

MODEL PREDICTIVE CONTROL OF A WAVE ENERGY CONVERTER FARM FOR ENERGY OPTIMIZATION USING EACH DEVICE'S NEIGHBORS AS ITS WAVE FORECAST

Jacob Fontaine^{1,*}

¹Department of Ocean Engineering, University of Rhode Island, Narragansett, RI, USA

ABSTRACT

Optimal control of a wave energy converter (WEC) needs the wave excitation about one period ahead, which a single device cannot measure. We show in simulation that a WEC farm can supply this preview from the inertial measurement unit (IMU) and power take-off (PTO) force each device already carries. Heaving buoys are modeled in six degrees of freedom with the RM3 reference mooring resolved by a lumped-mass line model. A Fourier neural operator (FNO) recovers the incident elevation with a normalized error of 0.12 at a 2-s lag and the fairlead tension of the three mooring legs with 0.22. A least-squares fit to the estimated elevations of neighboring devices forecasts the elevation one peak period ahead with errors of 0.21–0.32 behind the front row, against 0.6–1.0 from a device's own record. A model-predictive controller driven by this forecast raises the electrical energy of a 12-WEC farm by 78% over passive damping tuned to the sea state, with the peak mooring tension unchanged in that sea; the front row, whose forecast is no better than its own record, gains 51%, nearly 60% of what the rows behind it gain. The heave PTO cannot reduce storm mooring loads, which surge and pitch drive. Fed with noisy true elevations, the farm forecast and an FNO tension model warn of all six exceedances of an illustrative limit in one storm record 20–27 s ahead, with 14 of 25 alarms false. Over a year of measured sea states and wave directions at Humboldt Bay, with the estimator replaced by a noisy-truth stand-in and the mooring linearized, the controller yields 62% more annual energy for 12 WECs (64% for 50, modeled), which lowers the levelized cost of energy of the RM3 cost model by 34–35% with the PTO cost doubled.

Keywords: wave energy converter, array, wave forecasting, model predictive control, mooring, neural operator

1. INTRODUCTION

The energy a heaving wave energy converter (WEC) absorbs depends on how the power take-off (PTO) force is phased against the incoming wave. The optimum, complex-conjugate control law is non-causal [1], and its practical forms, from constrained optimal control [2] to model-predictive control (MPC) [3, 4], need

a forecast of the wave excitation over roughly one wave period [5]. A single device can only extrapolate its own past motion; such autoregressive forecasts stay accurate for about two periods in swell and degrade quickly in broadband seas [6], and learned forecasters of the excitation face the same limit [7]. Up-wave measurements extend the horizon [8], deterministic sea-wave prediction defines the space–time region in which an upwave record can predict the wave at a downwave point [9], and optimized buoy arrays predict about three periods ahead even in spread seas [10]. For a small WEC array with ideal motion sensing, an array-wide estimator and forecaster of the excitation beats independent per-device ones [11], and array MPC [12] and coordinated array control [13] assume the preview is available. No study has yet built the preview from the IMU and PTO force of moored devices, carried its error through forecast and control, and valued the result over a site year.

Vessel motions have long served as directional wave sensors [14], excitation estimators for WECs have been benchmarked on ideal measurements [15], and neural networks have been used to predict mooring-line tension from platform motions [16]; neural operators [17, 18] learn such maps between time series without a hand-built model. Wave forecasts have also been used to mitigate the loads on a floating structure [19], and survival in storms remains the design driver of WECs [20]. Array experiments and layout studies use spacings of 100–300 m [21, 22], so most devices in a farm have neighbors upwave that met the same wave group 15–40 s earlier. Reactive control pays for its gain with power that flows back through a non-ideal PTO [23], annual-energy estimates built on a parametric spectrum can misstate the yield as much as a poor dynamics model does [24], and the mooring of a floating WEC sets the loads the controller must respect [25].

We build this preview in simulation for a 12-device farm on lumped-mass moorings. A learned estimator turns each device's IMU and PTO force into elevation and mooring tension, and the estimates of neighboring devices feed a least-squares forecast that drives an MPC and a storm-load warning. A skill-to-gain model then carries the energy result over a measured year of sea states and directions. The estimates that feed the forecast come from passive runs of the same seas, so the chain is closed through the forecast and the controller but not through the estimator's own closed-loop motions, which Sec. 4 discusses.

*Corresponding author: j.fontaine98@gmail.com

2. METHODOLOGY

The chain runs from the farm model and its sensors (Sec. 2.1) through the two estimators and the farm forecast (Sec. 2.2) to the controller, the annual assessment and the storm-load warning (Sec. 2.3). Throughout, the normalized root-mean-square error (NRMSE) of an estimate is its root-mean-square error over a record divided by the standard deviation of the true signal, and a realization is one random draw of the wave phases.

2.1 Farm model and sensors

The WEC (Fig. 1) is a surface-piercing vertical cylinder (diameter 10 m, draft 5 m, mass 392 t with the mooring attached) in 70 m of water, within the 40–100 m design range of Reference Model 3 (RM3) at Humboldt Bay, California [26] and at the anchor depth of the WEC-Sim RM3 mooring [27]. Hydrodynamic coefficients come from NEMOH [28] on a 640-panel mesh with a water-plane lid; a fourfold refinement changes added mass, excitation and damping by at most 2.5% of their peaks, and radiation damping agrees with the Haskind relation to 2.4%. The time-domain model is the Cummins equation [29] in six degrees of freedom, with surge, heave and pitch coefficients expanded to sway and roll by axisymmetry and each radiation kernel replaced by a sixth-order state-space fit (error below 4% of peak). Viscous loads are relative-velocity Morison drag on the hull side and keel with a drag coefficient of 1.0. The PTO applies a vertical force u between the hull and a fixed reference, rated ± 1 MN, with a one-way conversion efficiency $\eta_{\text{PTO}} = 0.8$ and a stroke limit of 3 m; absorbed power is $P = -u\dot{z}$, and electrical power is $\eta_{\text{PTO}}P$ when $P > 0$ and P/η_{PTO} when the PTO motors. The time-domain model reproduces the frequency-domain heave response amplitude operator to 0.3%, and the float’s natural heave period is 5.5 s.

