

Structural Analysis of a Formula Student Spaceframe Using a Suspension-Integrated Torsional Model, Equivalent-Static Loading, and Modal Characterization

Tausif Nawaz^{a,*}, Abrar Shahriar^a, Miraz Morshed^a, Habib Al Muntasir^a, Jahid Hossen Tonmoy^a

^a *Mechanical Engineering Department, Military Institute of Science and Technology,
Dhaka 1216, Bangladesh*

*** Corresponding Author**

Email: tausifnawaz08@gmail.com

Abstract

Selecting an appropriate torsional stiffness requirement is essential for balancing structural rigidity and mass efficiency in Formula Student spaceframe chassis design. This study presents a suspension-informed framework for establishing a vehicle-specific torsional stiffness target and evaluating the structural performance of an AISI 4130 steel spaceframe using finite element analysis (FEA). A simplified equivalent-compliance model was developed using prescribed front and rear suspension roll stiffnesses of 133.07 and 221.37 Nm/deg, respectively. A parametric assessment of the equivalent chassis–suspension system established a nominal torsional stiffness target of 1600 Nm/deg based on diminishing returns, corresponding to 95.06% relative stiffness. An ANSYS Mechanical 2024 R1 beam-element model was employed to investigate torsional response, equivalent-static frontal, rear, and lateral loading, and free–free modal characteristics. A six-level mesh-convergence assessment yielded a torsional stiffness of 1651 Nm/deg at the selected 1 mm element size, exceeding the target by 3.2% and achieving a relative stiffness of 95.21%, with less than 1% variation upon further refinement. Under calculated frontal and rear loading conditions, maximum deformations of 21.906 and 8.191 mm were obtained, with reported minimum nominal factors of safety of 1.299 and 1.987, respectively. The lateral assessment produced a maximum deformation of 5.575 mm and a minimum nominal factor of safety of 1.837. Free–free modal analysis identified a first flexible natural frequency of 33.271 Hz. The results demonstrate the applicability of suspension-informed stiffness selection integrated with numerical structural assessment, while emphasizing the need for experimental validation and careful interpretation of equivalent-static loading assumptions.

Keywords: Formula Student Spaceframe; Torsional Stiffness; Finite Element Analysis; Mesh Convergence; Modal Analysis

1. Introduction

Formula Student is an international engineering competition in which university teams design, manufacture, and evaluate single-seat racing vehicles under specified technical and safety requirements. The chassis serves as the primary structural framework, supporting the suspension, powertrain, steering system, and driver-protection structures. Its design must accommodate diverse loading conditions while maintaining adequate stiffness, structural integrity, and low mass. Tubular steel spaceframes remain widely adopted because of their manufacturing accessibility, structural adaptability, and ability to distribute loads through interconnected members. However, effective chassis development requires quantitative evaluation of torsional rigidity, deformation, load transmission, and vibration characteristics beyond compliance with geometric and material requirements [1, 2].

Torsional stiffness is particularly significant because it characterizes chassis resistance to relative angular displacement between the front and rear suspension mounting regions. During cornering and asymmetric wheel loading, chassis deformation introduces additional compliance that may influence suspension response and wheel-load distribution. Although vehicle dynamics calculations commonly assume a rigid chassis, this assumption becomes less accurate when structural compliance is appreciable relative to suspension compliance. Riley and George [1] investigated this relationship using an analytical spring representation supported by finite element modelling and physical testing, establishing the importance of relating chassis rigidity to suspension characteristics. Using a coupled chassis–suspension finite element model of a Winston Cup race car, Thompson et al. [3] reported a 7.3% increase in effective front suspension roll stiffness when chassis torsional stiffness was increased by 130% relative to their baseline; further

stiffening produced little additional change. Deakin et al. [4] examined chassis rigidity through quasi-static calculations and dynamic vehicle simulations and showed that simplified stiffness-ratio relationships can be affected by suspension kinematics and the distribution of chassis flexibility.

Increasing torsional stiffness generally reduces angular deformation under a specified moment; however, excessive reinforcement may increase structural mass and manufacturing complexity without proportional improvements in vehicle performance. Velie [5] investigated chassis rigidity requirements through steady-state and transient vehicle dynamics simulations, relating stiffness selection to handling-related performance. Similarly, Bortoluzzi et al. [6] employed analytical modelling, finite element analysis (FEA), and experimental testing to examine chassis elasticity and its influence on suspension behavior. These investigations demonstrated that stiffness requirements should reflect vehicle-specific suspension characteristics rather than arbitrary structural targets. da Costa and Vilela [7] calculated the torsional stiffness of a Formula SAE frame using beam-element FEA and compared its effect on the distribution of lateral load transfer with suspension roll-stiffness distribution. Their assessment considered the compatibility of chassis and suspension stiffness rather than establishing a universal minimum torsional stiffness.

Further studies have addressed the relationship between chassis stiffness, structural configuration, and mass optimization. Jain [8] investigated computational approaches for improving Formula SAE chassis torsional rigidity through FEA, incorporating static and modal assessments. Ferreira et al. [9] examined stiffness-based structural optimization, including a tubular spaceframe benchmark used in evaluating composite chassis alternatives. Wheatley et al. [10] employed ANSYS and SolidWorks to investigate chassis modifications, reporting approximately 10% greater torsional stiffness with a 3.5% reduction in structural mass. Collectively, these studies emphasize the importance of balancing rigidity with structural efficiency rather than maximizing stiffness independently.

The numerical determination of torsional stiffness is influenced by structural representation, chassis triangulation, boundary conditions, and displacement measurement locations. Riley and George [1] examined alternative torsional loading configurations and their influence on calculated structural response. Krzikalla et al. [11] investigated Formula Student chassis torsional stiffness through numerical modelling and experimental testing, emphasizing the importance of consistent loading and restraint conditions. Gala et al. [12] combined an equivalent-stiffness model with numerical and experimental investigations and incorporated a mesh-convergence study to assess discretization sensitivity. Their findings highlight the importance of numerical verification and experimental correlation in obtaining reliable stiffness predictions. In a separate investigation, Krzikalla et al. [13] examined geometric and material simplifications in a beam-like frame model and reported simulation–test differences of up to 10% for their investigated configuration. Capata et al. [14] compared numerical and experimental torsional stiffness for Formula Student chassis and reduced the simulation–test discrepancy from 9.5% to 3.6% by better representing the physical test configuration.

Additional investigations have examined torsional testing procedures and structural configurations. Eakambaram et al. [15] employed finite element modelling to investigate tubular spaceframe torsional behavior and proposed a testing arrangement intended to support experimental verification. Their approach considered suspension attachment conditions and the influence of suspension compliance on measured deformation. Wolska et al. [16] compared three Formula Student spaceframe configurations using ABAQUS and investigated differences associated with structural geometry and stiffness calculation procedures. These studies reinforce the need for clearly defined loading conditions, numerical discretization, and measurement methodologies when evaluating or comparing chassis torsional stiffness.

Beyond torsional rigidity, Formula Student chassis structures must resist longitudinal and transverse loading. Frontal, rear, and lateral assessments are particularly relevant for identifying deformation patterns, critical members, and structural load paths. Wongchai [17] investigated frontal and lateral loading of Formula-type spaceframes using finite element models, demonstrating the influence of member arrangement on predicted stress and deformation. Rao et al. [18] similarly evaluated a Formula Student chassis under frontal, lateral, and torsional loading conditions, highlighting the importance of structural triangulation and efficient force transmission. More recently, Ali et al. [19] compared triangulated spaceframe configurations under multiple static loading scenarios, including frontal loading, lateral shear, and torsion. Their investigation demonstrated how structural arrangement influences calculated stiffness and stress distributions. Nevertheless, equivalent-static simulations cannot reproduce the transient contact forces, plastic deformation, energy absorption, or progressive failure associated with actual collisions. Their results must therefore be interpreted within the prescribed loading and material assumptions. Siegler et al. [20] used nonlinear LS-DYNA simulations to investigate frontal and lateral crashworthiness of a composite racing-car chassis. Saplinova et al. [21] reported numerical head-on and lateral impact assessments alongside torsional and structural analyses of a Formula Student tubular frame. Such collision-oriented analyses address different physical phenomena from a linear-elastic equivalent-static load case.

