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Integrated models of simulation and optimization for open pit mining planning

Paulo Lopes
https://linktr.ee/pftlopes

Abstract

Short term mine plans must coordinate heterogeneous loading and haulage equipment while maintaining material availability, plant feed and product quality. This study examines an integrated decision support architecture in which mixed integer linear programming allocates equipment and trips and discrete event simulation evaluates execution under queues, failures and interruptions. The historical case concerns an August 2016 iron ore plan in Brazil with seven mining regions, two processing plants, three ore stockpiles and 34 trucks. A revised reference formulation makes capacity units, material conservation, size dependent quality and lexicographic priorities explicit. An independent arithmetic audit of the original embedded chart data compares three 29 day scenarios without rerunning the simulator. Total movement reaches 2.449, 2.551 and 2.635 million tonnes, respectively. The third scenario moves 7.60% more material than the first, but the configurations are not a nested fleet reduction experiment. Daily ore production remains variable, and only 46.2-50.0% of the available dry route grade records and 16.7-38.5% of wet route records lie within the plotted iron target bands. Conservation checks hold for the cached series; they do not establish operational validation. The contribution is a reproducible reanalysis and a clarified modelling framework that identifies where scheduling, quality control and simulation evidence require stronger support before operational deployment.

Keywords: short term mine planning; mixed integer linear programming; discrete event simulation; blending; fleet allocation; lexicographic optimization

1 Introduction

Production plans connect a geological inventory to a sequence of executable material movements. A plan can satisfy aggregate tonnes and average grades while failing during execution because the required loading face is unavailable, a truck class cannot use a destination, or queues consume the available shift. These interactions make the distinction between an optimized allocation and an executable schedule central to short term open pit planning. The broader operations research literature treats mine planning as a connected set of extraction, processing and resource allocation decisions [1].

This paper studies a decision support system called Simulavra. A mixed integer linear programming model assigns loading equipment and truck trips to available minimum mining units and destinations. A discrete event simulator then executes the allocation, tracks interruptions and requests a new optimization when the current plan loses relevance. Figure 1 retains the original activity cycle representation, including competition for loading and unloading resources.

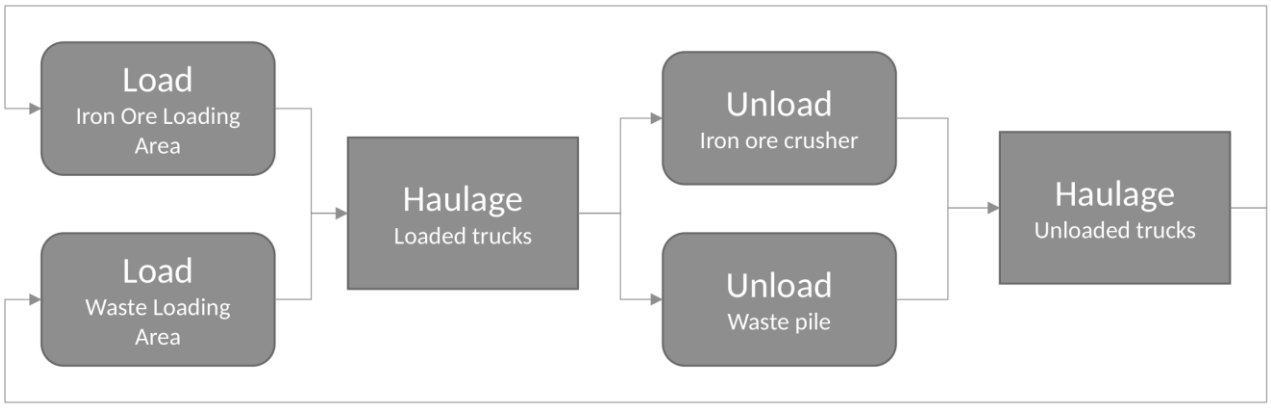


Figure 1. Original activity cycle diagram for loading, haulage and unloading. Source: supplied historical manuscript. The diagram describes the interaction structure rather than a validated capacity estimate.

The original case provides equipment specifications and numerical chart caches but does not provide the complete Arena model, optimizer code, dispatch observations or random seeds. The present contribution therefore has two explicit components: a clarified mathematical reference model and a reproducible audit of the historical numerical evidence. The revised equations are not represented as the equations that generated the historical runs. Appendix A preserves the original notation and all original mathematical objects so that the changes remain traceable.

2 Technical context

Concurrent simulation and optimization have a long history in operational mine planning. Fioroni et al. [2] coupled these techniques to address production and blending decisions. Figure 2 preserves the architecture recorded in the supplied manuscript; Figures 4 and 5 later retain the cited phosphorus examples from that earlier literature. These illustrations motivate the approach but are separate from the Brazilian case analysed here. Table 1 identifies the implementation scope claimed in the historical comparison without treating that comparison as a performance benchmark.

