From Poisson Analysis to Padé Approximation: An Electrostatic Saturation Current Correction Factor for MOSFETs Under Minimized Secondary Effects

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Abstract — The classical one-dimensional long-channel equation for a MOSFET's drain current can show very noticeable errors when applied to nanoscale devices, reaching a peak percentage error of 31.78% at a channel length of 50 nm. This paper investigates the discrepancy between the classical long channel equation and technology computer aided design (TCAD) simulations in the saturation region over a range of channel lengths and oxide thicknesses. As device dimensions vary, the threshold voltage rolls off. To counter this effect, the overdrive voltage was kept at a value of 0.436 V through the use of a TCAD simulation algorithm that continuously adjusted the gate bias. This approach helped to control variations in overdrive voltage, which usually resulted from threshold voltage roll-off, and, in turn, helped to facilitate the investigation of geometry dependent discrepancies. A very high saturation velocity was used to reduce the influence of carrier velocity saturation on the simulated drain current, allowing geometry dependent electrostatic deviations between the classical model and TCAD simulated drain current to be investigated with reduced interference from this transport limitation. A dimensionless correction factor which multiplies the drain current directly is developed from a quasi-static Poisson equation and represented using a Padé [4/4] approximation, with its coefficient calibrated against the TCAD result. This new correction factor greatly reduces the percentage error across every channel length. The resulting expression for the correction factor is pole free and avoids singularities for any device dimension.

Keywords — Correction Factor, Short Channel Effects, Electrostatic Effects, Compact Modelling, Padé Approximant.

I. INTRODUCTION

Historically, analytical compact models account for short channel effects through modifications to several physical variables which include threshold voltage, effective carrier mobility, subthreshold swing degradation and channel charge [1], [2]. Although established frameworks such as BSIM add all these effects into a unified drain current formulation [3], the interactions that occur between short channel electrostatics and carrier transport complicates the isolation of geometry dependent current discrepancies, making it difficult to understand how much of this deviation was caused by electrostatic effects. As a result of this, a dimensionless correction factor that can approximate these deviations caused by electrostatic effects without requiring a complete reformulation of the classical equation is needed.

This work develops a dimensionless geometry dependent correction factor for the classical one dimensional long channel MOSFET drain current equation in the saturation region [4].

Technology computer aided design (TCAD) simulation using Silvaco TCAD [5] provide the simulated drain current upon which the classical drain current equation is compared with. To reduce the influence of transport limitations and threshold voltage roll off while ensuring that the short channel electrostatic effects are properly captured, the simulation uses a very high saturation velocity parameter [6] and a continuously changing gate bias to constantly maintain the overdrive voltage as the channel length varies.

A long channel reference device was used to establish the 1D baseline under conditions consistent with the gradual channel approximation [7]. The analysis focuses on the saturation region operation and examines the deviations between the reference equation and the TCAD simulation data.

The resulting geometry dependent correction factor is represented using a dimensionless Padé [4/4] approximant [8], with its coefficient gotten from the simulation data. The corrected equation is then evaluated against the TCAD results over the investigated geometries to check how accurate it is and the extent to which it reduces the discrepancy of the classical model

II. Classical Drain Current Model, Reference Geometry and A Brief Introduction To The Correction Factor

Under the long channel approximation, the drain current equation is given by [4]:

$$I_{D,sat} = \frac{W}{2L}\mu C_{ox}(V_{GS} - V_T)^2 \quad (1)$$

where μ represents effective carrier mobility in the inversion layer, W is the effective channel width, L is the channel length, and $V_{GS} - V_T$ represents the overdrive voltage (V_{OV}). The gate oxide capacitance per unit area, C_{ox}, is characterized by:

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \quad (2)$$

with t_{ox} representing the oxide thickness and ε_{ox} denoting the permittivity of the gate dielectric material.