Dynamic characteristics provide another important dimension of chassis performance. During operation, the structure experiences time-dependent excitation from the powertrain, road irregularities, and suspension interactions. Modal analysis identifies natural frequencies and mode shapes, providing insight into structural vibration behavior and potential interactions with operating excitation frequencies. Kakria and Singh [2] integrated suspension modelling with static and modal finite element assessments of a Formula SAE structure, while Jain [8] incorporated modal analysis into chassis optimization. Rodrigues et al. [22] investigated the static and dynamic characteristics of a tubular sports-car chassis, reporting a torsional stiffness of 1639 Nm/deg and initial natural frequencies of 34.4 and 41.5 Hz. These studies support integrating static structural and modal analyses to evaluate complementary performance characteristics. However, identifying natural frequencies alone cannot establish resonance-free operation, as the response also depends on excitation harmonics, damping, and modal participation. Zainal et al. [23] identified natural frequencies and mode shapes of a racing-car chassis under approximately free-free conditions using experimental modal analysis and operational modal analysis, and compared the modal estimates obtained by different identification methods.

Previous investigations have established methods for suspension-informed stiffness selection, torsional testing, equivalent-static structural assessment, and modal characterization [1–4, 7–16, 20–23]. Building on these approaches, the present study addresses how a vehicle-specific torsional stiffness target can be selected from suspension roll stiffnesses and evaluated alongside the structural and modal responses of a Formula Student spaceframe. Particular attention is given to diminishing gains in relative stiffness, mesh sensitivity, and the influence of loading arrangements and suspension compliance on the interpretation of calculated torsional stiffness. The contribution is an integrated numerical case study linking target selection with structural assessment under explicitly stated modelling assumptions.

The present study investigates a normalized AISI 4130 steel Formula Student spaceframe using a suspension-informed analytical model and complementary finite element assessments. The primary objective is to establish a nominal chassis torsional stiffness requirement from the calculated front and rear suspension roll stiffnesses using a simplified equivalent-compliance formulation. A parametric assessment of the equivalent chassis-suspension system was conducted to establish a practical torsional stiffness target based on diminishing gains in relative stiffness, balancing chassis rigidity against the limited incremental benefits of further stiffening. An ANSYS-based beam-element model is subsequently employed to evaluate the chassis's torsional response, supported by a mesh-convergence study to assess numerical discretization sensitivity. Equivalent-static frontal, rear, and lateral analyses investigate deformation, combined beam stress, and nominal strength margins, while free-free modal analysis characterizes the natural vibration frequencies. The contribution of this study is a vehicle-specific assessment linking suspension-informed torsional stiffness target selection with a numerical model that retains suspension spring compliance. The comparison of front-loaded and rear-loaded configurations examines how loading and restraint arrangements influence effective torsional stiffness and local stress demand, while equivalent-static and modal analyses extend the assessment to complementary aspects of structural performance.

2. Methodology

2.1. Chassis geometry and properties

The study focused on a single chassis configuration constructed from normalized AISI 4130 chromium-molybdenum steel. The investigated structure comprises the front bulkhead, front and main roll hoops, roll-hoop bracing, side-impact members, and rear frame members interconnected to provide load paths between the primary structural regions and suspension mounting points. The chassis configuration was developed to balance structural rigidity, mass, driver accommodation, and manufacturability. Circular hollow steel sections of varying dimensions were assigned to the structural members according to their geometric and load-bearing requirements by the Formula SAE regulations [24]. A nominal yield strength of 435 MPa, determined through tensile testing of the actual chassis tubing using procedures referenced to ASTM A370-22 [25] and consistent with the manufacturer's specifications, was adopted for the structural safety-factor calculations. The geometric specifications, material properties, and key parameters adopted for the finite element analysis of the Formula Student spaceframe are summarized in Table 1.

Table 1. Geometric and material parameters used in the structural model

Parameter	Value
Chassis configuration	Tubular spaceframe
Structural material	Normalized AISI 4130 steel
Overall chassis length	2590 mm
Chassis mass	34.92 kg
Car mass with driver	260 kg
Nominal tube outside diameter	25.4 mm
Tube wall-thickness range	1.2–2.4 mm
Young's modulus	205 GPa
Poisson's ratio	0.29
Density	7850 kg/m ³
Tested yield strength	435 MPa
FEA software	ANSYS Mechanical 2024 R1
Car track	1200 mm

2.2 Equivalent suspension stiffness

A simplified equivalent-compliance model was employed to establish a vehicle-specific nominal torsional stiffness target. The front and rear suspension roll stiffnesses were obtained from the vehicle's suspension design calculations and adopted as fixed inputs to the simplified equivalent-compliance model. The corresponding values were 133.07 and 221.37 Nm/deg, respectively. In the adopted simplified series-compliance model, the equivalent suspension stiffness is expressed as

$$K_s = \frac{K_f \times K_r}{K_f + K_r} \dots \dots \dots (1)$$

where K_f and K_r are the front and rear suspension roll stiffnesses, respectively.

The chassis was represented as an additional torsional spring with stiffness K_c . The effective stiffness of the equivalent system was then calculated as

$$K_e = \frac{K_s \times K_c}{K_s + K_c} \dots \dots \dots (2)$$

The relative stiffness was defined as the ratio of the effective system stiffness to the equivalent suspension stiffness under the idealized rigid-chassis condition:

$$R = \frac{K_e}{K_s} \times 100\% \dots \dots \dots (3)$$

$$\text{Equivalently, } R = \frac{K_c}{K_s + K_c} \times 100\% \dots \dots \dots (4)$$

This formulation provides a measure of the contribution of chassis compliance to the effective stiffness response. To investigate the sensitivity of relative stiffness to chassis rigidity, a parametric calculation was performed for chassis torsional stiffness values between 600 and 3600 Nm/deg (Table 2).

Table 2. Parametric variation of chassis and relative stiffness

Chassis stiffness, K_c (Nm/deg)	Effective stiffness, K_e (Nm/deg)	Relative stiffness, R (%)	Relative increase in R (%)
600	72.999	87.83	-
1100	77.272	92.98	5.86
1600	79.007	95.06	2.24
2100	79.947	96.19	1.19
2600	80.536	96.90	0.74
3100	80.941	97.39	0.51
3600	81.235	97.74	0.36

As chassis stiffness increased, the relative stiffness approached its theoretical maximum, exhibiting progressively diminishing gains. At a chassis torsional stiffness of 1600 Nm/deg, the system achieved approximately 95.06% relative stiffness. Further increasing chassis stiffness to 2100 Nm/deg improved relative stiffness to only 96.19%, indicating a comparatively small incremental benefit. Consequently, 1600 Nm/deg was selected as a practical design target, balancing torsional rigidity against the diminishing benefits of further stiffness increases.

2.3. Finite element model development

The chassis was modelled and analyzed using beam-element modelling in ANSYS Mechanical 2024 R1. A three-dimensional line-body representation of the tubular frame was adopted, with the structural members idealized as interconnected beam elements. Riley and George [1] discussed finite element modeling strategies for Formula SAE chassis, while da Costa and Vilela [7] used beam elements to evaluate torsional stiffness against suspension characteristics. Krzikalla et al. [13] investigated the influence of beam-like model simplifications on experimentally measured torsional stiffness; Gala et al. [12] also examined discretization sensitivity in a numerical chassis model. The ANSYS BEAM188 formulation provides a three-dimensional, two-node beam representation based on Timoshenko beam theory, including transverse shear deformation. The standard formulation incorporates three translational and three rotational degrees of freedom at each node [26].

In the present model, circular hollow cross-sections and the specified material properties were assigned to the line bodies. Structural continuity was represented through the connections between intersecting members. The model therefore captures the global elastic behavior of the idealized frame without explicitly resolving the three-dimensional geometry of individual welded joints. Concentrated masses were introduced to represent major vehicle subsystems, including the driver, engine, pedal box, rear axle, and steering rack (Figure 1). The masses were positioned at their designated subsystem centers of gravity and incorporated into the mass representation of the chassis model. The numerical framework comprised static structural analyses for torsional stiffness and prescribed longitudinal and lateral loading, together with a free-free modal analysis for evaluating the natural vibration characteristics of the chassis.

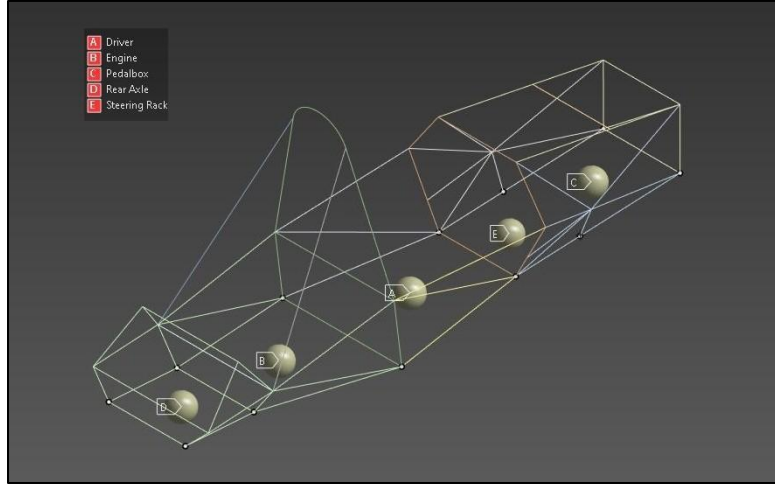


Figure 1. Distribution of major component masses within the tubular spaceframe chassis.