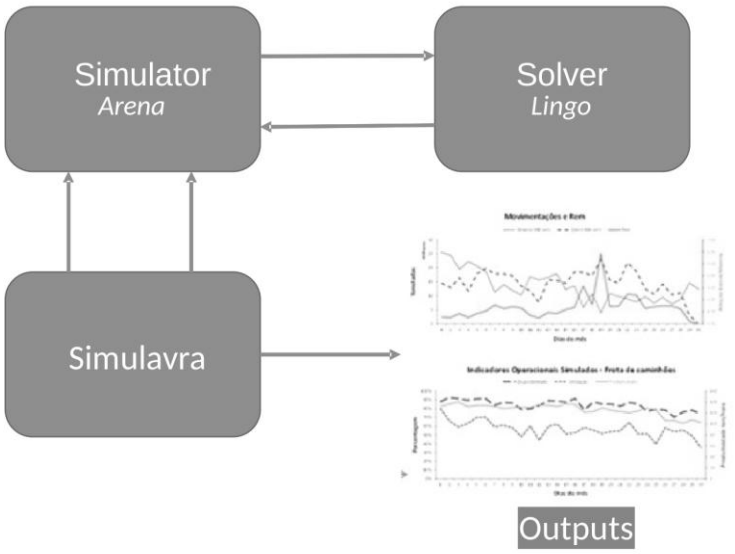


Figure 2. Original information exchange architecture linking the planning interface, optimization model and simulator. Source: supplied historical manuscript.

Queueing is one reason that a deterministic allocation can overestimate realized production. Coelho et al. [3] examined the gap between optimization and simulation under different congestion and operating time variability settings. Figure 3 is retained with its original attribution. Its observations concern that external experiment and cannot serve as calibration or validation of the present mine.

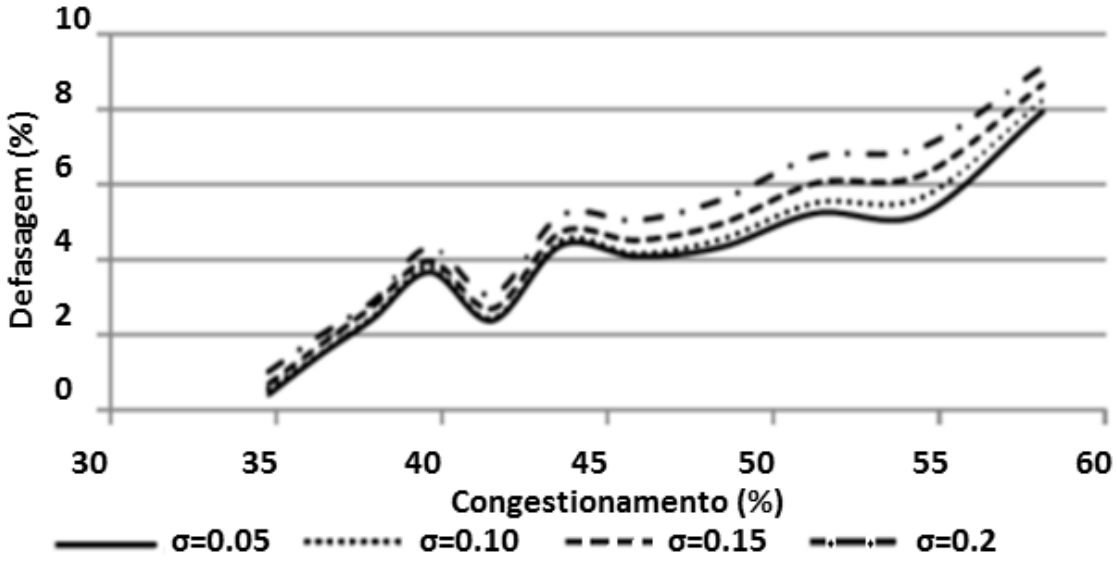


Figure 3. Original comparison of optimization and simulation under congestion and cycle time variability, reproduced in the supplied manuscript from Coelho et al. [3]. These are external literature results, not additional runs of the present case.

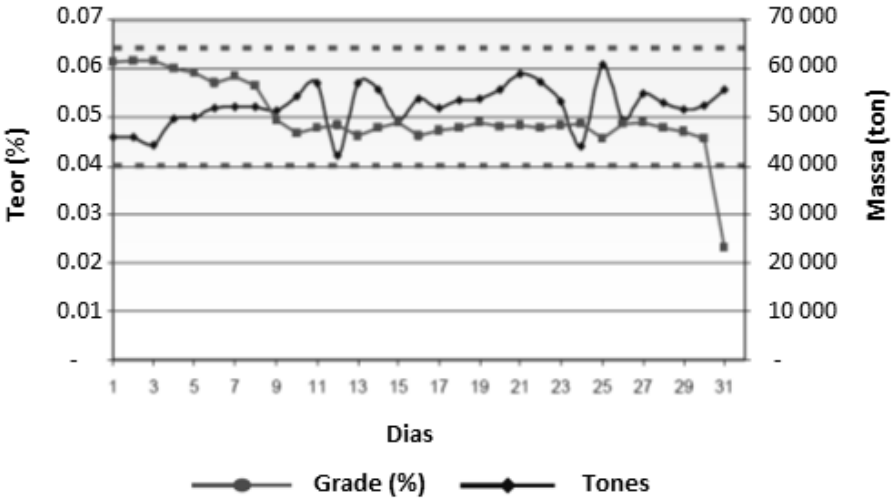


Figure 4. Phosphorus variation with grade restrictions in the earlier concurrent simulation and optimization study [2]. The original figure is retained as literature context.