While (1) provides high accuracy in determining the drain current of devices in the long channel region, its accuracy reduces as we go deep into the short channel region, where short channel effects (SCEs), drain induced barrier lowering and carrier velocity saturation become more significant [9]. A reference device with a channel length of $L_{ref} = 20\ \mu m$ and an oxide thickness of $t_{ox,ref} = 10\ nm$ was adopted to serve as the standard geometry where (1) remains valid. At this extended length, the electric field varies more in the vertical direction than in the lateral direction. Mathematically, the derivative of the electric field with respect to the x-axis is much lesser than the derivative of the electric field with respect to the y-axis:

$$\left|\frac{\partial \psi}{\partial x}\right| \ll \left|\frac{\partial \psi}{\partial y}\right|$$

Because the lateral field is reduced, this $20\ \mu m$ geometry serves as a suitable reference for the classical drain current equation, which is based on the gradual channel approximation [7].

To study how the drain current changes as the channel length and oxide thickness are reduced, a geometry dependent correction factor (F) is developed. Throughout the simulations, the channel width (W) and overdrive voltage ($V_{OV} = V_{GS} - V_T$) are kept constant across all device geometries.

To improve our study of the geometric scaling effects and avoid artificial discrepancies that occur from internal TCAD assumptions (such as finite depth of the inversion charge), the paper uses a proportional projection targeted at the $20\ \mu m$ reference device. By substituting $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$ into the classical saturation equation, it is evident that the drain current scales inversely with the product of channel length (L) and oxide thickness (t_{ox}).

Because the channel width (W), effective mobility (μ), oxide permittivity (ε_{ox}) and overdrive voltage (V_{OV}) are held constant across all simulated geometries, their product forms a proportional constant. Also, the expected drain current for any scaled device ($I_{D,new}$) can be projected directly from the reference current ($I_{D,ref}$) using the geometric scaling ratio:

$$I_{D,new} = I_{D,ref} \times \frac{L_{ref} t_{ox,ref}}{L_{new} t_{ox,new}} \quad (3)$$

where L_{ref} and $t_{ox,ref}$ represent the channel length and oxide thickness of the reference device while L_{new} and $t_{ox,new}$ represent the channel length and oxide thickness of the new device.

To correct the deviations of the classical drain equation as we scale deep into the short channel region, a dimensionless correction factor (F) is introduced.

$$F = \frac{I_{D,sim}}{I_{D,new}} \quad (4)$$

where $I_{D,sim}$ represents the drain current gotten from the TCAD simulation.

The percentage error, which shows how much the drain current gotten from the classical law deviates from the drain current gotten from the simulation, is calculated as:

$$Percentage\ error = \left|\frac{I_{D,new} - I_{D,sim}}{I_{D,sim}}\right| \times 100 \quad (5)$$

.

III. TCAD Simulation Setup and Current Error Analysis

In order to extract the geometry dependent correction factor (F) and quantify the deviation from the classical 1D model, a series of 12 MOSFET devices were simulated using Silvaco TCAD [5]. The devices were varied from the $20\ \mu m$ reference length down to a deep nanoscale channel length of $50\ nm$ with the gate oxide thickness varied from $10\ nm$ down to $1.2\ nm$.

As Dennard et al. [10] established that for devices deep in the short channel region, the classical 1D equation fails because the lateral electric field between the drain and source reduces the threshold voltage and this breaks the vertical field approximations.

To reduce the interference caused from parasitic and transport phenomena, strict constraints were applied across all the simulated devices. The channel width (W) was fixed at $40\ \mu m$ while the source and drain lengths were kept at $1\ \mu m$. As Taur et al. [11] demonstrated in the development of the Shift and Ratio extraction method, calculating the threshold voltage in devices that have a short channel length is very difficult because of a constantly changing mobility. To ensure the simulated drain current did not depend on these threshold voltage variations, a TCAD simulation algorithm was used to continuously adjust the gate bias, locking the overdrive voltage at exactly $0.436\ V$ for every device.

It was shown in [12] that contact resistances become the major resistance component at the $50\ nm$ gate length node. In order to prevent this parasitic resistance from influencing the drain current discrepancy, no additional source or drain contact resistance was introduced in the TCAD simulation.