2.4. Mesh optimization and convergence assessment

2.4.1. Mesh-refinement approach

A mesh-refinement assessment was defined for the present beam-element model, using torsional stiffness as the primary convergence quantity. The assessment employed multiple mesh resolutions ranging from coarse to fine. A six-level mesh-refinement study was conducted using nominal element sizes of 30, 20, 10, 5, 1, and 0.50 mm. The chassis geometry, material properties, loading conditions, and boundary conditions were maintained consistently across all mesh configurations to isolate the influence of mesh density on the calculated torsional stiffness.

2.4.2. Convergence criterion

Mesh convergence was evaluated by progressively refining the finite element mesh and calculating the percentage change in torsional stiffness between successive mesh densities (Figure 2). A change of less than 1% between successive refinements is proposed as the preliminary acceptance criterion for global torsional stiffness. Gala et al. [12] reported changes below approximately 0.1% following further refinement of their selected mesh. The general distinction between numerical verification and experimental validation is consistent with the ASME V&V 10 framework [27]; the less-than-1% convergence threshold adopted here remains a study-specific criterion.

Table 3. Mesh-refinement results for the investigated chassis

Nominal element size (mm)	Number of elements	Torsional stiffness (Nm/deg)	Change (%)
30	1343	1,899	-
20	1982	1,771	6.74
10	3922	1,702	3.90
5	7805	1,667	2.06
1	38810	1,651	0.96
0.5	77582	1,638	0.79

Therefore, the 1 mm mesh was selected for the final analysis, as it satisfied the adopted convergence criterion of less than 1% change in torsional stiffness and provided a suitable balance between numerical accuracy and computational cost (Table 3). Moreover, the corresponding torsional stiffness of 1651 Nm/deg is above the nominal design target.

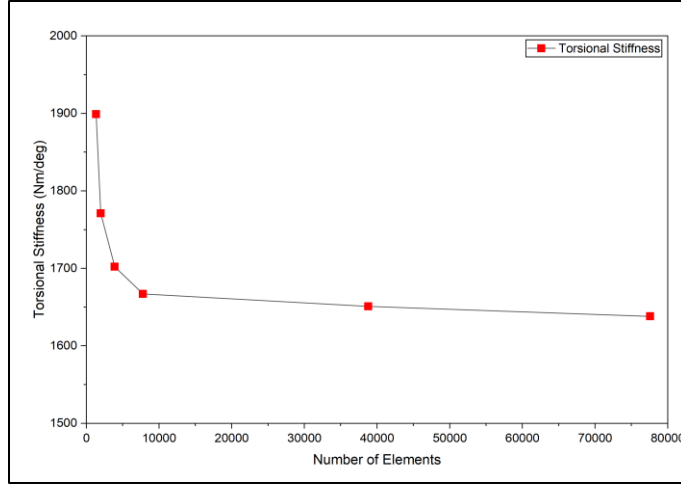


Figure 2. Mesh convergence study showing the variation of calculated chassis torsional stiffness with increasing finite element mesh density.

2.5. Torsional stiffness analysis

The torsional response of the chassis was investigated using static structural FEA under specified boundary conditions (Figure 3). The objective was to determine the resistance of the spaceframe to rotation about its longitudinal axis and subsequently compare the calculated stiffness with the suspension-informed analytical requirement. The influence of how suspension interfaces and test fixtures are represented when estimating torsional stiffness is illustrated by the numerical–experimental investigations of Krzikalla et al. [13] and Capata et al. [14]. Fixture design and displacement measurement for physical chassis torsion testing have also been described by Thompson et al. [28].

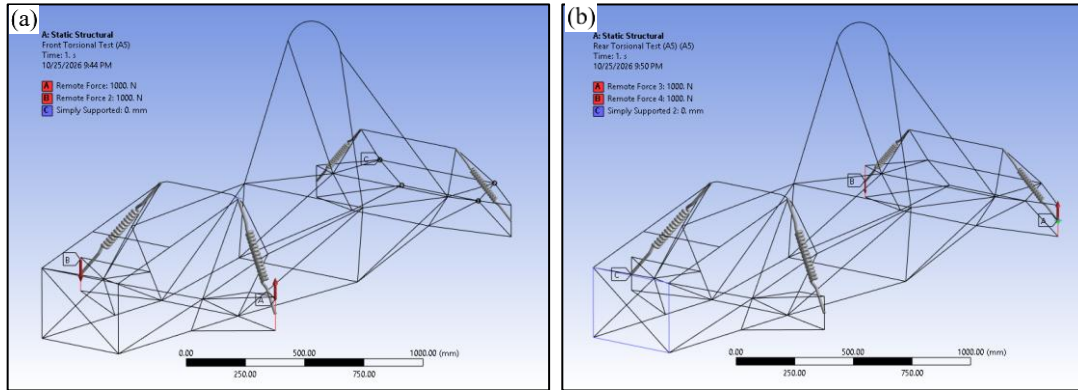


Figure 3. Boundary conditions for (a) Front torsional loading, and (b) Rear torsional loading.

For equal and opposite vertical forces of 1000 N separated by a transverse distance (b), the applied torque is:

$$T = F \times b \dots \dots \dots (5)$$

where F is the magnitude of one of the applied forces and b is the horizontal distance between the two suspension points.

The angular deformation may be calculated from the relative vertical displacement of the loading points:

$$\theta = \tan^{-1} \left(\frac{\delta_L - \delta_R}{b} \right) \dots \dots \dots (6)$$

where δ_L and δ_R represent the vertical displacements of the left and right loading points, respectively.

The resulting torsional stiffness is

$$K_t = \frac{T}{\theta} \dots \dots \dots (7)$$

The angular deformation is expressed in degrees to obtain torsional stiffness in Nm/deg.

Separate front and rear loading configurations are used to determine stiffness under different restraint conditions. For these assessments, the corresponding applied torque and differential displacement were calculated using consistent measurement locations.

The torsional stiffness obtained from the finite element analysis was compared with the nominal analytical target of 1600 Nm/deg, established in section 2.2.

The torsional loading magnitude was selected based on an extreme hypothetical cornering condition of 2.5g lateral acceleration. The car has a mass of 260 kg, including the driver, a center-of-gravity height of 0.30 m, and a track width of 1.20 m.

$$\text{Lateral load transfer moment, } M_{LT} = ma_y h_{cg} \dots \dots \dots (8)$$

$$\text{Total lateral load transfer, } F_z = \frac{M_{LT}}{t} \dots \dots \dots (9)$$

where, t is the track width.

$$\text{Front load-transfer fractions, } x_f = \frac{k_f}{k_f + k_r} \dots \dots \dots (10)$$

$$\text{Rear load-transfer fractions, } x_r = \frac{k_r}{k_f + k_r} \dots \dots \dots (11)$$

$$\text{Equivalent vertical load transfer on front upright, } F_{z,f} = x_f F_z \dots \dots \dots (12)$$

$$\text{Equivalent vertical load transfer on rear upright, } F_{z,r} = x_r F_z \dots \dots \dots (13)$$

Substituting values in equations (8-13), $F_{z,f} = 598.5 \text{ N}$ and $F_{z,r} = 995.7 \text{ N}$

Based on the estimated rear-axle loading, equal and opposite vertical forces of 1000 N were selected as a nominal severe-loading reference for both front-loaded and rear-loaded torsional assessments, ensuring consistent loading conditions.

2.6. Equivalent-static frontal loading

The frontal structural response was evaluated using an equivalent-static load derived from a prescribed nominal deceleration condition. A nominal deceleration of 20g was adopted in accordance with F.8.8.2 of Formula SAE regulations [24].

For an effective participating mass M_{eff} , the corresponding equivalent inertial force is

$$F_{front} = M_{eff} \times a \dots \dots \dots (14)$$

where F_{front} represents the applied frontal force (N), M_{eff} is the effective participating mass (kg), and a is the impact deceleration (m/s^2).