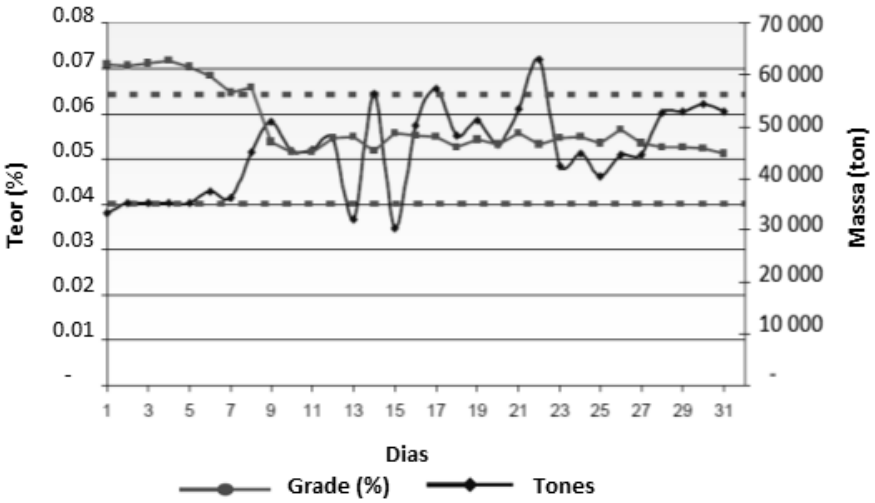


Figure 5. Phosphorus variation without grade restrictions in the earlier study [2]. Comparison with Figure 4 concerns that study only.

Table 1. Historical comparison of model scope recorded in the supplied manuscript. The comparison is descriptive and does not establish superiority over [2].

Aspect	Fioroni et al 2008	Historical case scope
Material classes	Ore and waste	Itabirite, canga, hematite and waste
Spatial unit	Mining area	MMU and material portions
Simulation activities	Loading, haulage and discharge	Also drilling, blasting and stockpile logic
Objectives	Goal programming	Reported sequential priorities
Mining sequence	Not explicit in the archived comparison	MMU availability released by the simulator

More recent work addresses the coupling of extraction, fleet allocation and downstream material handling explicitly. Both and Dimitrakopoulos [4] jointly consider stochastic short term scheduling and fleet management. Manríquez et al. [5] integrate multiple scheduling objectives with shovel allocation in stockpiles. These studies show why an objective hierarchy and a material flow model need to be specified together. They also provide a more appropriate methodological context than a comparison based only on software products.

Eustace and Hynard [6] discuss simulation and optimization in support of mine plan execution. Pathan et al. [7] connect shovel and truck performance with rockmass parameters, reminding planners that a single mean productivity does not fully describe loading behaviour. Tang et al. [8] study learned scheduling policies for dynamic ore flow blending under uncertainty. That development broadens the available control approaches; no comparison with those algorithms is possible from the archived runs used here. The present architecture remains useful as an interpretable reference against which new policies could be tested on the same state, routes, quality definitions and computational budget.

3 Mining system and material representation

3.1 Geological and operational units

The case separates mining regions, benches, minimum mining units and material portions within each unit. The minimum mining unit, abbreviated MMU, is the smallest face area considered for equipment allocation. The historical manuscript reports a representative footprint of 30 m by 50 m and a mean mass of 1,500 t. Bench thickness and the volume basis associated with that mass are not supplied. Consequently, the footprint cannot independently verify tonnage; an implementation must reconcile volume, density, dilution and the actual material portion definition before constructing the optimization inventory.

Iron ore quality can vary by particle size as well as by location. Figure 6 retains the particle size illustration. Figure 7 illustrates regional access, and Figure 8 shows the within bench extraction order. A region being represented separately does not establish physical independence: shared roads, geotechnical restrictions and equipment transfers can still couple regions.