Lundstrom [13] also proved that as transistors are varied below the $100\ nm$ channel length, the drain current becomes

capped by the speed limits of the electrons governed by velocity saturation and ballistic limits. Because these transport limits cap the current, they hide the underlying 2D geometric effects. Also, Taur et al. [11] observed that velocity saturation corrupts electrical measurements for channel lengths smaller than $0.25\,\mu m$, and this made the channel length to appear longer than it should be. Because traditional pseudo-2D models calculate all these effects simultaneously, it becomes very difficult to separate the structural geometry from the carrier speed limits. To solve this, our study deliberately used a very high saturation velocity ($1 \times 10^9 cm/s$) in the simulation. By removing this physical speed limit, we ensured the current was not capped by transport physics. This allowed us to mathematically isolate the 2D geometric effects and build an analytical correction factor.

Using the proportional methodology that was established in Section II, the expected drain current ($I_{D,new}$) was projected from the reference device for each device. This was then compared against the actual simulated drain current ($I_{D,sim}$). Table 1 lists the extracted simulation results, the projected 1D currents, the resulting percentage error and the calculated correction factor (F) across all 12 devices.

Table 1: Simulated vs. Expected Drain Currents and Extracted Correction Factor

Channel Length (μm)	Oxide thickness (nm)	$I_{D,sim}$ (μA)	$I_{D,new}$ (μA)	Percentage error (%)	Correction factor (F)
20.0	10.0	82.4688	82.4688	0.00	1
10.0	8.0	214.098	206.1700	3.70	1.0384
5.0	6.0	599.11	549.7900	8.23	1.0897
2.0	5.0	1840.60	1649.3776	10.39	1.1159
1.0	4.0	4741.52	4123.4440	13.04	1.1499
0.8	3.5	6855.38	5890.6340	14.07	1.1638
0.5	3.0	12996.11	10995.8500	15.39	1.1819
0.3	2.5	26551.51	21991.7010	17.17	1.2073
0.2	2.2	46116.11	37485.8540	18.71	1.2302
0.15	2.0	68635.21	54979.2530	19.90	1.2484
0.1	1.8	118036.09	91632.0890	22.37	1.2882
0.05	1.2	402928.81	274896.2670	31.78	1.4657

As observed in Table 1, the percentage error is 0% at the $20\,\mu m$ reference length but increases as the device goes deeper into the short channel regions, reaching a peak deviation of 31.78% at the $50\,nm$ channel length. This nonlinear increase in the correction factor shows the failure of the 1D approximation.

IV. Mathematical Derivation of The Correction Factor

The derivation begins with the 2D Poisson equation governing the depleted channel region of the MOSFET:

$$\frac{\partial^2\psi(x,y)}{\partial x^2} + \frac{\partial^2\psi(x,y)}{\partial y^2} = \frac{qN_A}{\epsilon_{si}} \qquad (6)$$

$$0 < x < L, 0 < y < X_d$$

where ϵ_{si} is the permittivity of silicon, N_A is the acceptor doping concentration and q is the positive elementary charge.

To reduce the dimension of this partial differential equation, we apply Young's parabolic potential approximation [14]. We assume the 2D potential can be modelled as a 1D surface potential $\psi_s(x)$ with a parabolic dependence on the vertical depth y.

$$\psi(x,y) = \psi_s(x) + c_1(x)y + c_2(x)y^2 \qquad (7)$$

The vertical electric field reaches a maximum value at the gate dielectric interface ($y = 0$). The depletion region is assumed to extend from the gate-silicon interface to $y = X_d$, where it meets a neutral bulk region. Under the depletion approximation, the vertical electric field is assumed to vanish at this boundary [7].

By differentiating the electrostatic potential with respect to y, the vertical electric field is determined.

$$\frac{\partial\psi(x,y)}{\partial y} = c_1(x) + 2c_2(x)y \qquad (8)$$

Assuming no free sheet charge at the oxide-silicon interface ($y = 0$), Gauss's law requires a continuous electric displacement field across the interface [7]. The electric field displacement at the silicon surface can then be represented by:

$$-\epsilon_{si}\frac{\partial\psi(x,y)}{\partial y}\Big|_{y=0} = -\epsilon_{si}c_1(x) \qquad (9)$$

while the electric displacement in the oxide is represented as:

$$\epsilon_{ox}\left(\frac{V_{GS}-V_{FB}-\psi_s(x)}{t_{ox}}\right) \quad (10)$$

where ϵ_{ox} represents the permittivity of the oxide.