According to equation (14), a 260 kg vehicle undergoing an average deceleration of 20g corresponds to an inertial force of 51 kN. The equivalent-static load was distributed equally across four designated vertices of the front bulkhead (12.75 kN per vertex), while the four rearmost chassis vertices were assigned simply supported boundary conditions to restrain the structure during the analysis. The resulting total deformation and combined beam stress distributions were examined to identify the displacement response and critical structural members under the imposed loading condition. The analysis used linear-elastic material behavior and therefore represented the structural response to an equivalent-static force rather than a transient collision event. For comparison, Siegler et al. [20] used nonlinear impact simulations to examine energy absorption and structural changes under crash loading, phenomena outside the scope of the present linear-elastic assessment.

2.7. Equivalent-static rear loading

A corresponding rear structural analysis was conducted using the same nominal deceleration magnitude of 20g. The equivalent-static load was distributed equally across four designated vertices of the rearmost part of the chassis (12.75 kN per vertex), while the four vertices of the front bulkhead were assigned simply supported boundary conditions to restrain the structure. Maximum total deformation, maximum combined beam stress, and nominal factor of safety were evaluated.

2.8. Equivalent-static lateral loading

A transverse force of 7 kN was adopted as the baseline lateral loading condition, derived from historical Formula SAE structural requirements. The load was selected with reference to Section AF4.3 of the 2016 Formula SAE regulations [29]. In the finite element model, the lateral impact load was applied as a remote force distributed over four designated edges in the side-impact region, while four corresponding edges on the opposite side of the chassis were assigned simply supported boundary conditions. The analysis was intended to determine the elastic structural response under a reproducible lateral loading condition. Its outputs included maximum total deformation, maximum combined beam stress, and nominal yield-based factor of safety.

2.9. Factor-of-safety assessment

The equivalent-static structural analyses were evaluated using maximum total deformation, maximum combined beam stress, and a nominal yield-based factor of safety. The ANSYS Beam Tool provides linearized beam-stress quantities associated with axial and bending contributions. The displayed Maximum Combined Stress result was therefore treated as a beam-based stress measure rather than automatically identified as von Mises equivalent stress [30].

The nominal factor of safety was determined using the assumed material yield strength:

$$FOS = \frac{\sigma_y}{\sigma_{ref}} \dots \dots \dots (15)$$

where σ_y is the adopted yield strength and σ_{ref} is the selected beam-stress quantity.

The available ANSYS post-processing setup uses the expression

$$FOS = \frac{\sigma_y}{|s_{max}|} \dots \dots \dots (16)$$

where s_{max} represents the stress quantity used in the user-defined expression in Ansys Mechanical, expressed in megapascals (MPa).

This formulation was used to assess the nominal elastic strength margin under each loading condition. The calculated factor of safety was interpreted in relation to the definition of the selected stress result. Accordingly, an FOS greater than unity indicates a positive margin relative to the selected nominal stress criterion.

2.10. Free-free modal analysis

Modal analysis was performed to evaluate the natural frequencies and mode shapes of the spaceframe chassis. The numerical analysis employed the stiffness and mass distributions of the finite element model, including the concentrated masses representing major vehicle components. For an undamped linear structure, the natural frequencies are obtained from the generalized eigenvalue problem:

$$(K - \omega_n^2 M) \varphi_n = 0 \dots \dots \dots (17)$$

where K is the global stiffness matrix, M is the global mass matrix, ω_n is the natural angular frequency, and φ_n is the corresponding mode shape. The natural frequency is calculated as:

$$f_n = \frac{\omega_n}{2\pi} \dots \dots \dots (18)$$

Free-free boundary conditions were used to examine the unconstrained vibration characteristics of the numerical chassis model. For an unconstrained three-dimensional structure, the first six eigenmodes correspond to rigid-body translations and rotations and are expected to have approximately zero natural frequency. The subsequent modes represent elastic structural deformation. The extracted flexible frequencies and corresponding mode shapes were used to identify the primary vibration characteristics of the chassis. The first flexible frequency was also considered in relation to the engine excitation range. However, a comparison of individual natural frequencies with engine rotational frequency alone is insufficient to establish the absence of resonance. A complete vibration assessment would require consideration of excitation orders, forcing amplitudes, damping, modal participation, and the dynamic behavior of the assembled vehicle.

3. Results and discussion

3.1. Suspension-integrated torsional stiffness requirement

The calculated roll stiffnesses of 133.07 Nm/deg (front) and 221.37 Nm/deg (rear) yielded an equivalent stiffness of 83.11 Nm/deg under the adopted simplified series-compliance representation. Based on the diminishing-returns trend, a chassis torsional stiffness of 1600 Nm/deg was selected as the nominal design target, corresponding to 95.06% relative stiffness (Figure 4). Increasing the chassis stiffness to 2100 Nm/deg improved relative stiffness to only 96.19%, representing an incremental gain of 1.13 percentage points despite a 31.25% increase in chassis stiffness. This threshold is a stated engineering design choice, not a universally validated vehicle-handling optimum. The calculation treats the front and rear suspension compliances through a simplified equivalent representation and does not resolve tire, linkage, or handling dynamics. Deakin et al. [4] showed that simplified stiffness-ratio criteria can depart from dynamic vehicle predictions because of suspension kinematics and chassis deformation distribution. Likewise, da Costa and Vilela [7] compared a suspension roll-stiffness distribution with lateral load-transfer distribution; their reported performance metric is not identical to the relative-stiffness ratio defined in this study.

As shown in Table 2, increasing the assumed chassis stiffness from 600 to 1600 Nm/deg raises the relative stiffness from 87.83% to 95.06%, whereas increasing it further to 3600 Nm/deg raises relative stiffness only to 97.74%. Consequently, the 1600 Nm/deg target should be interpreted as a nominal design threshold within the present idealized compliance model, not as a value experimentally verified for the complete vehicle.

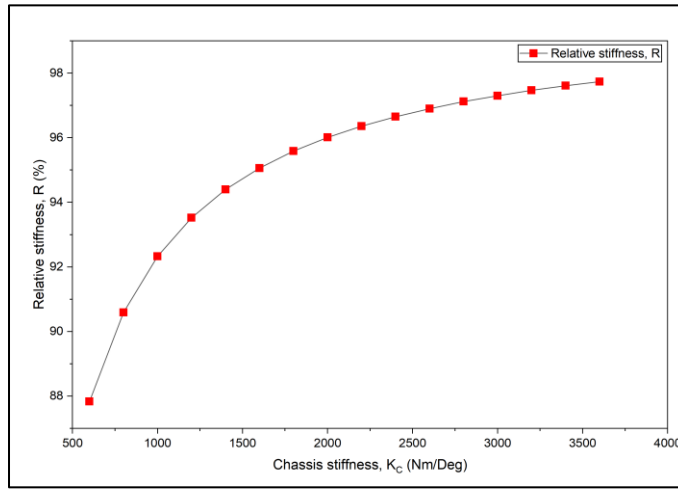


Figure 4. Relative stiffness vs. chassis torsional stiffness curve

3.2. Torsional response

The torsional response of the Formula Student spaceframe was investigated under two loading configurations to assess the influence of loading direction, boundary conditions, and suspension compliance on structural performance. In the front-loaded configuration, the rear end was restrained while equal and opposite vertical forces of 1000 N were applied at the front uprights. The loading and restraint locations were reversed for the rear-loaded configuration. Both analyses incorporated linear spring elements with axial stiffnesses of 40,000 N/m at the front and 60,000 N/m at the rear. The applied force couple generated a nominal torsional moment of 1200 Nm, based on a lateral separation of 1.20 m between the loading points.

Similar suspension-interface loading approaches have been employed in previous Formula Student chassis investigations by Riley and George [1], Krzikalla et al. [11], and Wolska et al. [16]. However, the present model retains finite suspension spring compliance, allowing the effective torsional response of the modelled chassis–suspension

assembly to be examined. Capata et al. [14] further demonstrated the importance of matching numerical loading arrangements with physical torsional-test conditions when validating predicted stiffness.

3.2.1. Front-loaded torsional response

Figure 5 presents the deformation and stress distributions obtained from the front-loaded configuration. The maximum total deformation was 49.550 mm, with the largest displacement occurring in the loaded front region. The selected upright deformation probe recorded a signed vertical displacement of +7.4467 mm, as shown in Figure 5d.