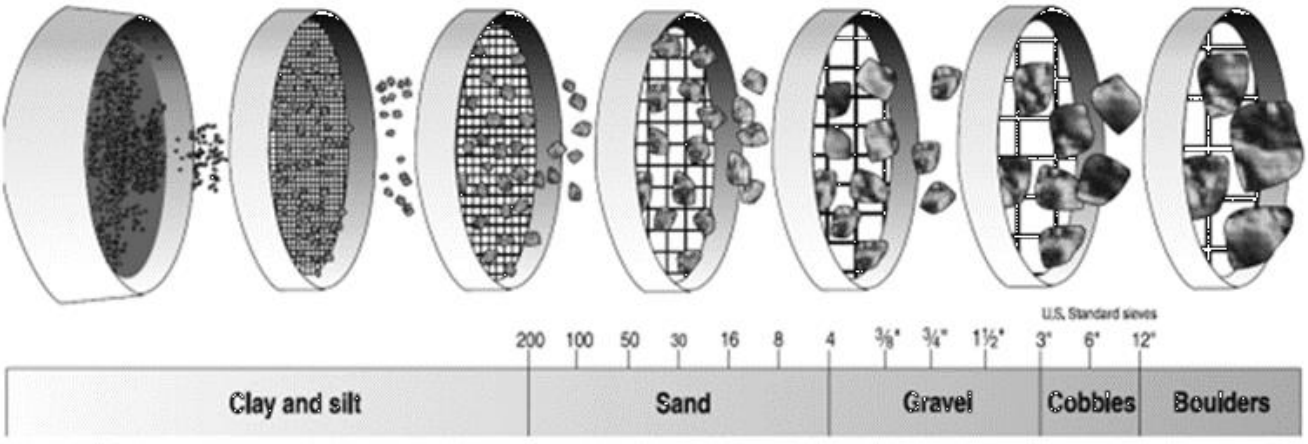


Figure 6. Original illustration of particle size ranges. Quality by size range requires both the mass fraction of the range and the constituent grade within that fraction.

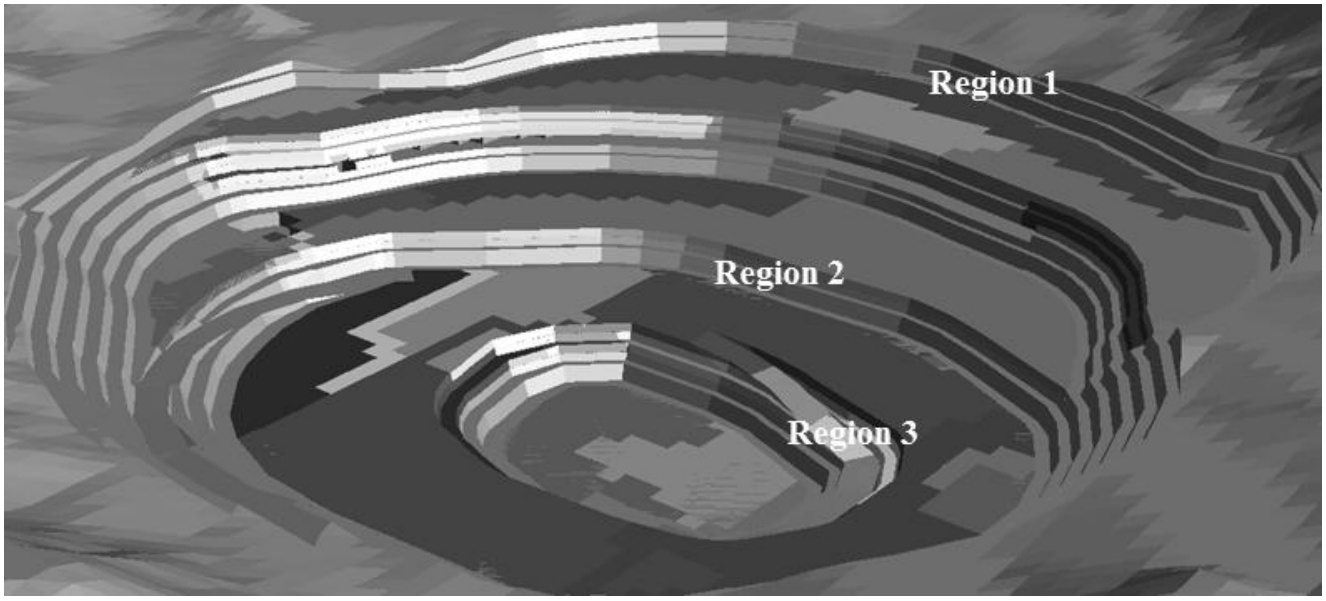


Figure 7. Original illustration of mining regions and extraction directions. The supplied manuscript attributes the illustration to MineSight material from 2012; a complete bibliographic record was not available. Retained as an archival illustration.

