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Optimization of iron ore blending in daily planning through linear integer programming

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Abstract

Iron ore blending requires a distinction between maximizing usable material and controlling the composition of daily plant feed. This paper develops a clarified two stage formulation and critically reexamines the historical two mine illustrations supplied with an earlier blending study. The model first selects material under linear mass and quality constraints, then schedules the selected material to fixed positive daily feed masses. Under divisible material flows, target deviations and successive day grade changes admit linear programming formulations using nonnegative deviation variables. Requiring whole blocks introduces binary assignments and yields a mixed integer linear program. Endogenous daily denominators generally retain a nonlinear fractional structure. Four reproducible synthetic checks demonstrate the practical consequences: divisible blending can remove a 10 percentage point mean daily deviation that remains under whole parcel scheduling; constant grade can coexist with changing contained metal mass; optional selection produces different continuous and integer optima; and target tracking and adjacent day stability can rank schedules differently. All original figures and equations are retained, with the latter identified as the historical formulation. The mine figures provide qualitative application context, but the absence of raw schedules, units for the monthly mass plots and solver logs prevents a new industrial performance claim. The contribution is a corrected, verifiable modelling framework with explicit conditions for linearity, feasibility and evaluation.

Keywords: iron ore blending; daily scheduling; linear programming; mixed integer programming; feed stability; fractional optimization

1 Introduction

Daily blending converts a set of available material parcels into a sequence of plant feeds. A monthly blend may satisfy average iron and contaminant specifications while individual days fluctuate enough to alter plant operation or produce off specification feed. A useful optimization model must therefore state which mass is selected, how that mass becomes available over time, what daily feed is required and which definition of quality variation is minimized.

Mine planning connects extraction, access, equipment and downstream processing decisions [1]. Treating the daily blend as an isolated arithmetic average can ignore whether materials are accessible together or whether a block can be split across days. Conversely, a highly detailed schedule remains misleading if the objective compares contained metal tonnes while its interpretation concerns grade percentages. The mathematical class of the model depends on these choices and cannot be inferred from the programming language or solver used to implement it.

This paper revisits the models and mine illustrations in the supplied manuscript. A published antecedent by Navarro Torres and Moraes [2] contains the same core blending approach and historical application material. The present paper is explicitly a methodological reformulation and verification study. It contributes an internally consistent distinction between selection and scheduling, corrected linear deviation models and reproducible counterexamples. It does not present the inherited mine figures as a new independent industrial experiment. Appendix A preserves all sixteen original displayed equations and the original inline analyte notation.

2 Technical background and research scope

Classical blending constraints express the quality of a mixture as contained constituent mass divided by total blend mass. A one sided quality bound can be made linear by cross multiplication when total mass is positive. An objective containing a variable denominator is a different problem. Charnes and Cooper [3] established a transformation for a class of linear fractional programs; this does not imply that a sum of daily ratios with shared scheduling decisions can be transformed by a single common scaling variable.

Recent mine scheduling studies also show that blending interacts with fleet and stockpile decisions. Both and Dimitrakopoulos [4] couple stochastic short term production scheduling and fleet management, while Manríquez et al. [5] account for multiple objectives and shovel allocation in stockpiles. These results motivate explicit accessibility and capacity assumptions in a daily blending model. Eustace and Hynard [6] connect simulation and optimization to plan execution, which is relevant when apparently feasible daily mixtures cannot be delivered at the required times.

Tang et al. [7] study dynamic ore flow blending through scheduling policies that account for disturbances and evolving operating conditions. Such approaches concern online control as well as composition. The deterministic formulations below serve a different, narrower purpose: they make the grade accounting and objective definitions transparent and provide an auditable baseline. No superiority over stochastic, heuristic or learned methods is claimed without a common dataset and evaluation protocol.

The historical application implemented optimization through Python, Gurobi and SolverStudio in Excel. That interface is useful for exposing planning inputs, but it does not establish linearity, optimality or field validity. The present verification uses a separate small implementation with SciPy and HiGHS [8,9]. The synthetic checks test formulation properties and are not replacements for the original industrial case study.

3 Problem definition and assumptions

3.1 Mass basis and material availability

Let B be an explicitly defined set of eligible parcels or blocks, J the quality attributes and k = 1,…,K the daily periods. Parcel i has available mass m_i in tonnes and a known grade fraction a_{ij} for attribute j. The same dry or wet mass basis must be used for availability, plant demand and assay values. A value of 0.60 denotes 60%, and a change of 0.10 is ten percentage points. Grade ranges for mineral names are site descriptors rather than universal classification rules.

The model assumes linear mass mixing with known coefficients. It does not represent nonlinear recovery, mineralogical interaction, moisture dependent processing or selective reclaim within a

stockpile. Fe, silica, phosphorus, alumina, manganese and loss on ignition are different analytical attributes; their numerical values cannot be added without a justified objective normalization. Oxide and elemental reporting conventions must also be reconciled before constructing coefficients.