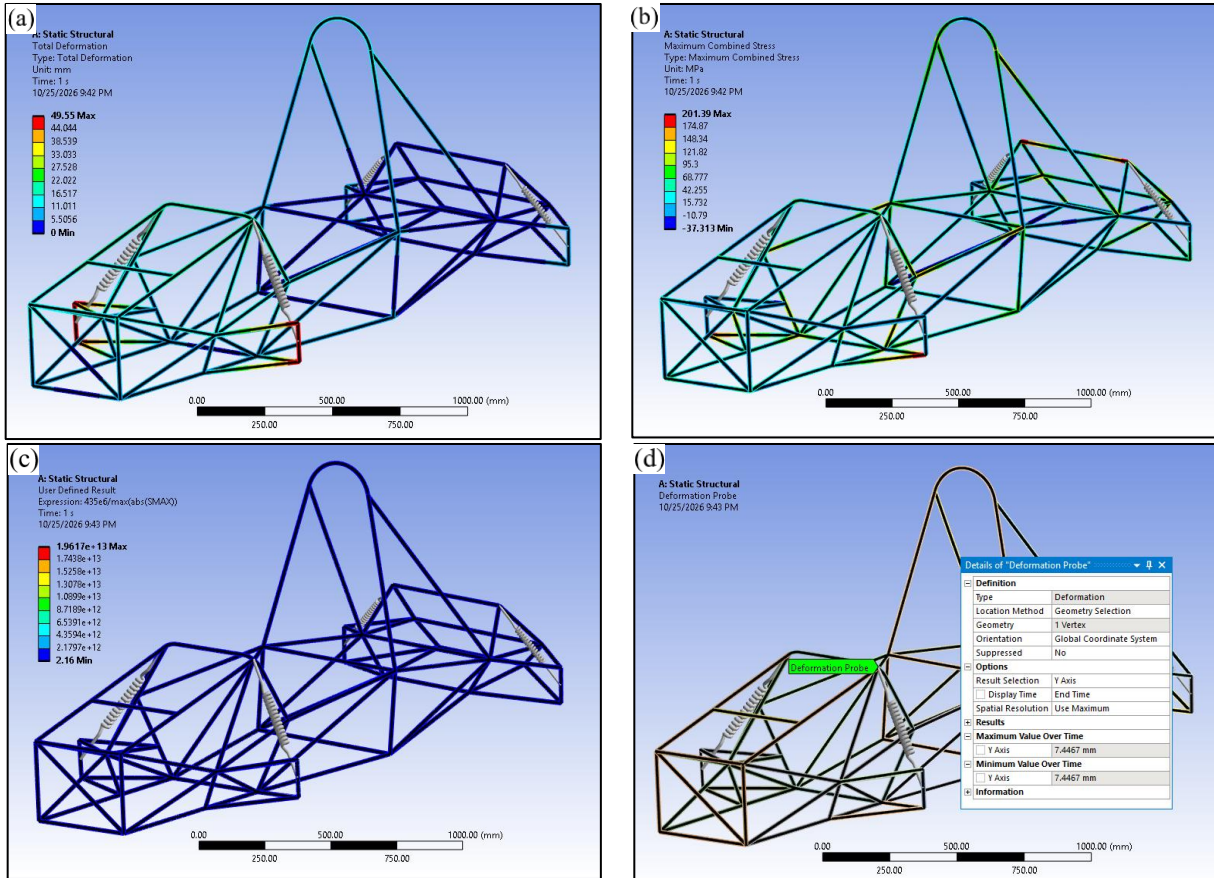


Figure 5. Finite element results for the front-loaded torsional configuration: (a) total deformation, (b) maximum combined beam stress, (c) factor-of-safety distribution, and (d) vertical displacement at the selected front upright.

The maximum combined beam stress reached 201.39 MPa, with elevated stresses concentrated near selected structural joints and load-transfer members. The corresponding minimum nominal factor of safety was 2.16, based on the

adopted yield strength of 435 MPa. This indicates that the maximum reported combined normal stress remained below the assumed material yield strength for the prescribed static loading condition.

3.2.2. Rear-loaded torsional response

The rear-loaded configuration was evaluated by reversing the loading and restraint locations. As illustrated in Figure 6, the highest deformation occurred toward the loaded rear region, while the restrained front exhibited comparatively smaller movement.

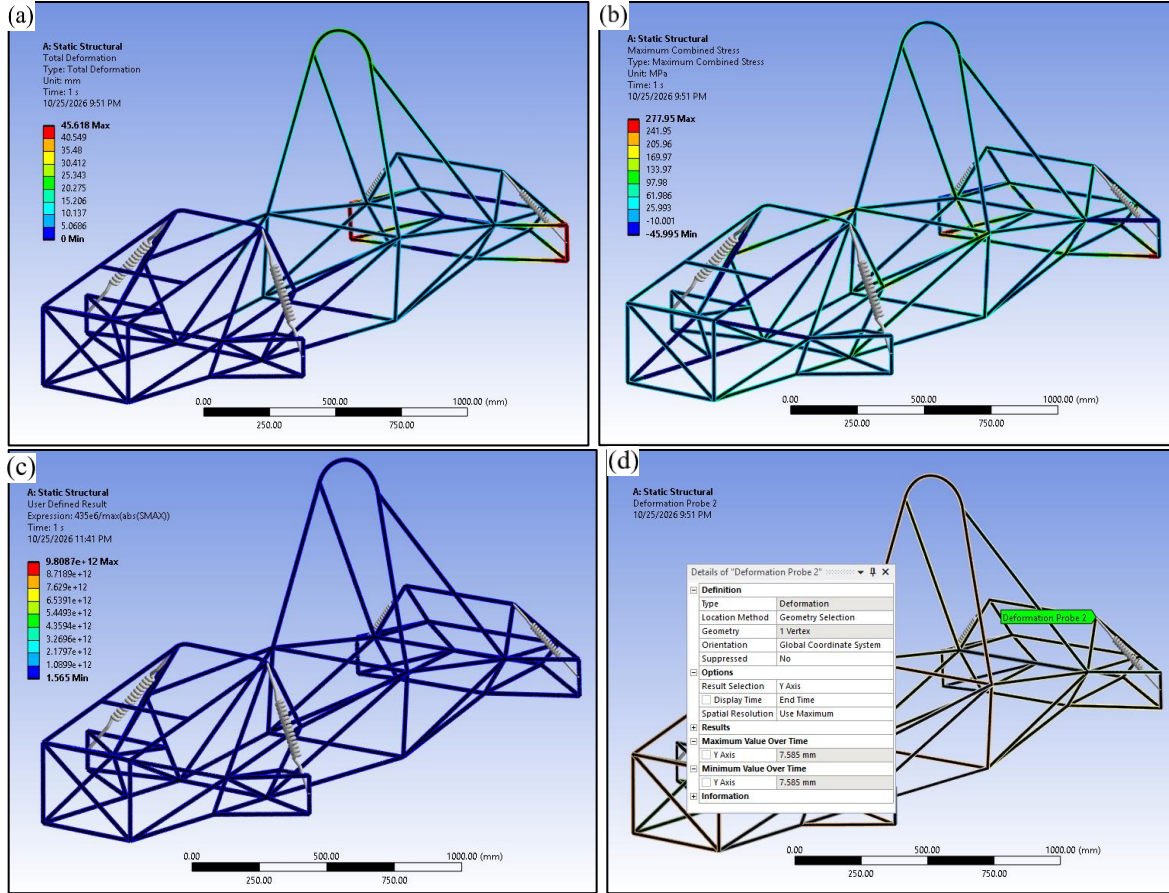


Figure 6. Finite element results for the rear-loaded torsional configuration: (a) total deformation, (b) maximum combined beam stress, (c) factor-of-safety distribution, and (d) vertical displacement at the selected rear upright.

The maximum total deformation was 45.618 mm, while the selected rear upright exhibited a vertical displacement of +7.5850 mm. The maximum combined beam stress increased to 277.95 MPa, corresponding to a minimum nominal factor of safety of 1.565. Although the rear-loaded configuration produced a lower maximum total deformation than the front-loaded configuration, its peak combined stress was approximately 38.0% higher. This difference suggests that the reversal of the loading and restraint conditions modified the internal force distribution and produced greater local stress demand in certain structural members. The contrasting deformation and stress responses highlight the importance of evaluating more than one loading configuration, particularly for nonuniform tubular spaceframes incorporating suspension interfaces and differently triangulated front and rear structures [11, 16].

3.2.3. Effective torsional stiffness assessment

The effective torsional stiffness for both cases was determined using equations (5)-(7). The comparative torsional responses of the chassis under front-loaded and rear-loaded configurations, including vertical displacement, angular

deformation, effective torsional stiffness, maximum combined beam stress, and nominal factor of safety, are summarized in Table 4.

Table 4. Comparative torsional response under front-loaded and rear-loaded configurations.