The two electric displacements are equated to give:

$$-\epsilon_{si}c_1(x) = \epsilon_{ox}\left(\frac{V_{GS}-V_{FB}-\psi_s(x)}{t_{ox}}\right) \quad (11)$$

with V_{GS} representing the gate to source voltage and V_{FB} representing the flatband voltage.

Hence,

$$c_1(x) = -\frac{\epsilon_{ox}}{\epsilon_{si}}\left(\frac{V_{GS}-V_{FB}-\psi_s(x)}{t_{ox}}\right) \quad (12)$$

At the bottom of the substrate($y = X_d$), the electric field vanishes.

$$\left.\frac{\partial\psi(x,y)}{\partial y}\right|_{y=X_d} = c_1(x) + 2c_2(x)X_d = 0 \quad (13)$$

$$c_2(x) = -\frac{c_1(x)}{2X_d} = \frac{\epsilon_{ox}}{2\epsilon_{si}}\left(\frac{V_{GS}^{'}-\psi_s(x)}{t_{ox}X_d}\right) \quad (14)$$

where

$$V_{GS}^{'} = V_{GS} - V_{FB} \quad (15)$$

Therefore,

$$\frac{\partial^2\psi(x,y)}{\partial y^2} = 2c_2(x) = \frac{\epsilon_{ox}}{\epsilon_{si}}\left(\frac{V_{GS}^{'}-\psi_s(x)}{t_{ox}X_d}\right) \quad (16)$$

Substituting these coefficients back into the original 2D Poisson equation collapses it into a 1D ordinary differential equation.

$$\frac{d^2\psi_s(x)}{dx^2} + \frac{\epsilon_{ox}}{\epsilon_{si}}\left(\frac{V_{GS}^{'}-\psi_s(x)}{t_{ox}X_d}\right) = \frac{qN_A}{\epsilon_{si}} \quad (17)$$

The characteristic scaling length which is defined as [7]:

$$\lambda = \sqrt{\frac{\epsilon_{si}}{\epsilon_{ox}}t_{ox}X_d} \quad (18)$$

By substituting λ^2 into (18) and recognizing that, the solution to the 1D Poisson equation is given by:

$$\psi_{1D} = V_{GS}^{'} - \frac{qN_A}{\epsilon_{si}}\lambda^2$$

The differential equation written in (18) can then be written as:

$$\frac{d^2\psi_s(x)}{dx^2} - \frac{\psi_s(x)-\psi_{1D}}{\lambda^2} = 0 \quad (19)$$

Using the substitution $\Delta\psi_s(x) = \psi_s(x) - \psi_{1D}$ and by also recognizing that ψ_{1D} has no spatial dependence ($\frac{d\psi_{1D}}{dx} = 0$), the differential equation in (19) becomes:

$$\frac{d^2\Delta\psi_s(x)}{dx^2} - \frac{\Delta\psi_s(x)}{\lambda^2} = 0 \quad (20)$$

Since the MOSFET possesses structural geometry about its centre, the lateral position (x) can be replaced using the substitution:

$$x' = x - \frac{L}{2}$$

$$-\frac{L}{2} < x' < \frac{L}{2}$$

and by applying the chain rule on this substitution, the differential equation in (20) is therefore rewritten as:

$$\frac{d^2\Delta\psi_s(x')}{dx^2} - \frac{\Delta\psi_s(x')}{\lambda^2} = 0 \quad (21)$$

The solution to this homogeneous differential equation is given by:

$$\Delta\psi_s(x') = Asinh\left(\frac{x'}{\lambda}\right) + Bcosh\left(\frac{x'}{\lambda}\right) \quad (22)$$

The constants in (22) can be obtained by evaluating $\Delta\psi_s(x')$ at boundaries $x' = \frac{-L}{2}$ and $\frac{L}{2}$. At these applied boundary conditions, the $sinh$ component of the equation cancels out, leaving:

$$B = \frac{\Delta\psi_D+\Delta\psi_s}{2\text{co}\ \ (\frac{L}{2\lambda})} \quad (23)$$

Where $\Delta\psi_s$ is the surface potential deviation at the source's end and $\Delta\psi_D$ is the surface potential deviation at the drain's end.