The mooring is the RM3 reference design from the WEC-Sim distribution [27]: three legs at 120° , each 240 m of 126 kg/m chain from an anchor at 267 m radius to a subsurface buoy (16.8 t, 33.5 m^3) and 40 m of chain to the hull. It is simulated with the lumped-mass model MoorDyn [30], which reproduces basin-measured fairlead loads within about 10% [31]; lumped-mass models of this kind capture snap events [32]. MoorDyn and the quasi-static MoorPy statics [33] agree within 4% at rest, and the coupled model is converged in segment count (peak storm tension unchanged from 30 to 120 segments). Figure 2 shows the fairlead tension and motions in an operational sea ($H_s = 3$ m, $T_p = 9$ s) and a storm ($H_s = 10$ m, $T_p = 14$ s) under passive damping; over the full storm record the peak tension is 15% of the 5.3 MN breaking load and the upwave leg momentarily falls to 2 kN without snapping, while at $H_s = 3$ m the peak is 6%. The mooring’s heave stiffness is about 1% of the hydrostatic stiffness.

Seas are linear, directional JONSWAP [34] fields with $\cos^{2s}$ spreading [35], 400 cosine-stretched, jittered frequency lines and 18 directions; extreme events are embedded as NewWave groups [36]. Wave directions are the direction of travel, whereas the Humboldt Bay buoy reports the direction waves come from. Each WEC is excited by the incident field at its own position, and WEC–WEC hydrodynamic interaction is neglected: in the farm-power sea state of Sec. 3.2 ($H_s = 2.5$ m, $T_p = 9$ s) the capture width of one device is 2–4 m against 150-m columns, so the shadow a

device casts on the row behind it is small, though park effects in small arrays accumulate over rows [37]. The farm has 4 rows $\times$ 3 columns (and 5×10 for the 50-WEC case) at 150 m [21, 22]; the mooring footprints of neighboring devices overlap at this spacing, which the isolated-device models do not represent. Each WEC measures specific force and angular rate with a marine-grade IMU on the deck (noise 1.9 mm/s^2 and $0.022^\circ/\text{s}$ at 20 Hz, plus a constant bias per record of 2 mm/s^2 and $0.01^\circ/\text{s}$, one standard deviation) and its PTO force (noise 1% of rating).

2.2 Estimation and forecasting

The physics estimator is an augmented Kalman filter (AKF). It extends the six-degree-of-freedom model (radiation states, hydrostatics, linearized mooring stiffness, an equivalent linear viscous damping tuned once) with a directional bank of harmonic oscillators, 24 frequencies $\times$ 8 directions, whose states give the elevation and, through the excitation transfer functions, all excitation forces and moments. The oscillators are driven with a JONSWAP prior at the record’s true H_s , T_p and peak enhancement and a uniform directional prior, so the filter finds the wave direction itself. The measurement model maps the state to the IMU specific force, including the gravity component from tilt, and to the angular rates; the PTO force is a known input. We run the filter forward (AKF filter) and with a Rauch–Tung–Striebel smoother over the record (AKF smoother). Fairlead tension follows from the estimated motions through the quasi-static leg-tension Jacobian of the mooring. As a reference for what a linear map can do, a frequency-domain dynamic line model of the same mooring is also driven with the true motions.

The data-driven estimator is a one-dimensional FNO [17] (four spectral layers, 64 modes, width 64) that maps 102-s windows of the seven noisy channels at 5 Hz to the elevation, the heave excitation and the three fairlead tensions. It is trained on 96 closed-loop records of 30 min (H_s 1–4 m in 1-m steps, T_p 6–12 s in 2-s steps, peak enhancement 1 and 3.3, three realizations per cell, random mean direction and spreading $s \in \{5, 10, 20\}$) in which the PTO runs passive or reactive control with random gains plus a dither force. Channels are standardized, sensor noise is redrawn every epoch, and the network is trained for 60 epochs with AdamW (learning rate 10^{-3} , cosine schedule) on the mean-square error; the hyperparameters are fixed a priori and there is no validation split. An estimate at time t uses the window ending at t (zero lag), at $t + 2$ s (the 2-s-lag estimate), or the whole record (the full-record estimate). The 32 test records use new wave phases on the same sea-state grid.

The farm forecast fits, every second, the last 120 s of the estimated elevations of all WECs within 350 m of the target by a regularized least-squares sum of directional wave components. The components span 0.6–2.6 times the peak frequency at a spacing of $1/(3 \times 120)$ Hz, finer than the window’s own resolution, because a grid matched to the window is periodic in it and cannot extrapolate; directions span $\pm 60^\circ$ about the mean wave direction in 7.5° steps. The regularization is the JONSWAP- $\cos^{2s}$ variance of each component at the sea state’s true H_s , T_p and s , with the estimation noise variance as the data weight. The mean direction is a known input; the AKF recovers it to 2.6° (Sec. 3.1) and could supply it. The fitted field is propagated to the target WEC, and

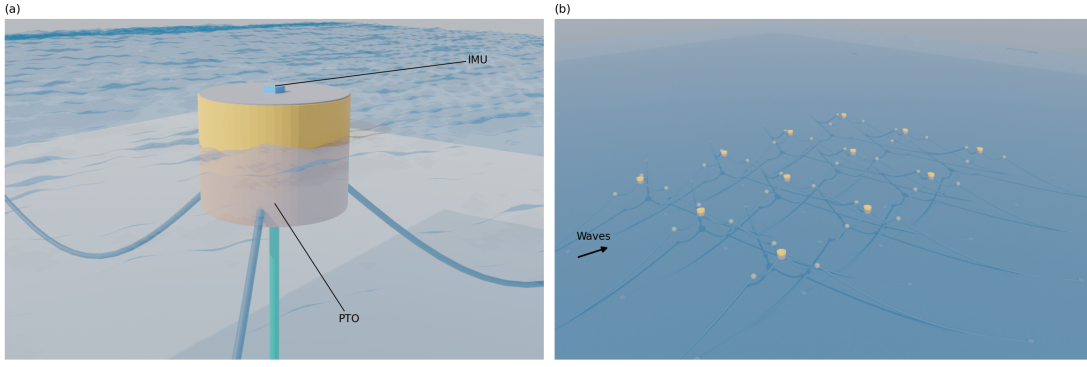


FIGURE 1: THE MODELED SYSTEM, RENDERED FROM THE SIMULATION GEOMETRY IN THE FARM-POWER SEA STATE ($H_s = 2.5$ M, $T_p = 9$ S): (A) THE WEC WITH ITS DECK IMU, THE FAIRLEAD CHAINS OF THE RM3 MOORING (THREE LEGS AT 120° , EACH A 40-M CHAIN TO A SUBSURFACE BUOY AND A 240-M CHAIN TO AN ANCHOR ON THE 70-M SEABED) AND THE PTO FORCE PATH TO A FIXED REFERENCE, WHICH IS AN IDEALIZATION (SEC. 4); (B) THE 12-WEC FARM AT 150-M SPACING WITH ITS OVERLAPPING MOORING FOOTPRINTS AND THE HEAD-ON WAVE DIRECTION.