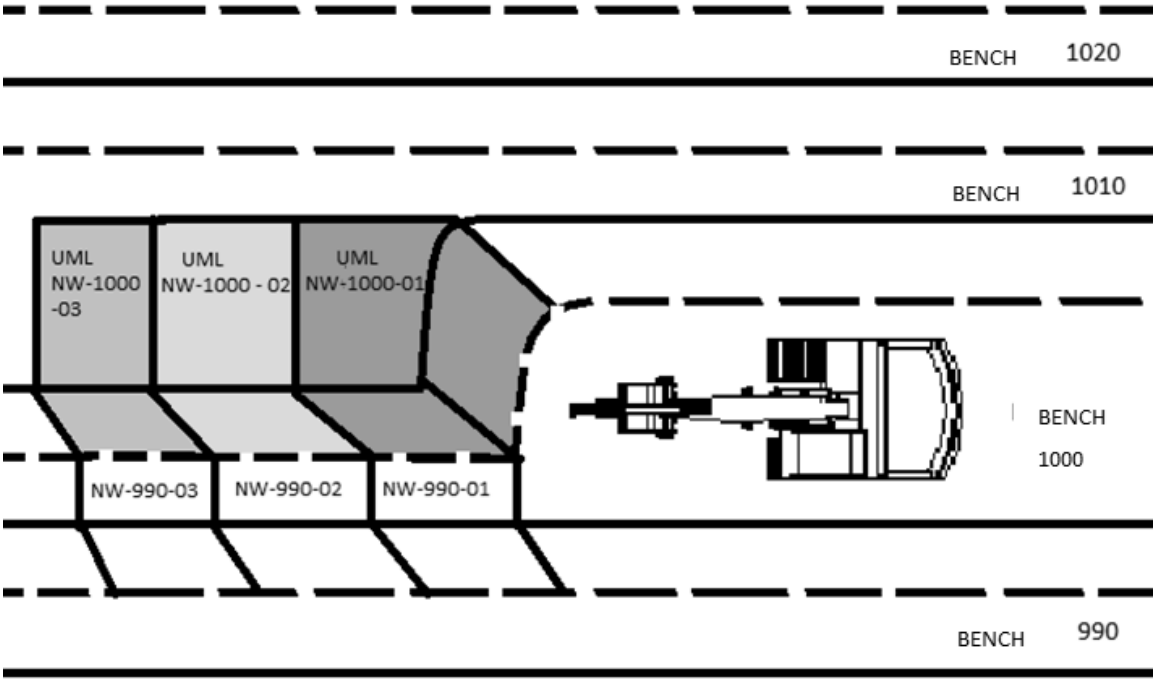


Figure 8. Original MMU and bench sequence example. The simulator releases accessible units to the optimizer; the drawing is a schematic, not a complete pit precedence graph.

Each MMU can contain several ore types and waste. Figure 9 preserves the separation into material portions. Allocation occurs at the parent MMU, while the conservation and chemistry calculations must use the individual portions. Combining these levels without an explicit mapping could allocate the same face several times or consume material that is no longer available.

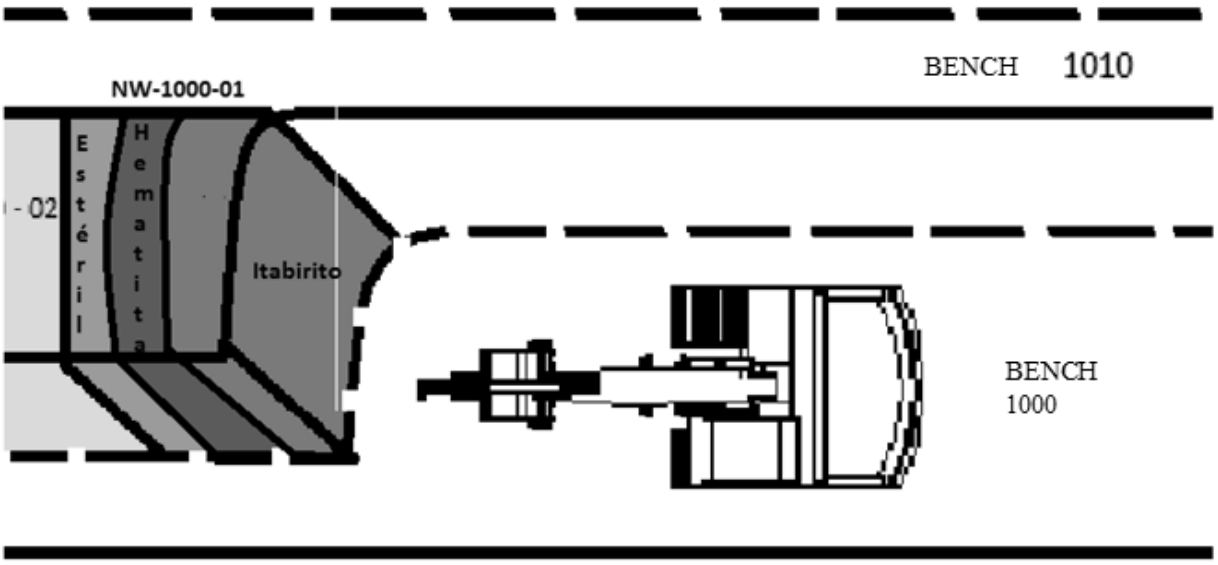


Figure 9. Original MMU subdivision into material portions, including itabirite, hematite and waste.

3.2 Destinations and equipment compatibility

Material is sent to processing plants, temporary ore stockpiles or a waste destination, as illustrated in Figure 10. The historical case includes seven mining regions, two plants and three ore stockpiles used when plant interruptions require diversion. In the recorded compatibility rules, CAT775 and Scania trucks are loaded by CAT992 or CAT390 units; CAT785 and MT33000 trucks use L1350, RH170 or CAT7295 units. These rules are inputs to the case and should not be generalized to every equipment configuration.