The local inversion charge can therefore be written as:

$$Q_{inv} = C_{ox}(V_{OV} - V(x') + \Delta\psi_s(x')) \quad (24)$$

We can then write the drift current equation as:

$$I_{DS}dx' = \mu W Q_{inv}(x')dV \quad (25)$$

By evaluating the integral of (25):

$$\int_{\frac{-L}{2}}^{\frac{L}{2}} I_{DS}dx' = \mu W C_{ox}\int_0^{V_{OV}}\left(V_{OV} - V(x')\right)dV + \mu W C_{ox}\int \Delta\psi_s(x')dV \quad (25)$$

The third integral is transformed by using

$$dV = \frac{dV}{dx'}dx'$$

and then by assuming that the channel potential varies linearly along the channel length, the transformation becomes:

$$\frac{dV}{dx'} \approx \frac{V_{OV}}{L}$$

After evaluating the integral, the drain current is then given by:

$$I_{DS} = \frac{\mu W C_{ox}}{2L}V_{OV}^2 + \frac{2\mu W C_{ox}B\lambda V_{OV}}{L^2}\sinh(\frac{L}{2\lambda}) \quad (26)$$

By substituting the value of B as seen in (23) and recognizing that the first part of (26) is the classical 1D drain equation in the saturation region and defining $\beta = \frac{2}{V_{OV}}(\frac{\Delta\psi_D+\Delta\psi_s}{2\cosh(\frac{L}{2\lambda})})$, the classical 1D drain equation can be factored out making the correction factor to be defined as:

$$F = 1 + \beta(\frac{2\lambda}{L}\tanh\left(\frac{L}{2\lambda}\right)) \quad (27)$$

As seen in (27), the correction factor approaches a value of one as the channel length grows without bound and approaches a finite value of $1 + \beta$ as the channel length goes deeper into the short channel region.

To make this analytical equation practical for compact modelling, the hyperbolic function is expanded using a Maclaurin series. Also, converting the complex hyperbolic expression into a polynomial series is of great advantage for computer simulations. In industry standard circuit simulators, evaluating transcendental functions like the hyperbolic function for millions of transistors is computationally expensive and can trigger numerical overflow or even convergence failures.

Therefore, we rewrite the correction factor as a Maclaurin series while restricting it to its fourth power.

$$F = 1 + \beta\left(1 - \frac{1}{12}\left(\frac{L}{\lambda}\right)^2 + \frac{1}{120}\left(\frac{L}{\lambda}\right)^4\right) \qquad (28)$$

While mathematically correct for the 1D to 2D reduction, the constants $\frac{1}{12}$ and $\frac{1}{120}$ originate from initial boundary assumptions, especially Young's parabolic approximation [14] and the uniform electric field simplification ($\frac{dV}{dx'} \approx \frac{V_{OV}}{L}$). In deep nanoscale regions, these assumptions tend to lose accuracy. Replacing these mathematical coefficients with variable fitting parameters (C_2) and (C_4) converts the pure analytical equation into a physics informed semi-empirical model.

Therefore, we can rewrite (28) as:

$$F = 1 + \beta\left(1 - C_2\left(\frac{L}{\lambda}\right)^2 + C_4\left(\frac{L}{\lambda}\right)^4\right) \qquad (29)$$

.