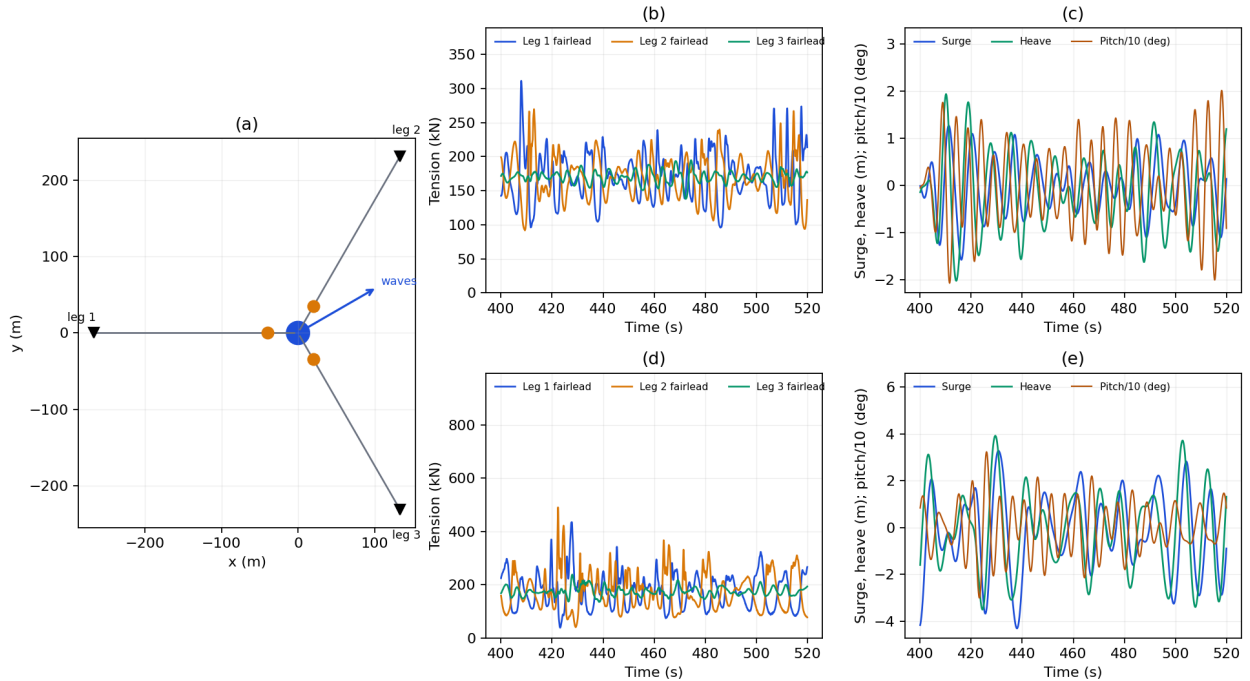


FIGURE 2: RM3 MOORING ON THE 10-M WEC: (A) PLAN VIEW WITH ANCHORS, SUBSURFACE BUOYS AND HULL; 120-S WINDOWS OF FAIRLEAD TENSION (B, D) AND SURGE, HEAVE AND PITCH (C, E) AT $H_s = 3$ M, $T_p = 9$ S AND $H_s = 10$ M, $T_p = 14$ S, WAVES TRAVELING TOWARD 30° FROM THE x AXIS, PASSIVE DAMPING 2×10^5 AND 5×10^5 NS/M.

its excitation over the control horizon follows from the excitation transfer functions. A single fit over the whole 50-WEC farm (1.35 km wide) fails because a finite direction grid cannot hold phase across that width, and the local fit is used throughout. The own-record forecast is the same fit on the target's own estimated elevation with a single direction. A forecast made at time t uses estimates up to $t - 2$ s, the estimator's lag. Where the FNO estimates are replaced by a stand-in, it is the true elevation plus white noise with 10% of the elevation's standard deviation at the forecast's 0.4-s input sampling; most of that noise lies above the fitted frequency band, and the in-band part is about 4% of the standard deviation.

2.3 Control, annual assessment and storm-load warning

Each WEC runs a receding-horizon controller on a heave model (radiation states, hydrostatic and mooring stiffness) discretized at 0.25 s. The cost is the negative absorbed power plus a quadratic conversion loss ru^2 , $r = (1 - \eta_{\text{PTO}})/b_{\text{ref}}$, which penalizes reactive power, subject to the force rating and the stroke limit [2, 3]; the excitation over the horizon is the farm forecast. The quadratic program is solved with OSQP [38] every 0.25 s from the true heave state, and the first force of the plan is applied. The baseline is passive damping tuned to each sea state; its damping maximizes the frequency-domain power for the sea state's T_p and is confirmed by a time-domain sweep. The con-

troller is scheduled on the sea state's H_s and T_p , known here, in the same way: the horizon is $\max(10\text{ s}, T_p)$, and the loss scale is $b_{\text{ref}} = c b_{\text{opt}}(T_p)$ with $c \in \{0.25, 0.5, 1\}$ chosen per sea-state bin on a tuning realization not used for evaluation.

The farm power result uses the 12-WEC farm at $H_s = 2.5\text{ m}$, $T_p = 9\text{ s}$, peak enhancement 3.3, $s = 10$, head-on, in three realizations. Every device is simulated for 600 s in the MoorDyn-coupled plant from 400 s into the sea record, and the first 50 s are discarded. The forecast input is the 2-s-lag FNO elevation at every WEC, computed once from passive runs of the same seas (damping $2.5 \times 10^5\text{ N s/m}$), so the estimator does not see the MPC's motions; its elevation error on those runs is 0.07–0.08.