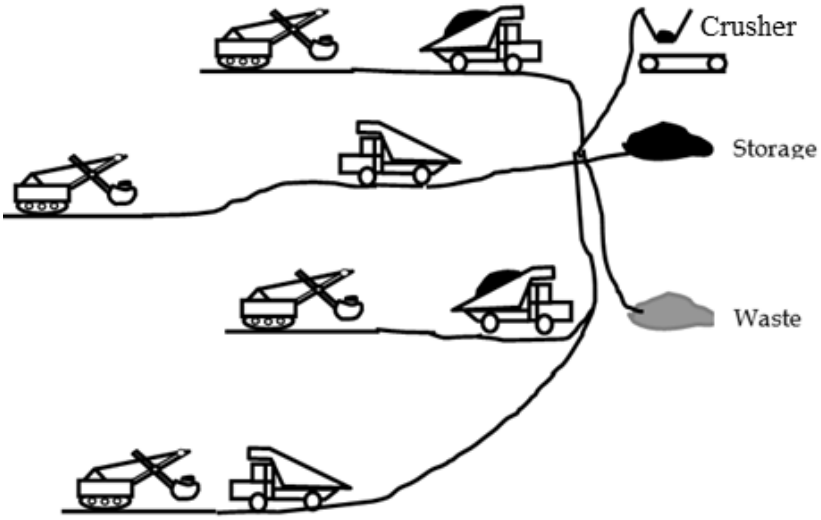


Figure 10. Original material flow representation between MMUs and destinations. In the revised formulation, incompatible combinations are excluded from the allowed route set.

Table 2. Original loading equipment specifications. The last column is bucket volume, not truck payload. Productivities are the historical model inputs rather than independently measured capacities in this revision.

Loader type	Productivity (t/h)	Bucket volume (m³)
CAT390	457	6.0
CAT992	626	11.0
CAT7295	1450	19.0
L1350	1200	19.9
RH170	1100	21.0

Table 3. Original haulage fleet. The four classes comprise 34 trucks. Their summed nominal payload is 2,776 t per simultaneous full fleet load and is not an hourly throughput.

Truck class	Nominal payload (t)	Number of trucks
CAT775	64	13
CAT785	135	10
MT33000	135	2
Scania	36	9

4 Reference optimization formulation

4.1 Sets variables and units

The reference model describes one residual planning interval of H calendar hours. Let I be the currently accessible material portions, U their parent MMUs, J the individual loaders, K the truck classes and D the destinations. The parent of portion i is u(i). The allowed set A contains only compatible combinations (i,j,k,d). This set incorporates material acceptance, truck and loader matching and route availability. Closed routes are omitted. Portions requiring drilling or blasting are unavailable until released by the simulator.

Table 4 defines the principal quantities. Grades are fractions on a common mass basis; displayed percentages are obtained by multiplying by 100. The model assumes that the mass represented by a trip is known for the route and material. If partial final loads are needed, a bounded trip load variable or an explicit partial load rule is required. The equality below otherwise represents full nominal payload trips.

Table 4. Principal notation for the revised residual interval model.

Symbol	Meaning	Unit or domain
m_i, q_{ik}	Remaining portion mass and nominal truck trip mass	t
n_{ijkd}, f_{ijkd}	Trips and transported mass on an allowed route	Integer trips; t
z_{uj}, α_j	Loader assignment and current loader availability	Binary
H, P_j, τ_{ijkd}	Residual time, loader rate and complete cycle time	h; t/h; h/trip
a_j, a_k, N_k	Operating availability fractions and available truck count	[0,1]; integer count
M_d, Q_{dt}	Interval delivery and constituent mass at destination	t
a_{it}, b_{ih}, a_{iht}	Whole portion grade, size fraction and within size grade	Mass fractions
delta, e	Delivery deviations and contained mass quality deviations	t; constituent t

$$f_{ijkd} = q_{ik} n_{ijkd}, \quad n_{ijkd} \in \mathbb{Z}_{\geq 0}, \quad f_{ijkd} \geq 0 \quad \big((i,j,k,d) \in A\big) \qquad \text{(C1)}$$

Here n is the integer number of trips and f is the corresponding transported mass. Summation over A is understood in subsequent equations, and variables for incompatible tuples do not exist. A tight trip bound $\overline{N}_{ijkd}$ is computed from available material and equipment time, rather than assigned an arbitrary large constant. The assignment variable z is binary, alpha is a binary loader availability indicator, and L_u is the number of loaders physically permitted at an MMU.

$$\begin{gathered} \textstyle\sum_u z_{uj} \leq \alpha_j, \quad \sum_j z_{uj} \leq L_u, \quad z_{uj} \in \{0,1\}, \\ n_{ijkd} \leq \overline{N}_{ijkd} z_{u(i)j} \quad \big((i,j,k,d) \in A\big) \end{gathered} \qquad \text{(C2)}$$

$$\textstyle\sum_{j,k,d} f_{ijkd} \leq m_i \quad (i \in I) \qquad \text{(C3)}$$

These constraints prevent a loader from serving multiple faces simultaneously in the interval and prevent outgoing tonnes from exceeding the remaining portion mass. Relocation during an interval requires additional time indexed assignment and movement constraints. The compact assignment model does not itself describe those moves.