V Padé Approximant and Physics Informed Parameter Extraction

The correction factor presented in (29) grows without limit as the channel length increases. This contradicts its definition because as the channel length increases, it is expected that the correction factor approaches a value of one. To resolve this issue, a new dimensionless variable ($x = \frac{L}{\lambda}$) is introduced. To convert the resulting polynomial into a universally bounded function suitable for continuous device simulation, it is transformed into a Padé [4/4] rational approximant [8]. The correction factor is then constructed as:

$$F(x) = \frac{a_0 + a_2x^2 + a_4x^4}{b_0 + b_2x^2 + b_4x^4} \qquad (30)$$

By mathematically imposing that $a_4 = b_4$, we force the model to satisfy the condition of the correction factor approaching a value of one as the channel length of the device grows beyond bound ($\lim_{x\to\infty} F = 1$).

To optimize this equation for parameter extraction and reduce the number of variables required, we normalize the fraction by dividing both the numerator and denominator by a_4. This yields a stable four parameter model as seen in (31):

$$F(x) = \frac{a_0 + a_2x^2 + x^4}{b_0 + b_2x^2 + x^4} \qquad (31)$$

With this model, the correction factor approaches a value of a_0/b_0 as the channel length approaches zero. The physical coupling magnitude β derived in Section IV is not discarded. Instead, the weight of β is factored into the coefficients of the Padé approximant.

The coefficients are then extracted from the TCAD simulation data. Two methodologies are presented for extracting these coefficients: a deterministic algebraic extraction and a non-linear regression method.

A. Deterministic Four-Point Algebraic Extraction

By equating the Padé model to the TCAD extracted correction factor (F_i) at specific nodes x_i, the rational function is rearranged into a strict linear equation:

$$F_i(b_0 + b_2x_i^2 + x_i^4) = a_0 + a_2x_i^2 + x_i^4$$
$$F_ix_i^4 - x_i^4 = a_0 + a_2x_i^2 - F_ib_2x_i^2 - F_ib_0 \qquad (32)$$

To solve for the four unknowns, a 4×4 linear matrix system is constructed using four distinct geometric anchor points ($L = 2.0\ \mu m, 1.0\ \mu m, 0.5\ \mu m, 0.05\ \mu m$).

$$\begin{bmatrix} 1 & x_1^2 & -F_1 & -F_1x_1^2 \\ 1 & x_2^2 & -F_2 & -F_2x_2^2 \\ 1 & x_3^2 & -F_3 & -F_3x_3^2 \\ 1 & x_4^2 & -F_4 & -F_4x_4^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_2 \\ b_0 \\ b_2 \end{bmatrix} = \begin{bmatrix} (F_1 - 1)x_1^4 \\ (F_2 - 1)x_2^4 \\ (F_3 - 1)x_3^4 \\ (F_4 - 1)x_4^4 \end{bmatrix} \qquad (33)$$

B. Global Non-Linear Regression (Levenberg-Marquardt Method)

The mathematical basis of the regression is the minimization of the Sum of Squared Residuals (SSR), defined by the objective function:

$$S(p) = \sum_{i=1}^{n}\left(F_{TCAD,i} - F_{Pade}(x_i, p)\right)^2 \qquad (34)$$

where $p = [a_2, a_4, b_2]^T$ is the parameter vector, n is the total number of simulated devices, $F_{TCAD,i}$ is the benchmark simulation target and $F_{Pade}(x_i, p)$ is the theoretical correction factor predicted by the normalized [4/4] Padé equation. The algorithm continuously interpolates between the two classical minimization approaches: the Gauss-Newton algorithm (for fast convergence near the minimum) and the Method of Gradient Descent (for navigation when far from the minimum). The parameter update step (Δp) at each iteration is governed by normal equations:

$$(J^TJ + \gamma I)\Delta p = J^Tr \qquad (35)$$

where J is the Jacobian matrix containing the first order partial derivatives of the Padé function with respect to the coefficients, r is the residual vector, I is the identity matrix, J^T is the transpose of the Jacobian matrix and γ is the algorithmic damping parameter.

When γ is large, the matrix J^TJ is dominated by the identity matrix, and the update rule behaves like Gradient Descent. As the SSR approaches its minimum, γ is dynamically reduced, and this transitions the update step into the Gauss-Newton method for rapid quadratic convergences.