Parameter	Front-loaded	Rear-loaded
Left-side vertical displacement (mm)	+7.446	+7.585
Right-side displacement (mm)	−7.246	−7.385
Differential vertical displacement (mm)	14.692	14.970
Angular deformation (deg)	0.711	0.724
Effective torsional stiffness (Nm/deg)	1710.73	1678.96
Maximum total deformation (mm)	49.55	45.618
Maximum combined beam stress (MPa)	201.39	277.95
Minimum nominal factor of safety	2.16	1.565

The front-loaded response was approximately 1.84% stiffer than the rear-loaded response relative to their arithmetic mean. The two configurations yielded a provisional mean effective torsional stiffness of 1694.84 Nm/deg. The provisional effective stiffness estimates obtained from both torsional configurations numerically exceed 1600 Nm/deg. The comparison shows that similar effective torsional stiffnesses can coexist with substantially different peak stresses under reversed loading and restraint arrangements. For the investigated chassis–suspension assembly, this finding indicates that a global stiffness value alone does not fully describe local structural demand. Evaluating both configurations therefore provides additional information about the sensitivity of the structural response to the adopted loading and restraint conditions [1, 11, 16].

3.3. Equivalent-static frontal and rear loading analysis

In the frontal equivalent-static assessment, the reported maximum total deformation was 21.906 mm. The deformation contour shows that the largest deflection occurs toward the loaded front-frame region, with a nonuniform distribution of displacement through the remainder of the spaceframe (Figure 7a). The maximum reported positive combined beam stress was 334.94 MPa (Figure 7b), and the ANSYS user-defined safety-factor field yielded a minimum of approximately 1.299.

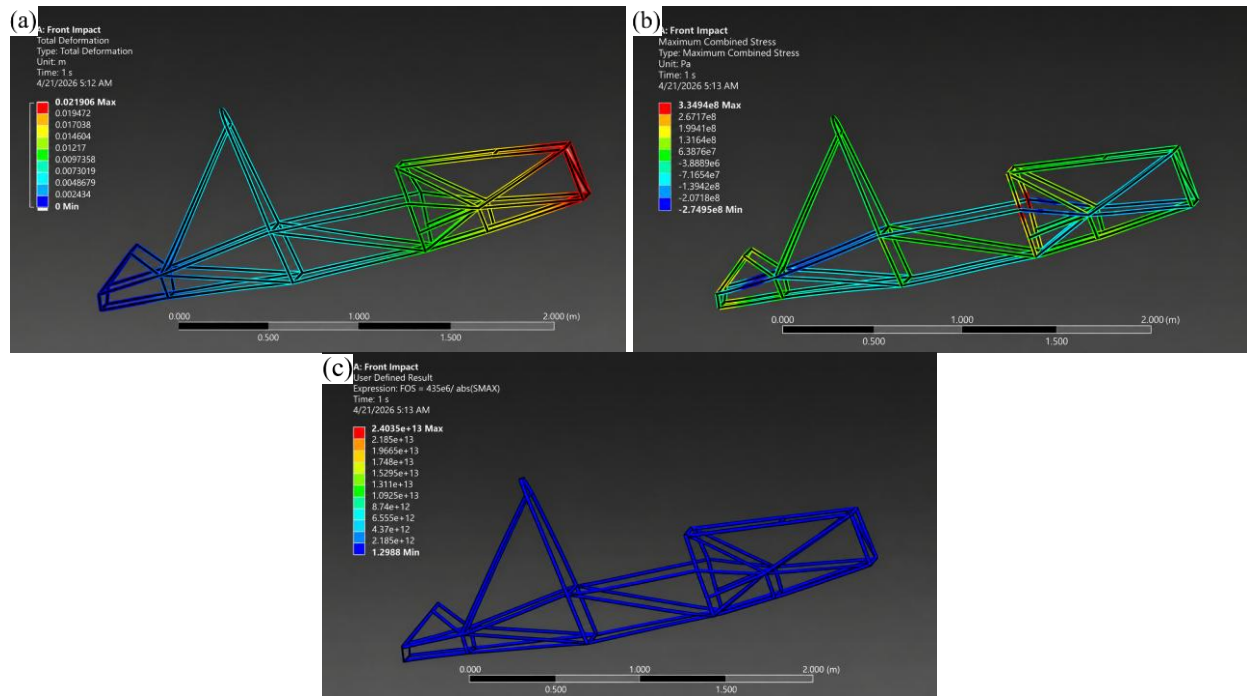


Figure 7. Frontal equivalent-static loading response: (a) total deformation, (b) maximum combined beam stress, and (c) factor of safety.

The corresponding rear structural loading case produced a maximum total deformation of 8.1906 mm and maximum positive combined beam stress of 218.93 MPa. The user-defined minimum nominal FOS was approximately 1.987. The rear displacement pattern differs from the frontal case, with deformation distributed according to its particular load application and support locations (Figure 8).

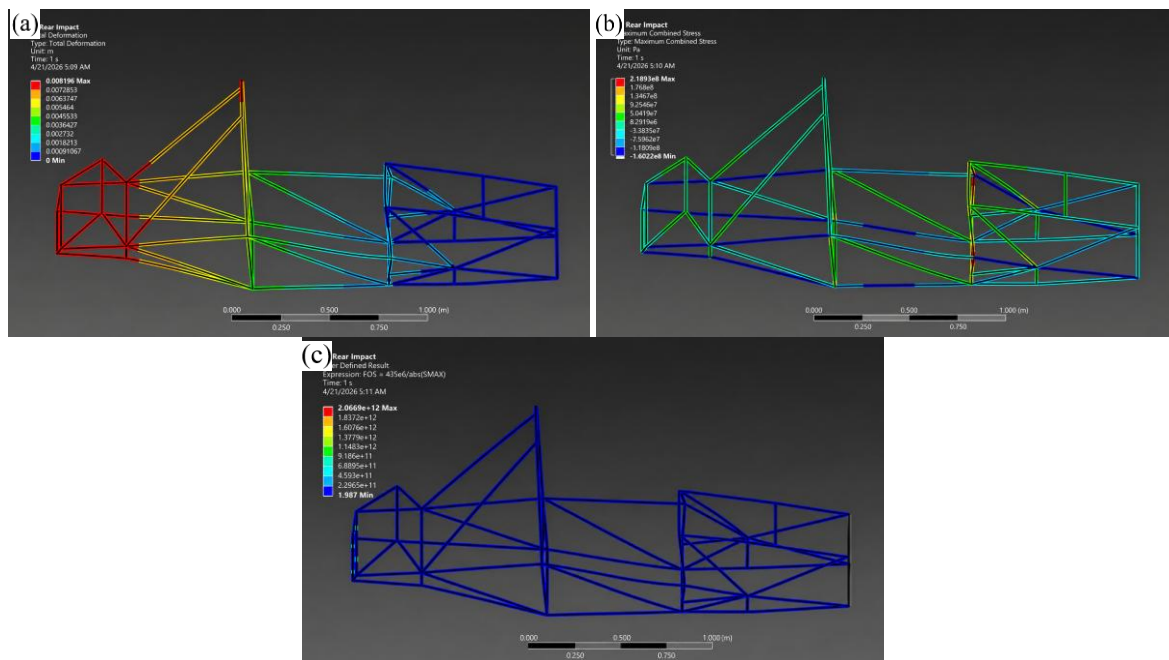


Figure 8. Rear equivalent-static loading response: (a) total deformation, (b) maximum combined beam stress, and (c) user-defined nominal factor of safety.

3.4. Lateral structural loading analysis

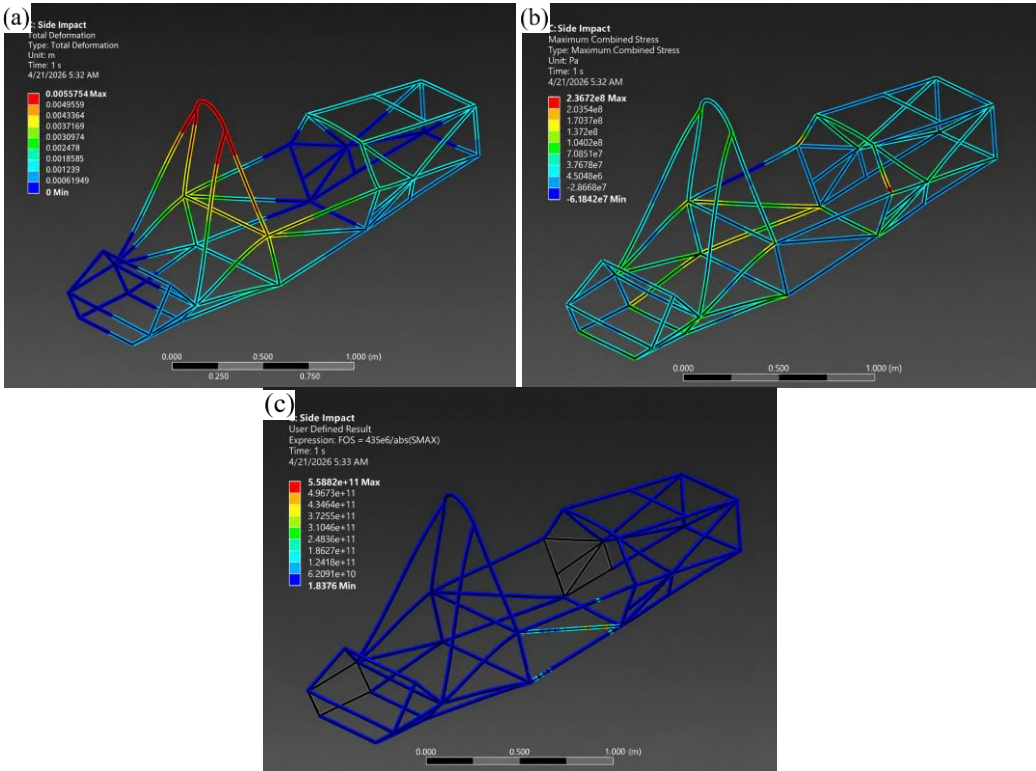


Figure 9. Lateral response under 7 kN loading: (a) total deformation, (b) maximum combined beam stress, and (c) factor of safety.