4.2 Equipment time and destination capacity

Let a_j and a_k be available operating fractions within the residual calendar interval, excluding time already consumed. P_j is loader throughput in t/h, tau is the expected complete truck cycle time in hours per trip, and N_k is the current number of available trucks in class k. The following necessary capacity conditions use loader tonnes and fleet hours separately.

$$\textstyle\sum_{i,k,d} f_{ijkd} \leq P_j H a_j \quad (j \in J) \qquad \text{(C4)}$$

$$\textstyle\sum_{i,j,d} \tau_{ijkd}\, n_{ijkd} \leq N_k H a_k \quad (k \in K) \qquad \text{(C5)}$$

Expected cycle times should be defined consistently: a travel only time omits loading and discharge, whereas a complete cycle may already include average service and delay. Delays must not be subtracted twice through both availability and cycle time. Queue growth, individual truck arrival order and simultaneous road occupancy remain simulation responsibilities. Meeting these aggregate bounds does not prove that every trip can be scheduled without conflict.

For destination d, M_d is interval delivery, T_d is requested delivery and C_d is receiving capacity in t/h. Positive and negative delivery deviations are measured in tonnes. A mandatory minimum can be imposed as a hard bound where justified, but declaring every requested tonne mandatory may make an otherwise informative diagnostic model infeasible.

$$\begin{aligned} &M_d = \textstyle\sum_{i,j,k} f_{ijkd}\,, \quad M_d - T_d = \delta_d^+ - \delta_d^-, \\ &0 \le M_d \le C_d H, \quad \delta_d^+, \delta_d^- \ge 0 \quad (d \in D) \end{aligned} \qquad \text{(C6)}$$

4.3 Chemical quality and particle size

Quality constraints are written as contained mass balances. For a blend accumulated over the interval, M_d^0 and Q_{dt}^0 are its initial mass and contained constituent mass. The fraction grade a_{it} is assumed to use the same moisture and analytical basis as the specification. Q is conserved contained mass, not a percentage. With lower and upper target fractions L_{dt} and U_{dt}, the following formulation is linear.

$$B_d = M_d^0 + M_d, \quad Q_{dt} = Q_{dt}^0 + \textstyle\sum_{i,j,k} a_{it}\, f_{ijkd} \qquad \text{(C7)}$$

$$L_{dt} B_d - e_{dt}^- \le Q_{dt} \le U_{dt} B_d + e_{dt}^+, \quad e_{dt}^-, e_{dt}^+ \ge 0 \qquad \text{(C8)}$$

The nonnegative e variables relax operational target bands in units of contained constituent tonnes. Essential acceptance limits should instead be enforced without these slacks. A zero mass destination has no defined average grade; a delivery requirement or an operating indicator must establish positive mass before reporting a grade. For a continuously reclaimed stockpile, these balances describe neither mixing kinetics nor an arbitrary sequence of withdrawals. Sequential inventory and reclaim equations are needed when the within interval stockpile state affects the delivered blend.

For size range h, b_{ih} is the mass fraction of portion i in that range and a_{iht} is its within range constituent fraction. The correct balance weights chemistry by both size fraction and portion tonnes. Existing stockpile mass and constituent inventory are also specific to the size range.

$$\begin{aligned} M_{dh} &= M_{dh}^0 + \textstyle\sum_{i,j,k} b_{ih}\, f_{ijkd}, \\ Q_{dht} &= Q_{dht}^0 + \textstyle\sum_{i,j,k} a_{iht}\, b_{ih} f_{ijkd} \end{aligned} \qquad \text{(C9)}$$

The band constraints in Equation C8 can then be applied to Q_{dht} and M_{dh}. For an exhaustive, nonoverlapping size partition, the b_{ih} fractions sum to one for each portion; an incomplete partition requires an explicit unclassified fraction. This construction prevents the common error of averaging within range grades using the whole ROM tonnes as weights. Moisture, loss on ignition and oxide reporting conventions must be reconciled before using the resulting coefficients.

4.4 Stripping and lexicographic priorities

Let O and W be the total new ore and waste extraction and let O^0 and W^0 be the extraction already credited to the relevant stripping account. A required minimum cumulative waste to ore ratio r can be enforced by cross multiplication. This constraint is meaningful only when the ratio account and its time horizon have been defined and ore tonnes are positive.

$$W^0 + W \ge r(O^0 + O), \qquad O^0 + O \ge O_{\min} > 0 \qquad \text{(C10)}$$

The historical account states four priorities: minimize plant delivery deviations; minimize quality deviations without degrading the first objective; maximize waste extraction; and favour extraction

from regions with larger remaining inventories. A single unscaled sum of production, grade and stripping deviations does not reproduce this hierarchy. The original aggregate objective is preserved in Appendix A as a historical record.