The annual assessment uses one year of hourly sea states at Humboldt Bay (NDBC station 46244, 2023; 8744 h) [39], binned in 1 m of H_s and 2 s of T_p (58 occupied bins). Each bin is simulated as a JONSWAP sea (peak enhancement 3.3, $s = 10$, one mean direction) at its mean H_s and T_p for 350 s, the first 50 s discarded, on one realization different from the tuning one. The runs cover headings of 0 and 30° and four representative devices, the front corner, the front center, a row-3 edge and the row-3 center, which stand for 2, 1, 6 and 3 devices of the 12-WEC farm. They use the linearized mooring and the 10%-noise stand-in, and no operating limit is applied. To avoid simulating every heading, a skill-to-gain model gives the gain of WEC i as $G_i = G_{\text{hi}}(H_s, T_p) \phi(e_i)$, with one free G_{hi} per bin and ϕ falling linearly from 1 at $e_i = 0$ to a floor f_{min} at $e_i = e_1$. Here e_i is the NRMSE of the device's heave-excitation forecast over the control horizon, pooled over leads and twelve forecast instants on one realization with the stand-in, and it is computed for every device at headings of 0–90° in 15° steps and every T_p bin (other headings follow from the symmetry of the grid), for the 12-WEC farm and separately for the 50-WEC farm. The 60 parameters are fitted to the 464 power-matrix runs, weighted by the hours in each bin, and the model is checked against direct simulations of every device at 60 and 90° in the two most energetic bins, which are not used in the fit. Each hour of the year then takes the passive power of its bin and the modeled gain at its measured heading relative to the farm axis, which is aligned with the commonest wave direction. The leveled cost of energy (LCOE) follows from the RM3 cost breakdown [26, 40] at 10 and 50 units, with the annual energy scaled by the farm's gain.

For storm loads, the heave PTO's authority over the fairlead tension is measured by forcing the hull in pure heave in MoorDyn, which gives the heave-caused tension as a linear map of heave and heave velocity; the remainder of the tension is regressed on surge, pitch and their rates. Storm tension is then forecast with an FNO of the same architecture, trained on the true six-component excitation of the MoorDyn records (the 96 operational records and six storm records at $H_s = 10\text{ m}$, $T_p = 14\text{ s}$ with random direction and spreading, the storms oversampled fourfold) and tested on two held-out storms. Its 102-s input window runs from 72 s in the past to 30 s ahead; at each decision second the farm forecast fills that window at the target, and the FNO returns the three tensions. The warning case is a separate $H_s = 10\text{ m}$, $T_p = 14\text{ s}$, $s = 5$, head-on record with an embedded NewWave group of crest $0.93H_s$ (wave height $1.86H_s$) focused on the row-3 center device; the crest reaches the scored back-row center device

21 s later with a height of 10.2 m. The forecast is fed with the 10%-noise stand-in at all 12 WECs, because the elevation FNO is trained on $H_s \leq 4\text{ m}$. The alarm compares the forecast maximum tension over leads of 2.5–27.5 s, the first forecast step beyond the nowcast to the last reliable one, with 80% of a 600-kN limit (11% of the breaking load, chosen so that the record contains several exceedances). A linear finite-impulse-response map from excitation to tension is the baseline.

3. RESULTS

We report the estimation and forecast skill (Sec. 3.1), the farm power, the control force and the mooring load (Sec. 3.2), the annual energy and cost (Sec. 3.3) and the storm loads (Sec. 3.4).

3.1 Estimation and forecast skill

Figure 3 scores both estimators on the 32 test records. At zero lag the FNO recovers the elevation with NRMSE 0.28 and the AKF filter 0.50; on the full record the FNO reaches 0.06 and the AKF smoother 0.25; at the 2-s lag used downstream the FNO reaches 0.12. For the heave excitation the figures are 0.30 against 0.49 at zero lag, 0.04 against 0.18 on the full record and 0.16 at the 2-s lag. The estimators differ most on the mooring tension: the AKF route errs by more than the tension's standard deviation (NRMSE 1.3–1.7), whereas the FNO reaches 0.22 at the 2-s lag (0.20–0.25 per leg) and 0.11 on the full record, and the frequency-domain line model driven by the true motions reaches 0.29–0.42. The line model's 0.29–0.42 points to the quasi-static map (Sec. 4). The AKF recovers the mean wave direction to 2.6° on average (Fig. 3e).

Figure 4 maps the elevation forecast error at every WEC of the 12-WEC farm for leads of one, two and three peak periods in the farm-power sea state, with the 2-s-lag FNO elevations as the only input. The front row has no upwave neighbors and little skill at one period (0.66–0.82 at the corners, 1.0 at the center), no better than its own record. From the second row on, the error at one period is 0.21–0.32; it rises to about 0.6 at three periods in the second row and stays near 0.3 to three periods in rows three and four, the range the storm warning's 25-s lead needs at $T_p = 14\text{ s}$. The own-record forecast gives 0.6–1.0 at one period everywhere, and its spread across devices in statistically identical seas shows the realization noise in these maps. The excitation-forecast error e_i of the skill-to-gain model, computed over the control horizon with the stand-in, is 0.57 and 0.68 at the front corners, 0.55 at the front center, 0.12 at the row-3 edge and 0.15 at the row-3 center, with row means of 0.60, 0.20, 0.13 and 0.13. At this sea state the farm forecast from the FNO elevations and from the stand-in give the same one-period error, 0.21–0.32 behind the front row.

3.2 Farm power

Figure 5 gives the farm power at $H_s = 2.5\text{ m}$, $T_p = 9\text{ s}$, scored in the MoorDyn-coupled plant (passive damping $5.7 \times 10^5\text{ N s/m}$). The farm produces 78.2% more electrical energy with the MPC and the farm forecast (77.4–79.3% across the three realizations): 86–88% in rows two to four and 51% in the front row (64% at its center device, 42% at the corners). The controller enlarges the heave motion within the stroke limit (Fig. 5c), and the peak fairlead tension changes by –1.0% on average (at most +4%)