The candidate set must be available within the planning horizon. It is not automatically the complete ultimate pit block model. When access depends on extraction order, the schedule needs precedence constraints or a separately verified eligible set. A blend that would require geographically or temporally inaccessible material is an upper bound on an operating opportunity rather than an executable mine plan.

Table 1. Principal notation and domains. Grades are fractions; tonnes and percentage points are reported separately in the results.

Symbol	Meaning	Unit or domain
m_i, x_i	Available and selected parcel mass	t
a_{ij}, L_j, U_j	Parcel grade and horizon quality limits	Mass fraction
D_k, f_{ik}	Prescribed daily delivery and parcel delivery by day	t
v_i, y_{ik}	Whole block selection and day assignment	Binary
e_{ik}	Given material availability on day k	Binary parameter
g_{jk}, q_j, s_j	Daily grade, target and positive normalization scale	Mass fraction
d+, d−, Delta+, Delta−	Nonnegative grade error variables	Mass fraction
T_k, C_{jk}, r+, r−	Variable daily tonnes, contained mass and residuals	t; constituent t

3.2 Selection and scheduling decisions

The first decision selects how much of each eligible material to use over the horizon. The second assigns the selected material to days. Maximizing tonnes under quality constraints is a resource use objective. It is not, without prices, recovery, mining costs and handling costs, an economic optimization of reserve value. Likewise, a mathematical opportunity to blend low grade material does not establish a change in the mineral reserve classification.

The two stages must exchange a clear decision. One valid implementation fixes the selected parcel masses x_i from Stage 1 before scheduling. Another fixes only an acceptable total mass and allows the composition to be revised jointly. Those approaches have different feasible regions. The reference formulation below fixes selected masses and explicitly checks whether daily scheduling is feasible. If the second stage is infeasible, the selection stage must be revisited or the two stages solved together.

4 Corrected mathematical formulations

4.1 Stage 1 Material selection

For divisible material, x_i is a continuous selected mass. Plant horizon capacity is C, and L_j and U_j are the permitted aggregate grade fractions. A missing one sided bound can simply be omitted. Maximizing usable tonnes gives the following linear objective and capacity conditions.

$$\max M = \sum_{i \in B} x_i \quad \text{(B1)}$$

$$0 \le x_i \le m_i \quad (i \in B), \qquad \sum_i x_i \le C \quad \text{(B2)}$$

For a positive selected mass, the grade bounds are equivalent to linear contained mass inequalities.

$$\sum_i (a_{ij} - L_j)\, x_i \ge 0, \qquad \sum_i (a_{ij} - U_j)\, x_i \le 0 \quad (j \in J) \quad \text{(B3)}$$

The zero mass solution satisfies the cross multiplied constraints but has no defined grade. Where a nonempty blend is required, a positive minimum production requirement should be imposed and its feasibility checked. Optional selection requires x_i to be allowed to equal zero; forcing every candidate block into the horizon changes the problem.

If whole blocks are indivisible, replace the continuous domain with the following linking condition. The resulting model is a mixed integer linear program, even though its objective and constraints are linear.

$$x_i = m_i v_i, \qquad v_i \in \{0,1\} \quad (i \in B) \quad \text{(B4)}$$

The LP optimum is an upper bound on the corresponding whole block maximum tonnage when all other assumptions are identical. This bound can help identify whether operational indivisibility has a material cost. It does not authorize splitting a block that must be mined and delivered as one unit.

4.2 Stage 2 Scheduling to prescribed daily masses

Suppose Stage 1 returns selected masses $x_i^\star$ and the planner prescribes positive daily feed masses D_k with a sum equal to the selected total. Continuous variable f_{ik} is tonnes of parcel i delivered on day k. Eligibility e_{ik} is one only if the material can be supplied that day. Capacity and availability give the following constraints.

$$\sum_k f_{ik} = x_i^\star, \quad \sum_i f_{ik} = D_k, \quad 0 \le f_{ik} \le m_i e_{ik},$$
$$D_k > 0, \quad e_{ik} \in \{0,1\} \text{ is given}, \qquad \sum_k D_k = \sum_i x_i^\star \quad \text{(B5)}$$

Prescribed daily masses make each denominator constant. Daily grade g_{jk} is therefore an affine expression in the flow variables, and any required daily quality band can be imposed directly.

$$g_{jk} = \frac{1}{D_k} \sum_i a_{ij} f_{ik}, \qquad L_{jk} \le g_{jk} \le U_{jk} \quad \text{(B6)}$$

The model assumes that the selected parcel can be divided across days and that the delivery schedule respects all relevant logistics through its eligibility and capacity inputs. Daily mine capacity, crusher throughput and stockpile reclaim capacity are separate physical constraints when they concern different flows. They should not be represented by one unexplained upper bound.