Figure 9 shows a maximum total deformation of 5.575 mm and a maximum positive combined beam stress of 236.72 MPa. The ANSYS user-defined minimum factor of safety is approximately 1.837.

3.5. Free–free modal characteristics

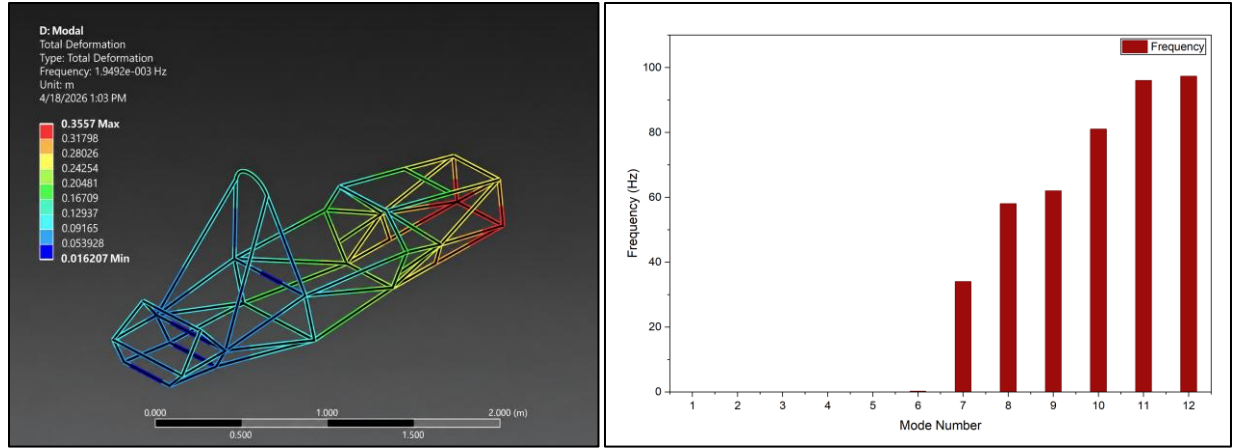


Figure 10. (a) Total deformation mode shape of the first rigid-body mode obtained from free–free modal analysis, (b) Natural Frequency Distribution Across the First 12 Vibration Modes

The dynamic characteristics of the Formula Student spaceframe chassis were evaluated through free–free modal analysis in ANSYS Mechanical 2024 R1 (Figure 10a). The numerical model incorporated the distributed structural mass and concentrated masses representing major vehicle components. The first six modes exhibited approximately zero natural frequencies, corresponding to the three translational and three rotational rigid-body degrees of freedom of an unconstrained three-dimensional structure. Figure 10a illustrates the first rigid-body mode, with a calculated frequency of 0.0019492 Hz. The near-zero frequency is consistent with the expected unconstrained behaviour, while the displayed deformation contours represent the relative modal displacement distribution rather than physical displacement under an applied load. Figure 10b illustrates the modal characteristics of the chassis, with the first flexible natural frequency occurring at 33.271 Hz (Mode 7), followed by natural frequencies of 56.376, 61.064, 80.954, 95.575, and 97.306 Hz for Modes 8–12, respectively. These frequencies characterize the elastic vibration behaviour of the numerical chassis under the adopted structural stiffness and mass distribution.

The obtained natural frequencies were compared with previously published modal investigations of racing chassis. Schweighardt et al. [31] performed free–free beam-element modal analysis of a Formula Student spaceframe and reported a first flexible natural frequency of 61.99 Hz, compared with an experimentally measured value of 55.81 Hz. Rodrigues et al. [22] reported an initial natural frequency of 34.4 Hz for a tubular sports-car chassis, while Gao et al. [32] obtained a first non-zero frequency of 32.241 Hz for a Formula racing frame. The first flexible natural frequency of the present chassis (33.271 Hz) is comparable in magnitude to those reported by Rodrigues et al. and Gao et al., although it is lower than the numerical value reported by Schweighardt et al.

Differences between the reported frequencies may be attributed to variations in chassis geometry, material distribution, boundary conditions, numerical modelling approaches, and representation of vehicle subsystem masses. The present model includes concentrated masses representing the driver, engine, and additional components, which influence the assembled mass matrix and corresponding natural frequencies. Furthermore, differences in mode shapes prevent direct equivalence between individual frequencies across different chassis configurations.

The first flexible frequency of 33.271 Hz also lies above the nominal engine idle rotational frequency range of 25–30 Hz. Nevertheless, this separation alone does not establish resonance-free operation, since engine excitation harmonics, modal participation, damping, and the operating conditions of the assembled vehicle may influence the dynamic response. Experimental modal testing and frequency-response analysis would therefore be required for further validation. For experimental follow-up, Zainal et al. [23] demonstrated the use of free–free operational and

experimental modal analyses to identify chassis frequencies and mode shapes, providing a relevant approach for comparison against future finite element predictions.

4. Conclusion

This study established a suspension-informed torsional stiffness requirement for a Formula Student spaceframe and evaluated its structural performance through torsional, equivalent-static, and modal finite element analyses using ANSYS Mechanical 2024 R1. The investigation integrated an analytical stiffness model with numerical assessments of an AISI 4130 steel tubular spaceframe. The principal findings are summarized as follows:

1. The front and rear suspension roll stiffnesses of 133.07 and 221.37 Nm/deg yielded an equivalent suspension stiffness of 83.11 Nm/deg. The parametric assessment demonstrated diminishing improvements in relative stiffness with increasing chassis rigidity. Accordingly, 1600 Nm/deg was selected as a nominal torsional stiffness target, corresponding to 95.06% relative stiffness. Increasing chassis stiffness to 2100 Nm/deg improved relative stiffness by only 1.13 percentage points, supporting the selection of a practical threshold within the adopted simplified compliance model.
2. The finite element model achieved a torsional stiffness of 1651 Nm/deg at the selected 1 mm mesh resolution, exceeding the nominal target by approximately 3.2% and corresponding to a relative stiffness of 95.21%. Further mesh refinement to 0.5 mm yielded 1638 Nm/deg, representing a variation of approximately 0.79% and satisfying the adopted convergence criterion. The front-loaded and rear-loaded torsional configurations also demonstrated that comparable global stiffness values could produce different local stress distributions under reversed loading and restraint conditions.
3. Frontal and rear loading analyses produced maximum deformations of 21.906 and 8.191 mm, with corresponding maximum combined beam stresses of 334.94 and 218.93 MPa. Under the baseline 7 kN lateral loading condition, the maximum deformation and combined beam stress were 5.575 mm and 236.72 MPa, respectively. The minimum nominal factors of safety were 1.299, 1.987, and 1.837 for frontal, rear, and lateral loading, respectively. The frontal configuration exhibited the lowest reported safety factor, indicating a comparatively smaller nominal elastic strength margin under the prescribed loading conditions.
4. Free-free modal analysis identified six approximately zero-frequency rigid-body modes, followed by a first flexible natural frequency of 33.271 Hz. The subsequent flexible frequencies ranged from 56.376 to 97.306 Hz, providing an initial characterization of the chassis vibration behavior under the adopted stiffness and mass distributions.

Overall, the study demonstrates the applicability of integrating suspension-informed torsional stiffness selection with numerical structural and modal assessments for Formula Student spaceframes. The findings emphasize the importance of considering both global stiffness and local stress demand when evaluating chassis performance. Nevertheless, the equivalent-static analyses do not establish crashworthiness, and the modal results cannot independently confirm resonance-free operation. Experimental validation, refined joint modelling, and nonlinear structural assessments are recommended to improve the reliability and practical applicability of the predicted responses.