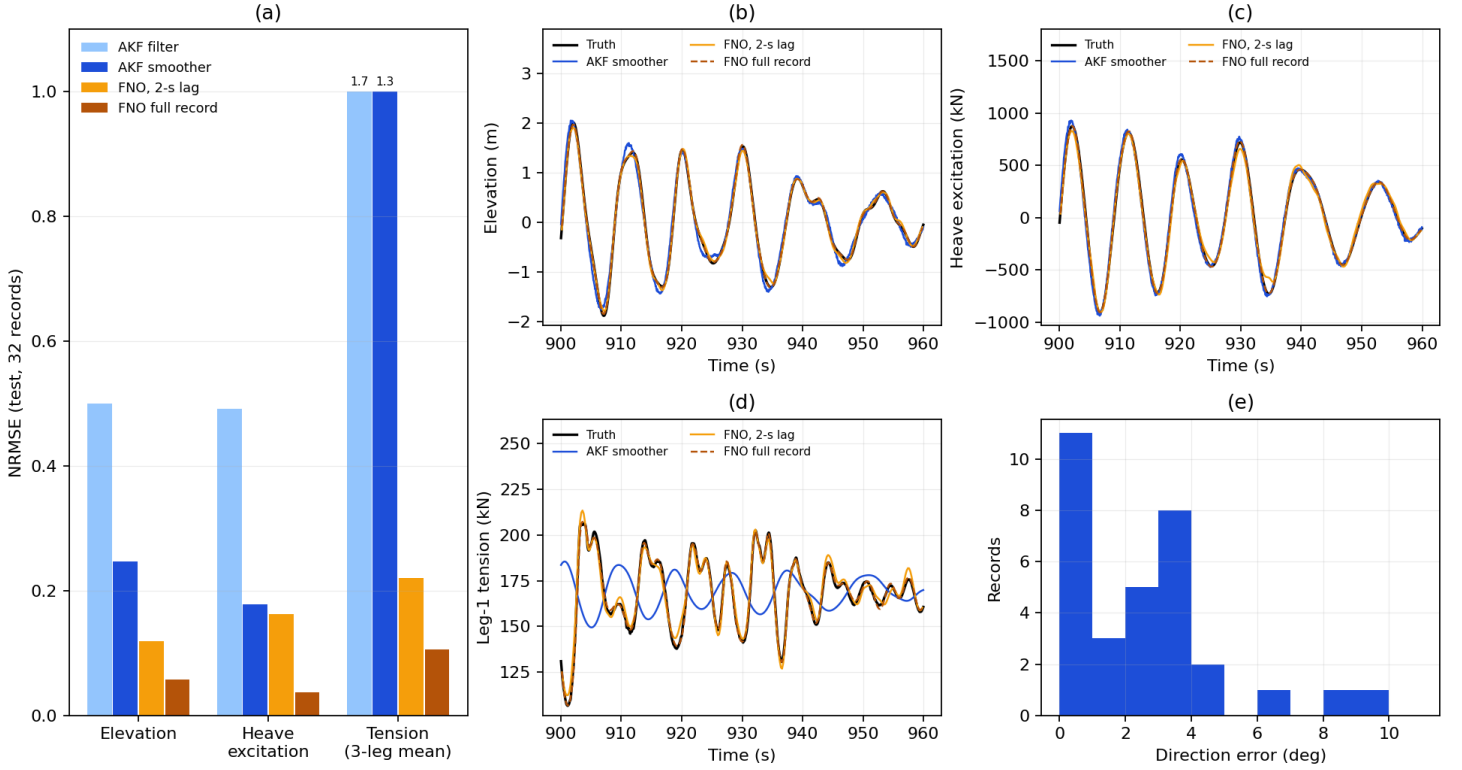


FIGURE 3: ESTIMATION FROM THE IMU AND PTO FORCE ON 32 TEST RECORDS: (A) NRMSE BY METHOD AND QUANTITY, THE TENSION BARS BEING THREE-LEG MEANS; (B–D) ELEVATION, HEAVE EXCITATION AND LEG-1 TENSION ON ONE TEST RECORD ($H_s = 3$ M, $T_p = 10$ S); (E) AKF WAVE-DIRECTION ERROR.

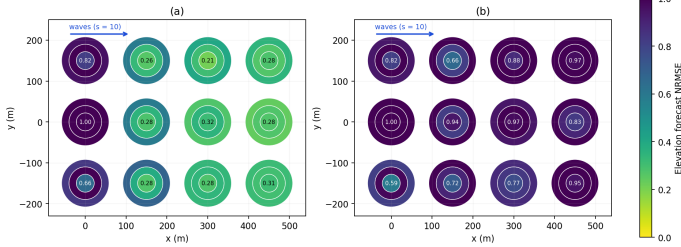


FIGURE 4: ELEVATION FORECAST NRMSE AT EVERY WEC OF THE 12-WEC FARM FROM THE 2-S-LAG FNO ELEVATIONS, $H_s = 2.5$ M, $T_p = 9$ S, HEAD-ON, THREE REALIZATIONS: RINGS ARE LEADS OF 1, 2 AND 3 T_p (INNER TO OUTER) AND THE NUMBER IS THE ERROR AT 1 T_p . (A) FARM FORECAST; (B) OWN-RECORD FORECAST.

and stays below 7% of the chain’s breaking load (Fig. 5d). The forced-heave map applied to the recorded heave explains why: the heave-caused tension has a standard deviation of 0.65 kN under passive damping and 1.2 kN under the MPC, against 30 kN for the leg-1 tension in both, because the subsurface buoys decouple the hull’s heave from the chain.

Figure 6 shows the force the controller commands at the back-row center WEC (Fig. 6a). At $T_p = 9$ s the float, with its 5.5-s natural period, is stiffness-dominated and its velocity leads the excitation; passive damping can only resist the motion, a straight line in Fig. 6d. The MPC force leads the damping force (Fig. 6b): part of it acts as a negative spring that cancels much of

the excess hydrostatic stiffness and brings the velocity into phase with the excitation. The PTO motors 30% of the time (Fig. 6c) and supplies 12% as much energy as it absorbs. Its force peaks at 0.95 MN against 0.6 MN passive, inside the rating, and the net electrical power rises from 51 to 95 kW. The motoring makes the farm’s instantaneous output peakier (coefficient of variation 0.59–0.68 against 0.37–0.44 passive, with brief net draws of up to 1 MW), but averaged over 10 or 60 s, where wave groups set the variability, the two controllers differ little. Delivering the 10-s average to the grid takes about 1 kWh of storage at farm level, the storage assumed in the doubled-PTO-cost case of Sec. 3.3.

3.3 Annual energy and cost

Figure 7a gives the passive power per WEC in each bin and Fig. 7b the head-on farm gain, which is positive in every bin. The front-corner device loses in one bin (mean $H_s = 4.4$ m, $T_p = 12$ s, 106 h), where the largest loss scale, chosen on the tuning realization, is unstable at that poorly forecast position (120% lost head-on, 151% at 30°); the farm still gains 41% there head-on, and the fit includes these runs. The same sensitivity appears without scheduling: a controller tuned at $T_p = 9$ s (fixed loss scale, 10-s horizon) loses up to 44% against passive damping for $T_p \geq 14$ s, where the horizon is shorter than one wave. The skill-to-gain model fits the runs with $R^2 = 0.78$ overall and 0.64 within bins, with $e_1 = 0.51$ and $f_{\min} = 0.44$. In the held-out check (Fig. 7c), the bin at mean $T_p = 8.0$ s (H_s 1–2 m) gives simulated gains of 42% at 60° and 64% at 90° against modeled 52 and 62%, and the bin at $T_p = 9.9$ s gives 70 and 85% against

