. Claude (Anthropic), a generative AI assistant, was used to help plan and debug the simulation set-up and to write the geometry scripts; to suggest the additional sectional and flow-field analyses; to write and run the post-processing and blade-element scripts that produce the figures and tables; to find and check references; to check calculations; and to draft and edit the text of the manuscript. The author reviewed and verified every result, value and statement and takes full responsibility for the content.

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Appendix A Iterative behaviour of the steady formulation

With the converged first-order solution as the starting field, switching momentum and both turbulence equations to second-order upwind produced a regular oscillation in the integral loads with a period of about 170 iterations, whose amplitude grew from $\pm 4\%$ to $\pm 20\%$ of rotor thrust over 2000 iterations while all scaled residuals rose by a factor of three to ten. Reducing the momentum under-relaxation factor from 0.7 to 0.5 delayed but did not remove the growth. Returning the turbulence equations to first-order upwind, with second-order momentum, gave 1600 iterations of smooth behaviour before the loads became irregularly oscillatory.

Three checks exclude a mesh defect. The mesh has no negative volumes and a minimum cell volume of $5.37 \times 10^{-12} \text{ m}^3$. The maximum velocity in the domain is 57.9 m/s against a tip speed of 47.1 m/s, consistent with an ordinary suction peak and tip-gap jet; an earlier mesh with

Table 10: Two-dimensional validation of the S1223 against the UIUC measurements [18, 19], at the production mesh resolution and wall treatment. Experimental uncertainty is 1.5 % on both coefficients. The reference data contain no drag measurements below $Re_c \approx 100,000$.

Re_c	α (deg)	C_l (exp.)	C_l (CFD)	ε_{C_l}	C_d (exp.)	C_d (CFD)	ε_{C_d}
61,000	-0.17	0.913	0.9202	+0.8%	—	—	—
	0.67	0.988	1.0113	+2.4%	—	—	—
	1.94	1.084	1.1343	+4.6%	—	—	—
	2.85	1.123	1.2247	+9.1%	—	—	—
	3.70	1.058	1.2973	+22.6%	—	—	—
100,600	-0.01	1.048	0.9525	-9.1%	0.0244	0.0369	+51.2%
	1.75	1.213	1.1296	-6.9%	0.0290	0.0424	+46.3%
	3.72	1.372	1.3173	-4.0%	0.0332	0.0499	+50.4%
	5.74	1.529	1.4716	-3.8%	0.0418	0.0591	+41.3%
	6.72	1.588	1.5379	-3.2%	0.0481	0.0679	+41.2%
200,400	0.00	1.083	0.9727	-10.2%	0.0191	0.0336	+75.8%
	3.77	1.428	1.3446	-5.8%	0.0227	0.0475	+109%
	7.93	1.753	1.6152	-7.9%	0.0314	0.0745	+137%
	11.87	2.018	1.6538	-18.0%	0.0397	0.1215	+206%
	15.19	2.195	1.5332	-30.2%	0.0454	0.1907	+320%

degenerate cells had returned 4.76×10^4 m/s. And the first-order solution on the same mesh converges to 0.1 % over 500 iterations without difficulty.

The interpretation is that the separated blade flow is genuinely unsteady. First-order upwind suppresses the unsteadiness through its numerical dissipation; without it, the steady equations, which contain no time derivative, have no mechanism to bound it. The pseudo-transient term supplies that mechanism and vanishes at convergence, so it does not alter the converged solution. With the coupled solver and a time scale factor of 0.5 all six scaled residuals fell monotonically over three consecutive blocks of 1000 iterations. With the settings tried, keeping the turbulence equations at first order was necessary for stability; it is a smaller compromise than it may appear, because the blade loading is governed by the momentum discretisation and the turbulence equations affect it only through the eddy viscosity.

Appendix B Sectional validation details

The two-dimensional domain extended 45 chords in every direction and was meshed at the production resolution and wall treatment; incidence was imposed by rotating the freestream vector, so that one mesh served every angle. The Reynolds numbers were 100,600, the lowest at which the reference data contain both lift and drag; 200,400, the design Reynolds number of the section [18]; and 61,000, the closest available to the narrow-chord cases, at which the reference data contain lift only (Table 10, Figures 8 and 9).