For whole selected blocks, delivery becomes m_i times a binary day assignment. This is the following replacement, not an additional continuous splitting option.

$$f_{ik} = m_i y_{ik}, \quad y_{ik} \in \{0,1\}, \quad \sum_k y_{ik} = v_i^\star \quad \text{(B7)}$$

Exact daily mass equalities may be impossible under block indivisibility, even when total capacity is adequate. A planner must either choose feasible D_k values, allow justified daily tolerances, or represent smaller operational parcels. If daily masses become endogenous while grade is still defined by their exact ratios, the target deviation objective generally ceases to be linear. Rounding an LP solution to blocks does not preserve its mass and quality guarantees.

4.3 Tracking a fixed grade target

Let q_j be a fixed grade target, s_j a positive fixed grade scale and w_j a nonnegative relative priority with sum one. The target may be a contractual set point or the weighted grade of the selected horizon blend. A lower or upper quality limit is not automatically a target. For positive prescribed D_k, absolute deviation has the following linear representation.

$$g_{jk} - q_j = d_{jk}^{+} - d_{jk}^{-}, \qquad d_{jk}^{+}, d_{jk}^{-} \geq 0 \quad \text{(B8)}$$

$$\min F_{\text{target}} = \frac{1}{K} \sum_{k=1}^{K} \sum_j \frac{w_j}{s_j} \left(d_{jk}^{+} + d_{jk}^{-} \right) \quad \text{(B9)}$$

For every attribute with positive objective weight, the minimized sum of nonnegative deviations equals the absolute grade difference. Binary sign indicators and a big M constant are unnecessary. If an attribute has zero weight, its auxiliary deviations need not be tight; its realized grade error should be computed directly when reporting results. Hard daily acceptance bounds remain active independently of the objective.

Normalization determines the tradeoff across attributes. Dividing by an explicitly chosen tolerance scale can make an error of one permitted grade interval comparable across attributes, whereas using raw fractions would allow the largest numerical attribute to dominate. Scales must remain positive and fixed. The chosen targets, scales and weights should be disclosed and examined through sensitivity analysis rather than described as intrinsic properties of the ore.

4.4 Controlling successive day changes

Target tracking and feed smoothness answer different questions. A schedule can remain close to a target but alternate between high and low days, or remain smooth at an unacceptable level. For k = 2,…,K, define nonnegative adjacent change deviations as follows.

$$g_{jk} - g_{j,k-1} = \Delta_{jk}^{+} - \Delta_{jk}^{-}, \quad \Delta_{jk}^{+}, \Delta_{jk}^{-} \geq 0 \quad (k = 2, \ldots, K) \quad \text{(B10)}$$

$$\min F_{\text{change}} = \frac{1}{K-1} \sum_{k=2}^{K} \sum_j \frac{w_j}{s_j} \left(\Delta_{jk}^{+} + \Delta_{jk}^{-} \right) \quad \text{(B11)}$$

The denominator K−1 counts the transitions that are actually included. When K is one, there are no within horizon adjacent transitions, so this objective is omitted. If the last feed grade before day 1 is known, a separate boundary transition can be included and its count documented. Undefined day zero grades must not be introduced implicitly.

The two objectives may be combined using disclosed weights or solved lexicographically, for example minimizing target deviation first and then minimizing adjacent changes within an allowed degradation epsilon. Solver priority and tolerance definitions affect that interpretation [10]. A smooth sequence that violates a hard quality bound remains infeasible. Conversely, minimizing a weighted sum of errors does not guarantee that every individual error is small; maximum deviation constraints may be appropriate when excursions are operationally costly.

4.5 Variable daily masses and a linear proxy

When daily throughput T_k is a decision variable, daily grade is contained constituent mass C_{jk} divided by T_k. The ratio is undefined on empty days and nonlinear when both numerator and denominator depend on the decisions.

$$T_k = \sum_i f_{ik}\,, \quad C_{jk} = \sum_i a_{ij}\, f_{ik}, \quad g_{jk} = \frac{C_{jk}}{T_k} \quad (T_k > 0) \qquad \text{(B12)}$$

A linear alternative controls deviation in contained mass relative to a fixed target, using the following identity and objective.

$$\begin{gathered} \sum_i \left(a_{ij} - q_j\right) f_{ik} = r_{jk}^{+} - r_{jk}^{-}, \quad r_{jk}^{+}, r_{jk}^{-} \geq 0, \\ \min F_{\text{mass}} = \sum_{j,k} \frac{w_j}{s_j} \left(r_{jk}^{+} + r_{jk}^{-}\right) \end{gathered} \qquad \text{(B13)}$$

For positive T_k, the absolute contained mass residual equals T_k times the absolute grade error. Consequently, this objective weights grade deviations by throughput. It is useful when that weighting matches the planning purpose, but it is not the same as the unweighted mean daily grade objective in Equation B9. Empty days incur zero residual and therefore require a separate feed requirement if idling is undesirable. Exact sum of ratio objectives instead require an appropriate nonlinear or specialized reformulation, or a justified discretization whose approximation error is evaluated.