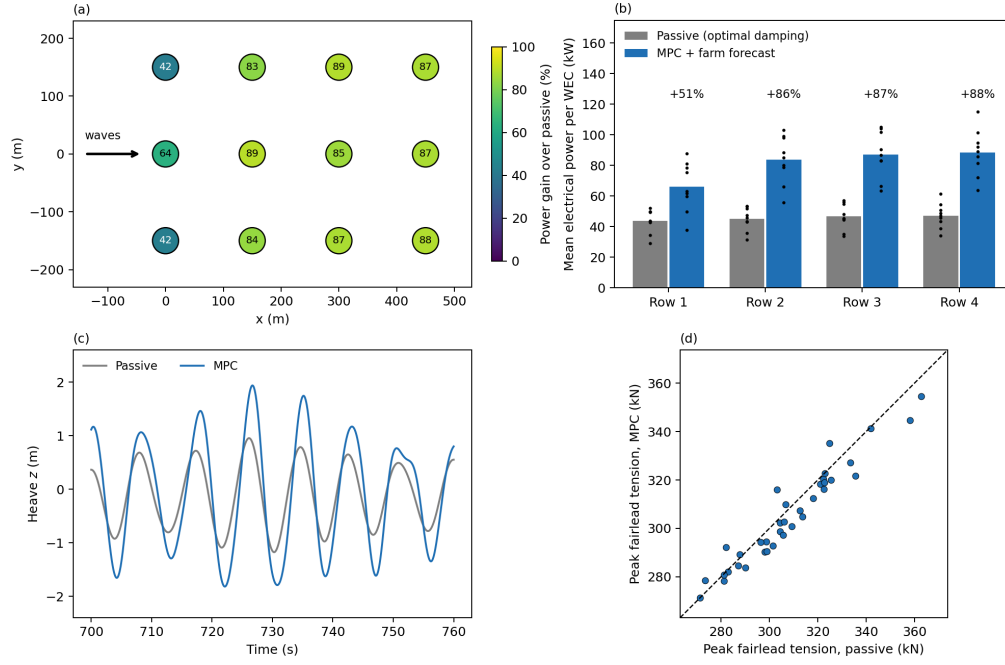


FIGURE 5: FARM POWER WITH PREVIEW CONTROL FROM THE FARM FORECAST ($H_s = 2.5$ M, $T_p = 9$ S, HEAD-ON, THREE REALIZATIONS): (A) ELECTRICAL POWER GAIN PER WEC OVER PASSIVE DAMPING; (B) ELECTRICAL POWER PER ROW, DOTS FOR INDIVIDUAL WECS AND REALIZATIONS; (C) HEAVE OF THE BACK-ROW CENTER WEC; (D) PEAK FAIRLEAD TENSION OF EVERY WEC AND REALIZATION, PASSIVE AGAINST MPC (BREAKING LOAD 5310 KN).

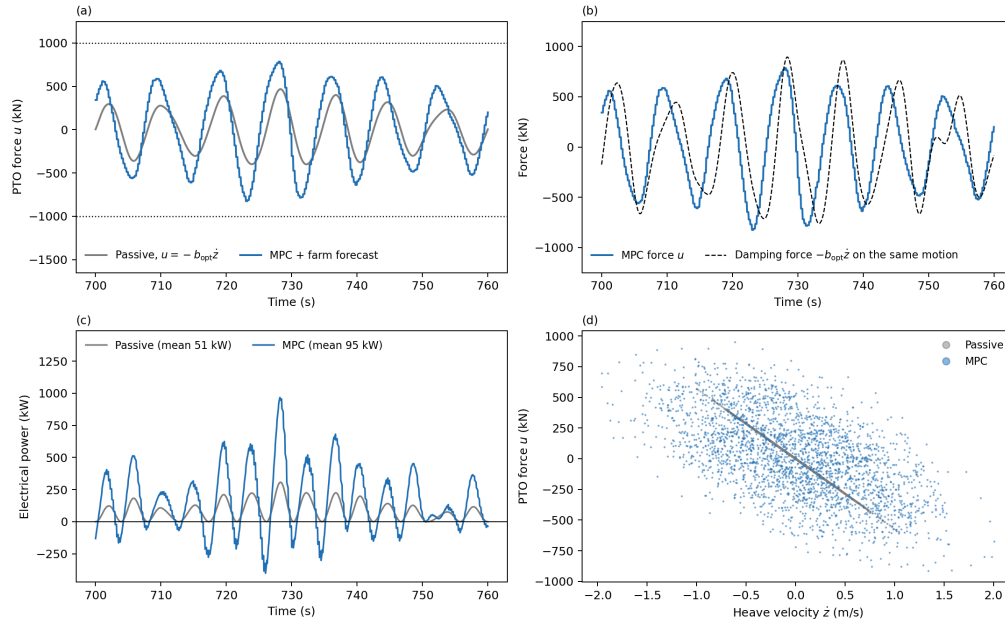


FIGURE 6: THE COMMANDED PTO FORCE AT THE BACK-ROW CENTER WEC ($H_s = 2.5$ M, $T_p = 9$ S; DOTTED LINES MARK THE 1-MN RATING): (A) FORCE, PASSIVE AGAINST MPC WITH THE FARM FORECAST; (B) MPC FORCE AGAINST THE DAMPING FORCE $-b_{opt} \dot{z}$ ON THE SAME MOTION; (C) ELECTRICAL POWER, NEGATIVE WHEN THE PTO MOTORS; (D) FORCE AGAINST HEAVE VELOCITY OVER THE WHOLE RUN.

70 and 81%.

Over the year the 12-WEC farm produces 4.73 GWh with passive damping and 7.65 GWh with the MPC and the farm forecast, 62% more; December to February carry nearly four times the energy of June to August, and the gain holds through

the year (Fig. 7e). Half the hours lie within 16° of the farm axis and 90% within 37° ; Fig. 7d shows the same distribution weighted by energy. With the waves always head-on the gain would be 65%; the farm gain falls to about 56% at $50-60^\circ$ and recovers to 61% side-on, where the farm is three deep and only