At $Re_c = 100,600$ the lift error falls monotonically as the section loads up and the lift-curve slope is 8.4 % high. The drag error grows with Reynolds number, from 46 % on average at 100,600 to 170 % at 200,400, because the computed drag barely responds to Reynolds number while the measured drag falls. Stall fails in both directions. At 200,400 the computation reaches $C_{l,max} = 1.66$ near 10.7° (the vertex of a quadratic through its three highest-incidence points), while the measured lift is still rising at the last point of that run, $C_l = 2.195$ at 15.19° , and the companion lift run at 198,100 peaks at 2.183 at 16.7° : maximum lift is under-predicted by about 24 %, and stall comes 4.5 to 6 degrees early. At 61,000 the measured lift reaches a local maximum of 1.123 at 2.85° and then falls while the computed lift keeps rising. The lift-curve slope errors, from fits over the linear part of each measured curve, are +25.4 %, +8.4 % and

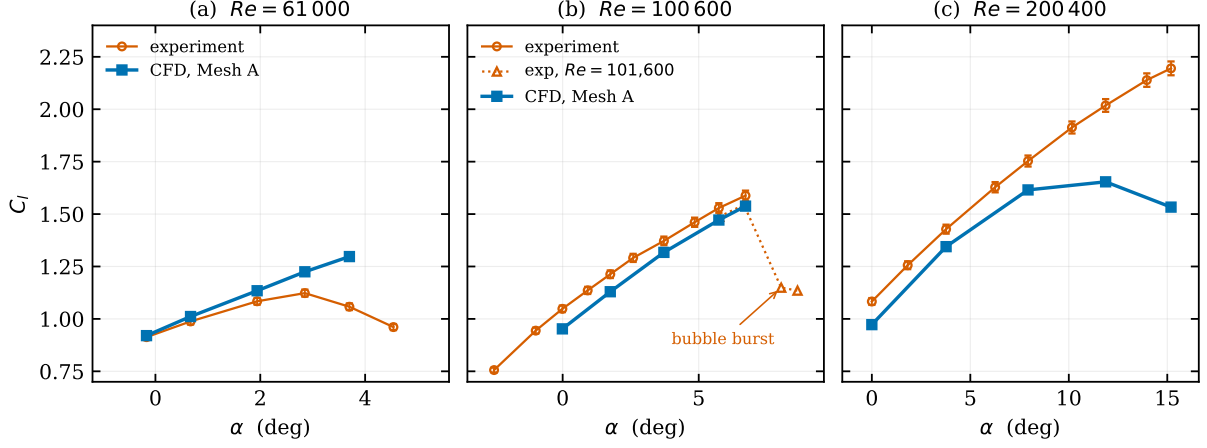


Figure 8: Sectional lift of the S1223 compared with the UIUC measurements at three chord Reynolds numbers. Error bars show the 1.5 % experimental uncertainty. The dotted extension in (b) is the $Re_c = 101,600$ lift run, showing the bubble burst that the fully turbulent closure cannot reproduce.

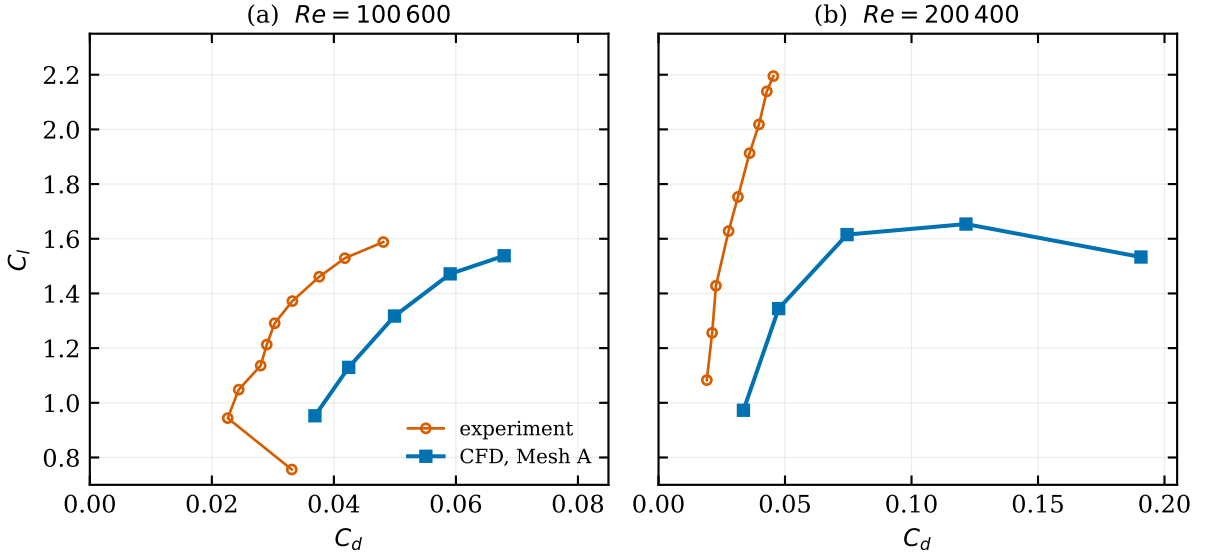


Figure 9: Drag polars at $Re_c = 100,600$ and $200,400$. The computed drag is nearly insensitive to Reynolds number because the closure contains no laminar separation bubble whose length can respond to it.