4.6 Precedence and stockpiles

For an indivisible block schedule, a basic end of period predecessor condition can be expressed as follows for every predecessor p of block i. It permits both blocks within the same day and therefore does not prove the within day extraction order.

$$\sum_{t=1}^{k} y_{it} \leq \sum_{t=1}^{k} y_{pt} \quad (p \text{ precedes } i;\ k = 1, \ldots, K) \qquad \text{(B14)}$$

Stricter access can require predecessor completion before the day starts, implemented by shifting the right hand cumulative sum to k−1. Bench geometry, drilling and blasting release, loader access and equipment hours require further constraints when they are decision dependent. For fractional parcel delivery, the binary predecessor expression cannot be copied without defining the extraction and release state.

Stockpiles can decouple extraction from delivery, but they require inventory balances, capacity, reclaim availability and an explicit mixing assumption. If withdrawal grade depends on a variable inventory composition, an exact well mixed reclaim model can introduce additional nonlinearities. This paper does not claim that the simple linear delivery model captures those effects. Discrete event simulation can test timing and queues once a physically consistent material accounting model has been established [6].

5 Audit of the historical formulation

All original equations are preserved in Appendix A and are designated O1-O16 in the discussion, while the revised equations are designated B1-B14. This separation avoids interpreting an unexecuted corrected model as the source of the inherited mine results. Table 2 summarizes the mathematical changes that materially affect the claims.

Table 2. Main formulation issues and their treatment. The original expressions remain available in Appendix A.

Original expression	Issue affecting interpretation	Revised treatment
O2 and O3	Daily and horizon indices are inconsistent; grade denominator must be positive	B1–B4 use optional selected mass and cross multiplied bounds
O4	Maximization contradicts stated minimization; variable denominator is fractional	B5–B9 prescribe positive D_k and minimize linear deviations
O5 and O13	Every candidate block is forced into the schedule	B5 or B7 links scheduling to explicit selected masses or blocks
O8 to O10	Opposite equalities sum to zero; numerator is contained mass rather than grade	B10–B11 use a signed grade change and nonnegative deviations
O11 and O12	Sign inequalities do not repair the zero sum objective	Absolute deviation minimization needs no sign binary or big M
O6 and O14	An upper capacity does not fix a denominator or guarantee feed	B5 fixes D_k; B12–B13 distinguish variable mass and proxy objectives

Original Equation O4 displays maximization of a sum of absolute target errors, while its accompanying prose describes minimization. More fundamentally, its daily denominator depends on binary assignments. Correcting the sign alone would not yield a linear program. Equations B5-B9 state the additional fixed positive daily mass assumption under which target tracking is linear.

Original Equations O8 and O9 define opposite signed differences in contained constituent mass. Their sum is identically zero if the equalities hold. The objective in O10 therefore cannot represent a positive absolute change as written. If both variables were also constrained nonnegative, those equalities would force zero differences, often causing infeasibility. The following big M inequalities do not repair that construction. Equations B10-B11 use one signed equality and two nonnegative deviation variables to obtain the absolute grade change under minimization.

Original Equations O5 and O13 assign every candidate block to exactly one day. This is consistent only if the candidate set has already been restricted to the chosen blocks, or if mining all candidates is mandatory. It conflicts with optional reserve opportunity selection when applied to the full eligible set. The revised stage interface fixes or links the selection explicitly. Feasibility must also be checked against daily capacities and access, rather than inferred from a monthly average.

6 Reproducible synthetic verification

The four checks below are deliberately small enough to inspect independently. They are synthetic, use no confidential mine data and do not estimate operational gains. The supplied script solves the LP cases with SciPy linprog and the integer cases with SciPy milp through HiGHS [8,9], checks solver termination and verifies the expected values numerically. The recorded execution uses SciPy 1.17.0. Enumeration verifies the scheduling objective comparison.

Table 3. Executed synthetic formulation checks. Percentage point quantities are absolute grade differences multiplied by 100.

Check	Assumptions	Executed result
V1	Two 100 t parcels at 40% and 60%; two fixed 100 t days; 50% target	LP mean error 0 pp; whole parcel MILP mean error 10 pp
V2	100 t and 200 t days, both at 50% Fe	Grade change 0 pp; contained Fe change 50 t
V3	Two 100 t parcels at 40% and 60%; capacity 150 t; minimum Fe 55%	LP selects 133.333 t; whole block MILP selects 100 t
V4	Three 100 t parcels at 40%, 50%, 60%; all six orders	Mean target error always 6.667 pp; total adjacent change 20 or 30 pp

6.1 Divisibility changes the attainable target

In V1, two 100 t parcels have grades of 40% and 60%, and two days each require exactly 100 t with a target of 50%. The LP sends 50 t from each parcel to each day and obtains zero target deviation. If each parcel is indivisible and occupies one day, the only schedules deliver 40% and 60% in either order. The mean absolute target error is then ten percentage points, and the single adjacent change is twenty percentage points. This is a difference in feasible operating assumptions, not a solver failure.

6.2 Contained mass is not grade

In V2, a 100 t day and a 200 t day both have 50% Fe. Contained Fe rises from 50 t to 100 t while the grade change is zero. An objective based on the absolute change in contained mass would penalize this schedule despite perfectly constant chemistry. This dimensional counterexample explains why the original adjacent difference equations cannot be interpreted as grade stabilization without a constant mass assumption.