References

- [1] Riley, W. B., and George, A. R. (2002). "Design, Analysis and Testing of a Formula SAE Car Chassis." *SAE Technical Paper 2002-01-3300*. <https://doi.org/10.4271/2002-01-3300>
- [2] Kakria, S., and Singh, D. (2015). "CAE Analysis, Optimization and Fabrication of Formula SAE Vehicle Structure." *SAE Technical Paper 2015-01-0072*. <https://doi.org/10.4271/2015-01-0072>
- [3] Thompson, L. L., Soni, P. H., Raju, S., and Law, E. H. (1998). "The Effects of Chassis Flexibility on Roll Stiffness of a Winston Cup Race Car." *SAE Technical Paper 983051*. <https://doi.org/10.4271/983051>
- [4] Deakin, A., Crolla, D., Ramirez, J. P., and Hanley, R. (2000). "The Effect of Chassis Stiffness on Race Car Handling Balance." *SAE Technical Paper 2000-01-3554*. <https://doi.org/10.4271/2000-01-3554>
- [5] Velie, H. D. (2016). *Chassis Torsional Rigidity Analysis for a Formula SAE Racecar*. Department of Mechanical Engineering, University of Michigan, Ann Arbor, USA.
- [6] Bortoluzzi, L. I., Schommer, A., Martins, M., and Buenos, A. A. (2016). "Formula SAE Chassis Design to Improve Suspension Tuning." *SAE Technical Paper 2016-36-0239*. <https://doi.org/10.4271/2016-36-0239>

- [7] da Costa, J. A., and Vilela, D. (2014). "Formula SAE Frame Torsional Stiffness Study using FEA." *SAE Technical Paper 2014-36-0234*. <https://doi.org/10.4271/2014-36-0234>
- [8] Jain, A. (2014). "Computational Analysis and Optimization of Torsional Stiffness of a Formula-SAE Chassis." *SAE Technical Paper 2014-01-0355*. <https://doi.org/10.4271/2014-01-0355>
- [9] Ferreira, R., Sousa, L., and Infante, V. (2017). "A Composite Chassis for a Formula Student Vehicle." *International Journal of Automotive Composites*, 3(2/3/4), 294–322. <https://doi.org/10.1504/IJAUTO.2017.091426>
- [10] Wheatley, G., Popoola, S., and Collins, J. (2021). "Design, Optimization, and Analysis of the Chassis of a Racecar." *SAE Technical Paper 2021-01-5058*. <https://doi.org/10.4271/2021-01-5058>
- [11] Krzikalla, D., Mesicek, J., Petru, J., Sliva, A., and Smirau, J. (2019). "Analysis of Torsional Stiffness of the Frame of a Formula Student Vehicle." *Journal of Applied Mechanical Engineering*, 8(1), Article 315. <https://doi.org/10.35248/2168-9873.19.8.315>
- [12] Gala, B. A., Munani, D., Mandaliya, K. P., Talekar, T., Vasudevan, H., and Khatawate, V. H. (2025). "Numerical Investigation of Chassis Torsional Stiffness for Formula Student Vehicles." *Journal of Mines, Metals and Fuels*, 73(6), 1651–1662. <https://doi.org/10.18311/jmmf/2025/48770>
- [13] Krzikalla, D., Sliva, A., Měsíček, J., and Petrů, J. (2020). "On Modelling of Simulation Model for Racing Car Frame Torsional Stiffness Analysis." *Alexandria Engineering Journal*, 59(6), 5123–5133. <https://doi.org/10.1016/j.aej.2020.09.042>
- [14] Capata, R., Martellucci, L., Buccolini, D., De Felice, C., and Giannini, M. (2025). "Validation Method of Torsional Stiffness for a Single-Seater Car Chassis." *World Electric Vehicle Journal*, 16(11), Article 604. <https://doi.org/10.3390/wevj16110604>
- [15] Eakambaram, A., Baskara Sethupathi, P., Saibalaji, M. A., and Baskar, A. (2021). "Experimental Analysis and Validation of Torsional Stiffness of a Tubular Space Frame Chassis." *Materials Today: Proceedings*, 46, 7719–7727. <https://doi.org/10.1016/j.matpr.2021.02.238>
- [16] Wolska, N., Szyca, M., Estrada, Q., and Castaneda, L. (2024). "Formula Student Car Chassis – Research on the Torsional Stiffness." *MATEC Web of Conferences*, 391, Article 01004. <https://doi.org/10.1051/mateconf/202439101004>
- [17] Wongchai, B. (2020). "Front and Side Impact Analysis of Space Frame Chassis of Formula Car." *International Journal of GEOMATE*, 18(67), 168–174. <https://doi.org/10.21660/2020.67.5528>
- [18] Rao, P. K. V., Putsala, K. S. K., Muthupandi, M., and Mojeswararao, D. (2022). "Numerical Analysis on Space Frame Chassis of a Formula Student Race Car." *Materials Today: Proceedings*, 66, 754–759. <https://doi.org/10.1016/j.matpr.2022.04.077>
- [19] Ali, M. H. A., Mohamad, M. S., Rahman, M. T. A., Ayob, M. N., Rahman, M. K. F. A., Hamzas, M. F. M. A., Sanuddin, A., Fouzi, M. S. A., Siahaan, M. Y. R., and Kadir, M. S. A. (2025). "Comparative Finite Element Analysis of Triangulated Spaceframe Designs for Formula SAE Chassis under Static Load Conditions." *Journal of the Society of Automotive Engineers Malaysia*, 9(2), 139–149. <https://jsaem.my/index.php/journal/article/view/285>
- [20] Siegler, B. P., Butler, L., Deakin, A. J., and Barton, D. C. (1999). "The Application of Finite Element Analysis to Composite Racing Car Chassis Design." *Sports Engineering*, 2(4), 245–252. <https://doi.org/10.1046/j.1460-2687.1999.00038.x>
- [21] Saplinova, V., Novikov, I., and Glagolev, S. (2020). "Design and Specifications of Racing Car Chassis as Passive Safety Feature." *Transportation Research Procedia*, 50, 591–607. <https://doi.org/10.1016/j.trpro.2020.10.071>
- [22] Rodrigues, A. F. A., Gertz, L. C., Cervieri, A., Poncio, A. L. R., Oliveira, A. B., and Pereira, M. S. (2015). "Static and Dynamic Analysis of a Chassis of a Prototype Car." *SAE Technical Paper 2015-36-0353*. <https://doi.org/10.4271/2015-36-0353>
- [23] Zainal, M. N. A., Anuar, M. A., Mat Isa, A. A., Adenan, M. S., and Zamri, A. R. (2018). "Experimental Analysis of Racing Car Chassis for Modal Identification." *Journal of Mechanical Engineering*, SI 7(1), 203–216. <https://ir.uitm.edu.my/id/eprint/41736/>
- [24] SAE International. (2025). *Formula SAE Rules 2026*, Version 1.0. Section F.8.8.2: Impact Attenuator Functional Requirements.
- [25] ASTM International. (2022). ASTM A370-22: Standard Test Methods and Definitions for Mechanical Testing of Steel Products. <https://doi.org/10.1520/A0370-22>

- [26] ANSYS, Inc. (2024). *ANSYS Mechanical APDL Element Reference: BEAM188—3D Two-Node Beam*. Release 2024 R1.
- [27] ASME. (2019). Standard for Verification and Validation in Computational Solid Mechanics. ASME V&V 10-2019 (R2025). <https://www.asme.org/codes-standards/find-codes-standards/standard-for-verification-and-validation-in-computational-solid-mechanics>
- [28] Thompson, L. L., Lampert, J. K., and Law, E. H. (1998). “Design of a Twist Fixture to Measure the Torsional Stiffness of a Winston Cup Chassis.” SAE Technical Paper 983054. <https://doi.org/10.4271/983054>
- [29] SAE International. (2016). *2016 Formula SAE Rules*. Section AF4.3: Side Impact.
- [30] ANSYS, Inc. (2024). *ANSYS Mechanical User's Guide: Beam Tool*. Release 2024 R1.
- [31] Schweighardt, A., Vehovszky, B., and Feszty, D. (2020). “Modal Analysis of the Tubular Space Frame of a Formula Student Race Car.” *Manufacturing Technology*, 20(1), 84–91. <https://doi.org/10.21062/mft.2020.013>
- [32] Gao, J., Xuan, S., and Zhang, Q. (2023). “Modal Analysis of Formula Racing Frame.” *Journal of Engineering Mechanics and Machinery*, 8(1), 82–85. <https://doi.org/10.23977/jemm.2023.080109>