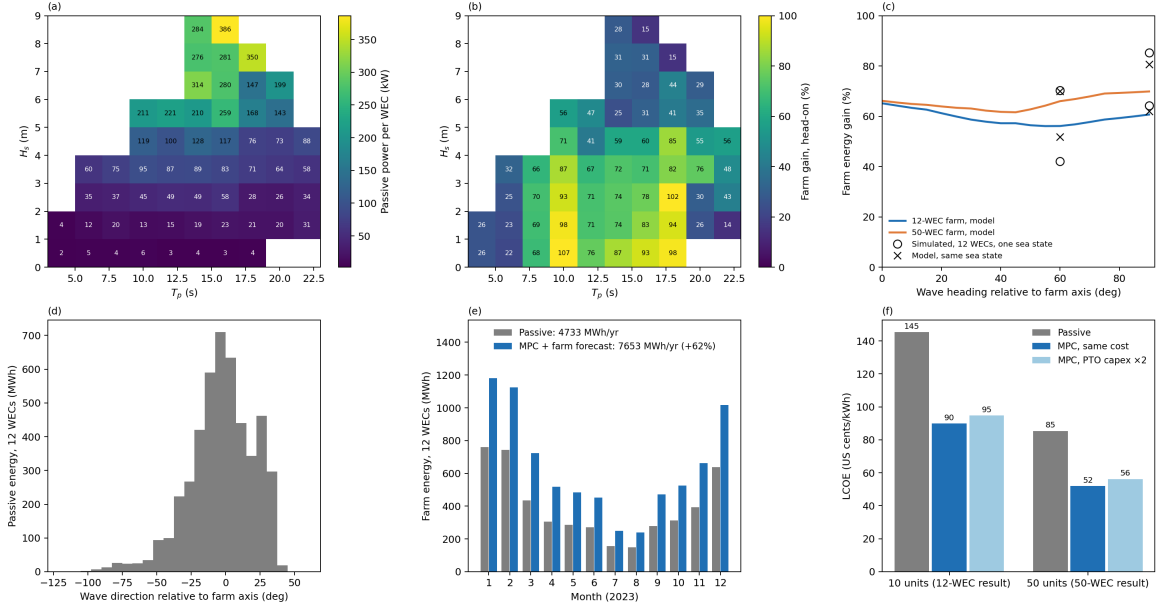


FIGURE 7: ANNUAL ENERGY AT HUMBOLDT BAY: (A) PASSIVE POWER PER WEC IN EACH SEA-STATE BIN; (B) SIMULATED 12-WEC FARM GAIN FOR HEAD-ON WAVES; (C) FARM GAIN AGAINST WAVE HEADING, LINES FROM THE SKILL-TO-GAIN MODEL WEIGHTED BY THE YEAR'S ENERGY, CIRCLES FROM DIRECT SIMULATION AND CROSSES FROM THE MODEL IN THE TWO HELD-OUT SEA STATES; (D) DISTRIBUTION OF THE MEASURED WAVE DIRECTIONS RELATIVE TO THE FARM AXIS (300° FROM-DIRECTION), WEIGHTED BY PASSIVE ENERGY; (E) MONTHLY ENERGY OF THE 12-WEC FARM; (F) LCOE AT 10 AND 50 UNITS.

four devices are exposed. The 50-WEC farm gains 64% in the model, because a smaller share of its devices sits on an exposed edge. The gain is largest in the 8–18-s periods that carry most of the energy and smallest in short or very long periods and in the largest seas, where the force rating binds. With costs unchanged, the energy gain lowers the LCOE of the RM3 cost model from 145 to 90 US cents/kWh at 10 units, which here stand for the 12-WEC farm, and from 85 to 52 at 50 units, reductions of 38 and 39%; with the PTO capital cost doubled to pay for four-quadrant operation and storage, the reductions are 35 and 34% (Fig. 7f).

3.4 Storm loads

In the storm the heave-caused tension has a standard deviation of 2.8 kN against 74 kN for the leg-1 tension, of which surge and pitch account for 63 kN (Fig. 8a). Because a heave PTO cannot reduce storm tension, we test whether the farm forecast can warn of it. The linear map from the six excitation components to the fairlead tensions fails on storm tension, whose peaks reach ten standard deviations (NRMSE 0.48–0.67 on the two held-out storms with the true excitation); the tension FNO reduces this to 0.27–0.38, also with the true excitation. Figure 8b–d shows the warning at the back-row center WEC in the record with the embedded crest, which is not among the eight storm records used to train and test the tension FNO. The upwave leg's tension is forecast with an error of 0.31 at zero lead and 0.44 at 27.5 s, and the two downwave legs with 0.50 at zero lead and 0.55 at 25 s. The forecast follows the tension peaks of the record but underestimates the largest, 830 against 1150 kN. The alarm warns of all six exceedance episodes in the record 20–27 s before they begin, a lead capped by the window. Of the 25 alarm episodes, 14 are false, all in one 40-s stretch late in the record where the forecast

maximum hovers between 490 and 660 kN.

4. DISCUSSION

Every energy figure above compares the MPC with the farm forecast against passive damping, so it bundles the value of reactive control with the value of the preview. The front row separates them at one sea state. Its farm forecast is no better than its own record, and it gains 51% against an average of 87% in the rows behind, so at $T_p = 9$ s nearly 60% of a well-forecast device's gain is available without the farm and the preview supplies the rest. The own-record test is a $T_p = 9$ s wind sea, while the site's energy extends to $T_p = 18$ s, where autoregressive forecasts of swell hold for longer [6], and the own-record alternative may therefore be stronger over the year than at this sea state. The floor $f_{\min}$ of the skill-to-gain model is reached at $e_1 = 0.51$, a forecast that still explains three quarters of the excitation variance, so the model does not measure the gain of a device with no forecast at all. A controller on each device's own-record forecast, and one with perfect preview, would fix the share; neither is run here. The scheduled controller is also sensitive to its tuning, as the unscheduled loss and the losing bin of Sec. 3.3 show.

The annual numbers replace four parts of the Sec. 3.2 simulation. The mooring is linearized, which the heave-caused tension of Sec. 3.2 supports. The FNO estimates are replaced by the 10%-noise stand-in, matched to the FNO at one sea state with passive motions where the FNO's error is 0.07–0.08, while the FNO is trained on $H_s \leq 4$ m and $T_p \leq 12$ s and most of the site's energy arrives at T_p of 8–18 s. The power matrix is parametric, with one realization per bin and no operating limit, so it can misstate the yield [24] and credits power at H_s of 8–9 m while $H_s = 10$ m is treated as a storm. The heading model, with