6.3 Optional selection changes the horizon blend

In V3, the same two 100 t parcels compete for a 150 t horizon capacity, and the selected blend must contain at least 55% Fe. The continuous optimum uses 100 t of the 60% parcel and 33.333 t of the 40% parcel, for 133.333 t in total. Whole block selection permits only the 100 t high grade parcel. Requiring both blocks would violate capacity and yield only 50% Fe. This example verifies the separate roles of quality, capacity, divisibility and optional selection.

6.4 Target error and temporal smoothness differ

In V4, three intact 100 t parcels with grades of 40%, 50% and 60% fill three equal mass days. Every permutation has the same mean absolute error of 6.667 percentage points around 50%. The total adjacent variation is twenty percentage points for a monotone order and thirty for orders that alternate through an extreme. Target tracking alone cannot select the smoother schedule. The reported twenty and thirty are transition sums; their mean transition errors are ten and fifteen percentage points, respectively.

Together, these checks establish the intended behaviour of the small reference formulations and identify counterexamples to broader interpretations. They do not establish numerical scalability, field accuracy, robustness to assay error or performance of the historical solver implementation. Such claims need larger benchmark instances and the original or reconstructed industrial datasets.

7 Historical mine illustrations and discussion

7.1 Daily quality profiles

The historical material contains daily quality graphs for two mines over a nominal 30 day horizon. The original panels and their analyte labels are retained in Figures 1 and 2. Table 4 expands the labels; the original inline mathematical notation is also preserved in Appendix A. The paired panels use different vertical ranges. Attributes close to zero in a wide range panel cannot be evaluated reliably from point height alone; each comparison must use the relevant panel scale and analyte definition.

Table 4. Original analyte labels in the mine figures. Analytical basis and units must be taken from the original assay definitions before operational reuse.

Label	Historical meaning	Reporting clarification
FET	Total iron	Confirm elemental Fe and assay mass basis
SIT	Total silica	Confirm whether reported as SiO2
PT	Total phosphorus	Confirm elemental P convention
ALT	Total alumina	Confirm whether reported as Al2O3
MNT	Total manganese	Confirm elemental Mn convention
PPCT	Total loss on ignition	Historical wording is calcination loss

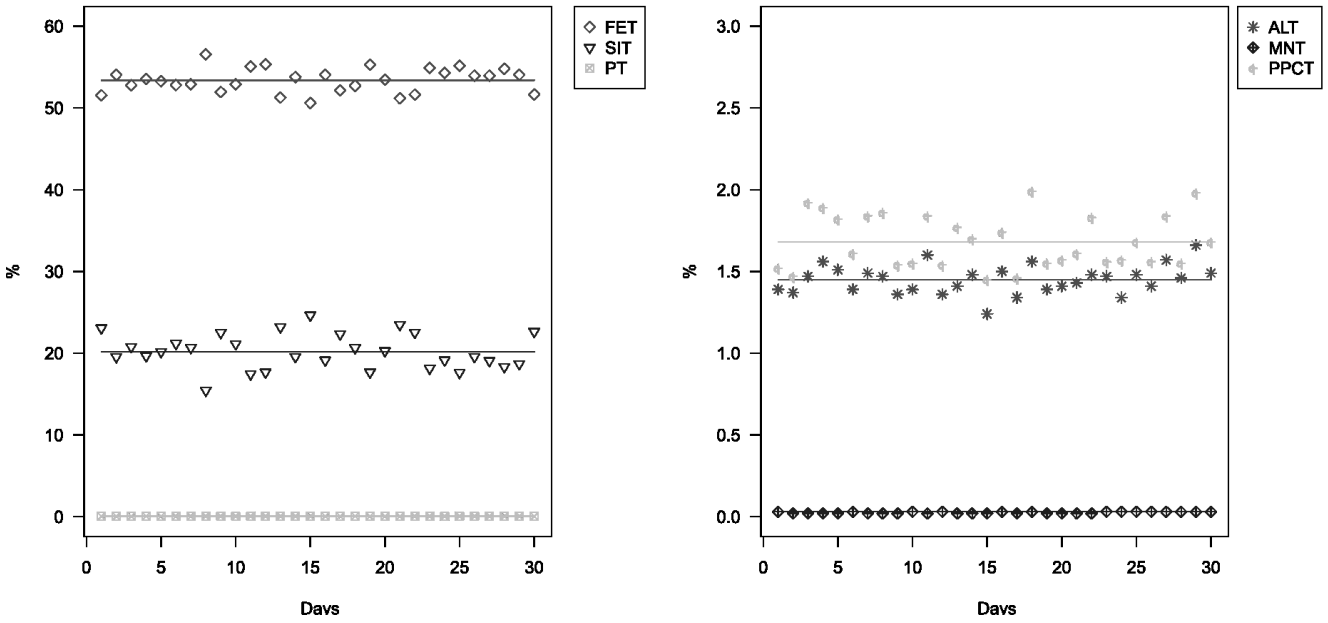


Figure 1. Original daily quality profiles for Mine 1, with both original panels retained. The figure is inherited application material associated with the earlier study [2]; it is not a result of the corrected formulations introduced here.