VI. Results and Discussion

To validate the structure and predictive accuracy of the proposed rational model, the formulated [4/4] Padé approximant was evaluated against the benchmark short channel correction factors (F_{sim}) extracted from the 2D TCAD simulations. The model was tested across all the devices. The deterministic four point parameter extraction yielded the coefficients $a_0 = 29{,}815.55, a_2 = 2{,}677.41, b_0 = 19{,}228.69$ and $b_2 = 2{,}317.61$ while the Levenberg-Marquardt (LM) non-linear regression yielded $a_0 = 6{,}067.18, a_2 = 3{,}757.10, b_0 = 2{,}707.97$ and $b_2 = 3{,}201.89$. The evaluation compares the two parameter extraction methodologies proposed in Section V. The complete error analysis is detailed in Table 2.

Table 2: Device Error and Predictive Accuracy

Device	L (μm)	t_{ox} (nm)	F_{sim}	4-Point $F(x)$	4-Point Error (%)	LM Reg. $F(x)$	LM Reg. Error (%)
1	20.0	10.0	1.0000	1.0079	0.79	1.0119	1.19
2	10.0	8.0	1.0384	1.0227	1.52	1.0331	0.51
3	5.0	6.0	1.0897	1.0529	3.38	1.0719	1.63
4	2.0	5.0	1.1159	1.1159	0.00*	1.1372	1.91
5	1.0	4.0	1.1499	1.1499	0.00*	1.1629	1.13
6	0.8	3.5	1.1638	1.1581	0.49	1.1676	0.32
7	0.5	3.0	1.1819	1.1819	0.00*	1.1783	0.31
8	0.3	2.5	1.2073	1.2213	1.16	1.1937	1.13
9	0.2	2.2	1.2302	1.2705	3.28	1.2159	1.16
10	0.15	2.0	1.2484	1.3140	5.26	1.2416	0.55
11	0.1	1.8	1.2882	1.3836	7.41	1.3040	1.22
12	0.05	1.2	1.4657	1.4657	0.00*	1.4633	0.17

*(*Indicates deterministic algebraic anchor points used to solve the 4x4 matrix)*

The deterministic four point extraction proves that the core structure of the model is sound. By tying the equation directly to four different devices, the formula produces a 0.00% error. However, between the $0.5\ \mu m$ and $0.05\ \mu m$ devices, the model overcompensates to ensure it hits its final target which creates a numerical "wobble." This results in a noticeable 7.41% spike at the $0.1\ \mu m$ device, showing that while the algebraic method is theoretically exact at specific points, it is too bouncy for continuous circuit simulation.
To fix this wobble, the Levenberg-Marquardt (LM) regression was used to smooth out the error. As shown in Table 2, this algorithm completely flattens the deep nanoscale spike, bringing the error at the $0.1\ \mu m$ device down from 7.41% to a highly accurate 1.22%. to achieve this overall smoothness, the algorithm makes small, strategic trade-offs such as allowing a minor 1.91% instead of forcing it to stay at absolute zero.
Finally, for any model to be useful in commercial software like SPICE, it must be completely safe from "divide by zero" errors, which cause simulations to crash. For this rational model, a mathematical singularity would only happen if the denominator ever equals zero. When calculating the roots of this denominator using the globally extracted regression coefficients, the results for x^2 are purely negative numbers. Because the variable x represents real transistor dimensions, x^2 will always be a positive number. Therefore, based on the physical data set used for this calibration, the denominator can never equal zero. This demonstrates that when the model is anchored towards realistic physical device behaviour, the extracted coefficients naturally prevent mathematical singularities, guaranteeing that the model remains perfectly stable and pole free across all practical dimensions

VII. Conclusion

This paper developed a correction factor for the drain current in the saturation region when only electrostatic effects were considered while other effects were minimized.

With this analytical foundation now established for core electrostatics, future work can confidently build upon it by incorporating the other physical realities of shrinking transistors such as velocity saturation, mobility degradation and quantum confinement.

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