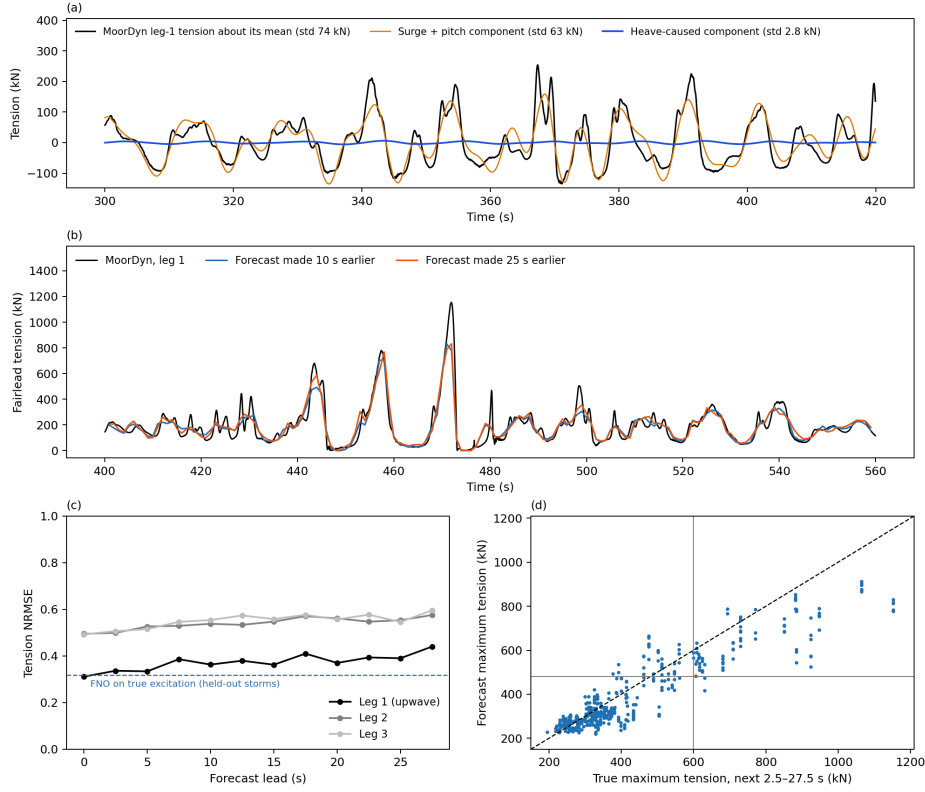


FIGURE 8: STORM MOORING LOADS, $H_s = 10$ M, $T_p = 14$ S: (A) LEG-1 TENSION ABOUT ITS MEAN IN THE FIG. 2 STORM (WAVES TOWARD 30°), ITS SURGE-PITCH COMPONENT FROM A REGRESSION ON SURGE, PITCH AND THEIR RATES, AND THE HEAVE-CAUSED COMPONENT FROM THE FORCED-HEAVE TEST; (B-D) THE WARNING IN THE HEAD-ON RECORD WITH AN EMBEDDED NEWWAVE CREST AT THE BACK-ROW CENTER WEC, FORECASTS FLOORED AT ZERO: (B) UPWAVE-LEG TENSION AND THE FORECASTS MADE 10 AND 25 S EARLIER; (C) ERROR AGAINST LEAD FOR THE THREE LEGS, THE DASHED LINE BEING THE TENSION FNO ON THE TRUE EXCITATION OF THE HELD-OUT STORMS; (D) FORECAST AGAINST TRUE MAXIMUM TENSION OVER THE NEXT 2.5–27.5 S AT EVERY DECISION SECOND, WITH THE 600-KN LIMIT AND THE 480-KN ALARM LEVEL.

one free value per bin, explains 64% of the within-bin variance and misses the held-out simulations by up to 10 points. The head-on bins either side of the farm-power sea state give 70% at $T_p = 8.2$ s and 93% at 9.9 s, about 81% interpolated to 9 s against the MoorDyn 78%, which brackets the substitutions together; a dedicated run of the annual pipeline at that sea state remains to be done. The LCOE reduction with unchanged costs is the energy ratio restated, $1 - 1/1.62$, and the doubled PTO cost is a placeholder for the four-quadrant drive and storage. The RM3 cost structure belongs to a two-body device about four times our hull’s mass and to the reference model’s published 2014 costs for a first-generation design (capacity factor 0.30, structure alone about 30 cents/kWh at 50 units), so only the relative reduction is a result of this study; for scale, our device yields 394 MWh per year with passive damping and 638 MWh with the controller, against the reference model’s 701 MWh.

For a real device, the PTO here reacts against a fixed reference, an idealization of a float on a bottom-fixed pile or deep spar, which can push and pull; a two-body device such as the RM3 float and spar loses part of the stroke to the reaction body. The controller uses the true heave state. The FNO estimates that feed the forecast come from passive runs, and the estimator has not seen the MPC’s larger motions. Both FNOs learn MoorDyn’s

tension, so in the field they would be trained on a pilot device with load cells, and the Sec. 3.1 estimate would give the other devices load monitoring; the controller carries no tension constraint. The AKF’s tension failure deserves a direct check of the quasi-static map on the true motions before it is attributed to the map alone. Waves are linear and WEC–WEC interaction is neglected, which is reasonable at 15 diameters and in operational seas but not in breaking storm crests, and MoorDyn’s snap-load peaks carry an uncertainty of up to tens of percent [41, 42]. The storm warning comes from one record at one device, with the forecast fed with the stand-in rather than onboard estimates, and the front row, with no upwave neighbors, would get the least warning. This device has no actuator with authority over surge- and pitch-driven loads within 25 s, so here the warning serves load monitoring; a device with a survival action such as submergence [20] could take it 20–27 s before the load arrives.

5. CONCLUSIONS

The IMU and PTO force that each WEC carries are enough to estimate the incident elevation and the fairlead tension of a dynamic mooring: a Fourier neural operator reaches NRMSE 0.12 and 0.22 at a 2-s lag, and an augmented Kalman filter recovers the wave direction. Fitted jointly across the farm, the estimated

elevations forecast the waves one peak period ahead at every device behind the front row with errors of 0.21–0.32, and to three periods in the rows further back, where a device’s own record has little skill even at one period.

Driven by this forecast, a model-predictive controller scheduled on the sea state raises the electrical energy of the 12-WEC farm by 78% over passive damping tuned to the same sea, with the peak mooring tension unchanged. With the estimator replaced by a noisy-truth stand-in and the mooring linearized, the gain over the Humboldt Bay year, with the measured wave directions, is 62%. The front row shows that nearly 60% of the gain at a well-forecast device is available from reactive control on a device’s own record; measuring the preview’s share against own-record and perfect-preview controllers, running the estimator inside the control loop, and validating the chain in a wave basin are the next steps.

The heave PTO has almost no authority over the storm tension of the RM3 mooring, which surge and pitch drive. Fed with noisy true elevations, the farm forecast with a learned tension model warns of every limit exceedance in a storm record 20–27 s ahead, enough for a device with a survival action; false alarms occur when the load hovers near the limit.

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