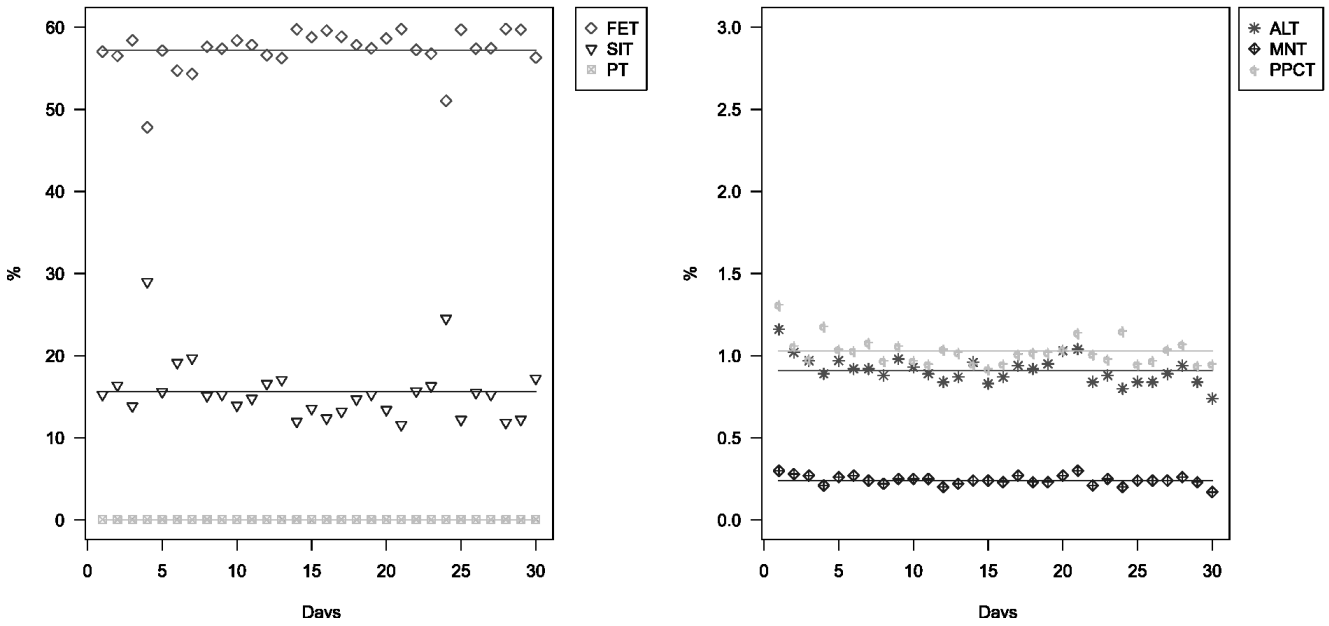


Figure 2. Original daily quality profiles for Mine 2, retaining both original panels and axes. The archive does not contain the underlying daily tonnages or complete solver output needed for an independent performance comparison.

These curves show that the historical application considered multiple attributes and daily ordering. They do not, by themselves, quantify a reduction in variability: the archive lacks a complete unscheduled baseline, daily feed tonnes, hard specifications and reproducible optimized assignments. A plotted line remaining visually near a mean also does not establish grade compliance or plant recovery. The corrected model can be tested on these mines only after the input data and the historical objective definitions have been reconciled.

7.2 Monthly mass figures

Figure 3 retains both original monthly mass panels. Its categories correspond to the analytes and the axes are labelled only as mass in the inherited figure. The mass basis and unit multiplier are not sufficiently documented to convert the bars into ROM tonnes, constituent recovery or plant capacity utilization. Summing bars for Fe, silica and other attributes would also mix constituent quantities that are not independent material streams.