−4.3% at $Re_c = 61,000$, $100,600$ and $200,400$, so the closure is least reliable at the low end of the range.

A blade-element propagation indicates that these errors affect the rotor loads less than the drag figure suggests: at the inflow angles of the outer span, 12° to 20° , sectional drag supplies only 6% to 10% of the torque, and the $Re_c = 100,600$ errors give a thrust deficit of about 5% and a torque error within 2%. This concerns the absolute level of the loads; the difference between configurations, which depends on the Reynolds-number trend, is treated in Section 5.3.

The reference data also show that the S1223 does not reach its design $C_{l,max} \approx 2.2$ anywhere in the Reynolds-number range of this rotor. In the $Re_c = 101,600$ lift run its lift rises to 1.544 at 6.66° and falls to 1.149 at 8.03° , a loss of 26% within 1.4° that is characteristic of a laminar separation bubble bursting. At 61,000 the section reaches only its local maximum of 1.12, and its highest recorded lift, about 1.39, needs an incidence near 20° .

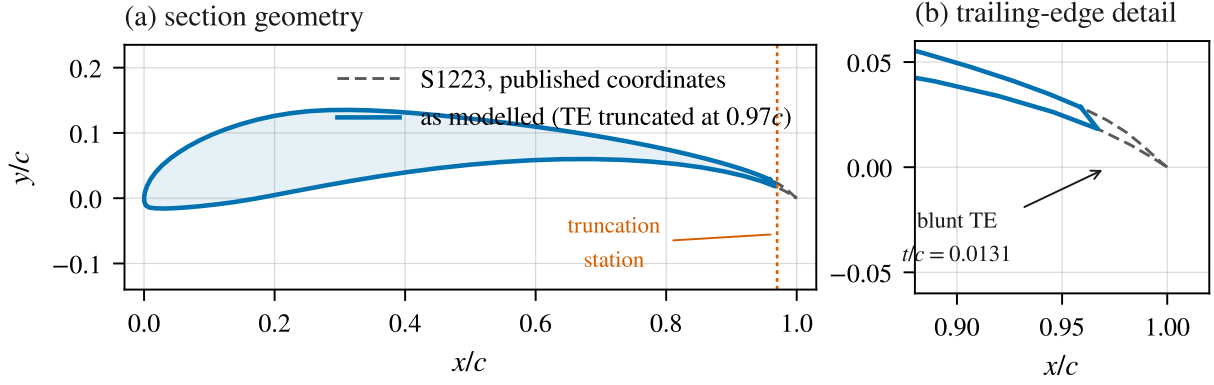


Figure 10: S1223 blade section. (a) Published coordinates compared with the modelled section, truncated at $0.97c$. (b) Detail of the blunt trailing edge, of thickness $0.0131c$.

Appendix C Geometry details

The published S1223 coordinates define a sharp, slightly cusped trailing edge that downstream CAD tools rejected and that would need impractically fine cells. The section was truncated at $0.97c$, giving a blunt trailing edge of thickness $0.0131c$, 0.26 mm to 0.59 mm depending on chord (Figure 10).

A blade made by extruding a pitched section along its span ends in a flat face, so the corners of the pitched chord project beyond the nominal tip radius by an amount that grows with chord: to 75.59, 76.33 and 77.34 mm (measured) for the 20, 30 and 40 mm chords and 77.94 mm (by the same construction) for 45 mm. Untrimmed, the tip clearance would have varied from 3.41 to 1.06 mm with chord, a confound in exactly the variable under study. Trimming to the 75 mm cylinder holds it at 4.00 mm in every case.

The straight duct throat extends 5 mm beyond the rotating-zone interfaces at either end. An earlier configuration placed the slope discontinuities of the duct profile on the interface planes, which produced degenerate cells there; separating the two features leaves throat radius, tip clearance, lip flare and exit area unchanged.