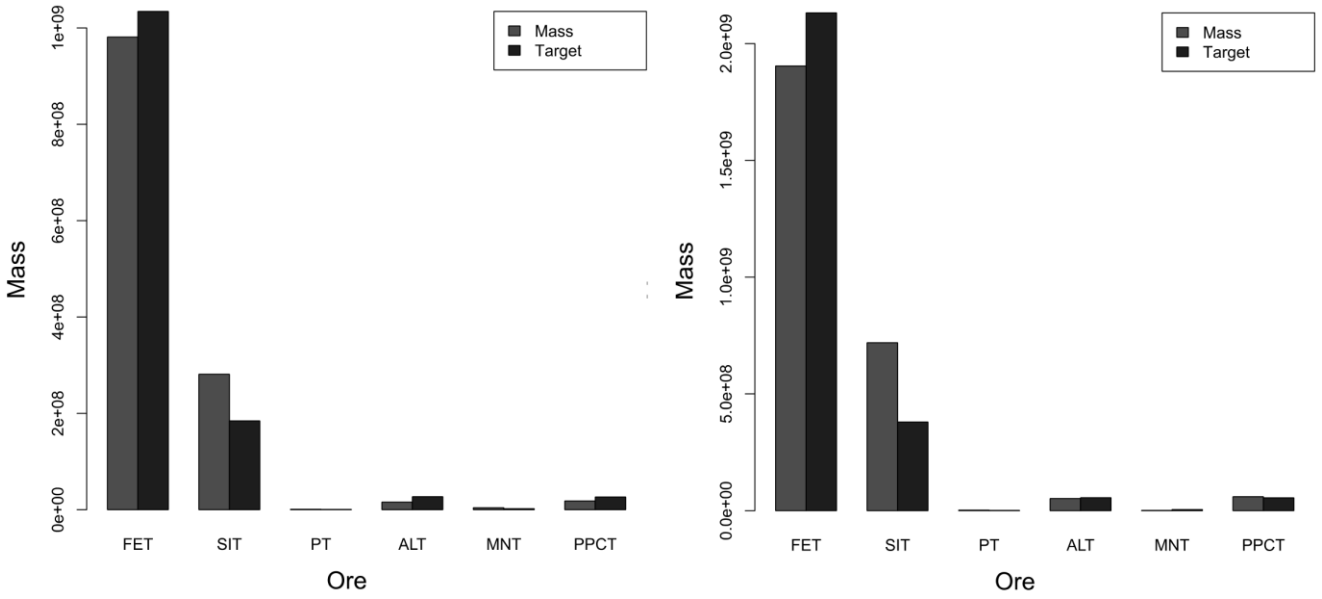


Figure 3. Original monthly mass comparison panels for Mine 1 on the left and Mine 2 on the right. Both panels, legends and displayed values are retained. The source does not establish a complete mass basis and unit definition for interpreting these bars as total ROM throughput.

The appropriate interpretation is qualitative evidence of the output interface used in the historical application. No numerical production gain, reserve increase or economic benefit is recalculated from these images. Such estimates require the schedule, tonnes, assay units, capacity definition and baseline used to generate them. Preserving the illustrations with this qualification is more informative than assigning unsupported precision to their visual bars.

7.3 Operational and computational evaluation

An industrial evaluation should report total selected and delivered tonnes, unmet daily demand, mass weighted horizon grade, unweighted daily grade error, adjacent day grade changes, maximum excursions and off specification tonnes. These quantities answer different questions. A schedule may improve its average error by omitting low throughput days or shift a deviation into a large tonnage day. Daily tonnages and the treatment of empty days must therefore accompany any grade statistic.

Computational reporting should include the number of parcels, days and attributes; continuous and integer variable counts; nonzero coefficients; solver and hardware; time limits; feasible incumbent and dual bound; gap; and any solution repair. Infeasibility analysis should distinguish insufficient quality opportunities, daily capacity conflicts and access restrictions. A solver status indicating a feasible incumbent should not be described as an optimal solution without the corresponding proof or accepted gap.

Assay uncertainty and differences between planned and realized tonnes affect both feasibility and the ranking of schedules. Candidate extensions include robust bands, scenario based grades and rolling updates after reconciled assays. These are proposed extensions, not capabilities demonstrated by the deterministic examples. The daily formulation should first be validated on known historical schedules, then compared with an explicit baseline on independent periods before use in production decisions.

8 Conclusions

Daily iron ore blending admits transparent linear formulations when material is divisible and daily feed masses are prescribed and positive. Whole block assignments introduce integrality. Variable daily masses generally retain fractional grade objectives unless the model uses an explicitly defined proxy or an appropriate specialized method. These conditions are essential to any claim that a blending problem is an LP.

The corrected reference separates optional material selection from day allocation and expresses absolute target errors and adjacent day changes with nonnegative deviations. Four executed synthetic checks show why divisibility, mass basis and objective choice change the solution. They also demonstrate that minimizing contained mass changes is not a general substitute for controlling grade changes.

The historical mine figures remain useful application illustrations, but the available record does not support a new quantified industrial gain or independent reproduction of the earlier solver runs. The resulting contribution is a verifiable methodological reformulation with preserved source material and a clear route to a future reproducible industrial evaluation.

Data provenance and acknowledgements

The supplement provides the synthetic verification code and numerical output. The complete two mine datasets, daily schedule assignments and original Gurobi logs were not supplied. Funding and competing interests were not documented in the files, and their absence is not asserted. The prior publication and inherited material must be disclosed in any submission of this revised methodological work.

References

[1] Newman A M, Rubio E, Caro R, Weintraub A, Eurek K (2010). A review of operations research in mine planning. Interfaces 40(3), 222-245. https://doi.org/10.1287/inte.1090.0492

[2] Navarro Torres V F, Moraes E (2019). Innovative optimization of iron ore blending in daily mine planning from a linear programming model. Revista de Medio Ambiente y Minería 4(2). https://www.scielo.org.bo/scielo.php?lng=es&pid=S2519-53522019000200003&script=sci_arttext

[3] Charnes A, Cooper W W (1962). Programming with linear fractional functionals. Naval Research Logistics Quarterly 9(3-4), 181-186. https://doi.org/10.1002/nav.3800090303

[4] Both C, Dimitrakopoulos R (2020). Joint stochastic short-term production scheduling and fleet management optimization for mining complexes. Optimization and Engineering 21, 1717-1743. https://doi.org/10.1007/s11081-020-09495-x

[5] Manríquez F, González H, Morales N (2023). Short-term open-pit production scheduling optimizing multiple objectives accounting for shovel allocation in stockpiles. Optimization and Engineering 24, 681-707. https://doi.org/10.1007/s11081-021-09701-4

[6] Eustace C, Hynard K (2024). Application of simulation and optimization to support mine plan execution. Mining Metallurgy and Exploration 41, 3789-3800. https://doi.org/10.1007/s42461-024-01118-8

[7] Tang H, Li C, Chen G, Han S, Mavrovouniotis M, Li M, Yuan L (2026). Robust scheduling policy learning for ore-flow blending in dynamic and uncertain environments. Computers and Operations Research 194, 107526. https://doi.org/10.1016/j.cor.2026.107526

[8] SciPy Developers (2026). scipy.optimize.linprog. SciPy 1.17.0 Manual. Accessed 29 October 2026. https://docs.scipy.org/doc/scipy-1.17.0/reference/generated/scipy.optimize.linprog.html

[9] SciPy Developers (2026). scipy.optimize.milp. SciPy 1.17.0 Manual. Accessed 29 October 2026. https://docs.scipy.org/doc/scipy-1.17.0/reference/generated/scipy.optimize.milp.html

[10] Gurobi Optimization (2026). Multiple objectives. Optimizer Reference Manual. Accessed 29 October 2026. https://docs.gurobi.com/projects/optimizer/en/current/features/multiobjective.html

Appendix A Preserved original mathematical formulation

The sixteen displayed equations below are retained exactly as mathematical objects from the supplied manuscript. Their original labels 1-16 are used as O1-O16 in the main text to distinguish them from the corrected B equations. They document the historical formulation and include the inconsistencies analysed in Section 5. No claim is made that these archived expressions constitute an executable, corrected LP.

O1 Original horizon mass objective

$$max \sum_i m_i y_i \qquad (1)$$

O2 Original upper grade constraint

$$\frac{\sum_i t_{ij} m_i y_i}{\sum_i m_i y_{ik}} \leq l_j \; \forall \, j \; \in \; Q^- \qquad (2)$$

O3 Original lower grade constraint

$$\frac{\sum_i t_{ij} m_i y_i}{\sum_i m_i y_i} \geq l_j \; \forall \, j \; \in \; Q^+ \qquad (3)$$

O4 Original target variation objective

$$max \sum_j \frac{w_j}{K} \sum_k \left| \frac{\sum_i t_{ij} m_i y_{ik}}{\sum_i m_i y_{ik}} - l_j \right| \qquad (4)$$

O5 Original all block assignment

$$\sum_k y_{ik} = 1 \; \forall \; i \in B \qquad (5)$$

O6 Original capacity bound

$$\sum_i m_i y_{ik} \leq u \; \forall k \qquad (6)$$

O7 Original binary assignment

$$y_{ik} \in \{0,1\} \; \forall i \in B, k \qquad (7)$$

O8 Original positive change equality

$$y_{jk}^{+} = \sum_i t_{ij} m_i y_{ik} - \sum_i t_{ij} m_i y_{ik-1} \; \forall j, k \qquad (8)$$

O9 Original negative change equality

$$y_{jk}^{-} = \sum_i t_{ij} m_i y_{ik-1} - \sum_i t_{ij} m_i y_{ik} \; \forall j, k \qquad (9)$$

O10 Original adjacent variation objective

$$min \sum_j \frac{w_j}{K} \sum_k \left(y_{jk}^{+} + y_{jk}^{-} \right) \qquad (10)$$

O11 Original sign inequality

$$y_{jk}^{-} \geq -M \left(1 - z_{jk} \right) \forall j, k \qquad (11)$$

O12 Original sign inequality

$$y_{jk}^{+} \geq -Mz_{jk}\ \forall j,k \quad (12)$$

O13 Original all block assignment

$$\sum_k y_{ik} = 1\ \forall\ i \in B \quad (13)$$

O14 Original capacity bound

$$\sum_i m_i y_{ik} \leq u\ \forall k \quad (14)$$

O15 Original assignment domain

$$y_{ik} \in \{0,1\}\ \forall i \in B \quad (15)$$

O16 Original sign indicator domain

$$z_{jk} \in \{0,1\}\ \forall j,k \quad (16)$$

Original analyte notation

FET: total iron SIT: total silica

PT: total phosphorus ALT: total alumina

MNT: total manganese $PPCT$: total calcination loss