The clarified reference uses the following vector. Weights pi are nonnegative delivery preferences; omega are nonnegative quality preferences; s_{dt} are fixed positive reference contained masses used for normalization. R_r is tonnes extracted from region r and β_r is a fixed weight computed from its remaining inventory at the optimization snapshot.

$$f_1 = \sum_d \pi_d \,(\delta_d^{+} + \delta_d^{-}), \quad f_2 = \sum_{d,t} \frac{\omega_{dt}}{s_{dt}} (e_{dt}^{+} + e_{dt}^{-}), \\ f_3 = -W, \quad f_4 = -\sum_r \beta_r \, R_r, \qquad \operatorname{lexmin}(f_1, f_2, f_3, f_4) \tag{C11}$$

Each stage is solved in order with the previous minimization oriented objectives bounded as follows. The tolerance ϵ_l has the units of f_l and is reported with the solution. In particular, fixing the third stage means preserving waste extraction, not fixing an unrelated stripping deviation. Hard stripping constraints remain active throughout.

$$\min f_s \quad \text{subject to } f_\ell \le f_\ell^{\star} + \epsilon_\ell \quad (\ell < s), \qquad \epsilon_\ell \ge 0 \tag{C12}$$

A time limited feasible incumbent is not a proven optimum. The implementation must report the incumbent, bound, gap and termination status for each stage and identify whether later stages preserve an optimum or an incumbent. Hierarchical solver priorities and tolerance semantics also require attention [12]. The historical archive does not contain those logs, so no claim of certified optimality or improved convergence is made for the reported mine runs.

5 Simulation and feedback control

5.1 Event logic

Figure 11 retains the original interdependencies between unit operations. The historical implementation follows the simulation and allocation work described by Martins [10] and used Arena 14.7 for discrete event simulation and LINGO for mathematical programming, with Simulavra handling inputs and outputs. Trucks are entities competing for loading, roads and discharge resources. Shift changes, maintenance, weather interruptions and waiting for blasting alter the available resources and the time remaining for execution.

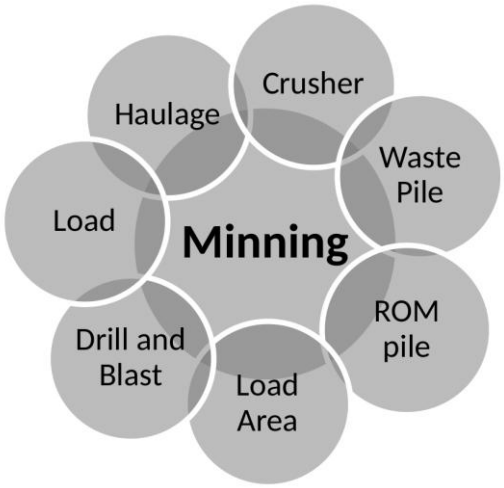


Figure 11. Original logical relationships between the mining unit operations represented in the simulator.

The haulage cycle comprises loading, loaded travel, discharge and empty return. Figure 12 retains the transport schematic attributed to Quevedo [11]. Figures 13 and 14 preserve the implemented truck and loader logic. The historical text reports dispatch derived empirical distributions but gives no fitted parameters, sample sizes or goodness of fit results. Their inclusion in a simulator therefore cannot by itself establish statistical adequacy.

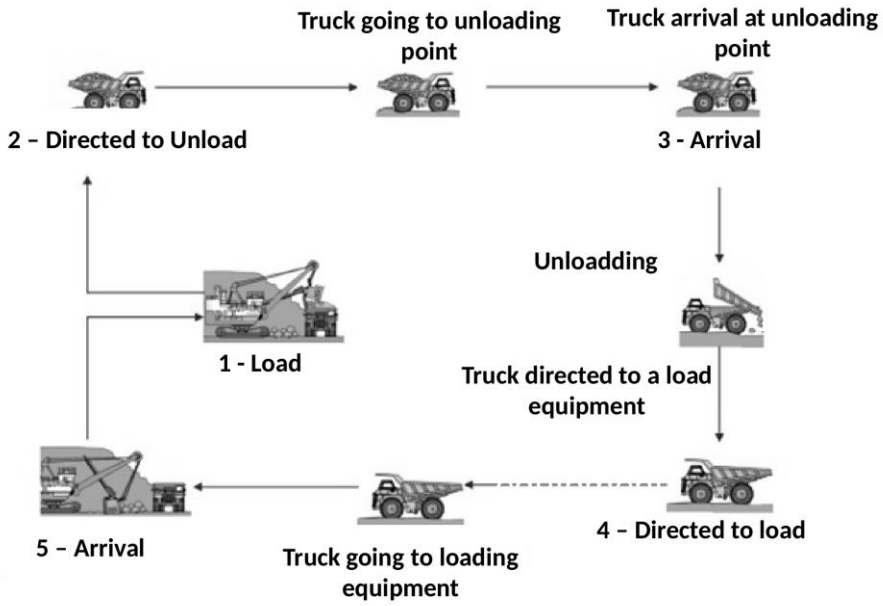


Figure 12. Original loading and transport cycle schematic, attributed in the supplied manuscript to Quevedo [11].

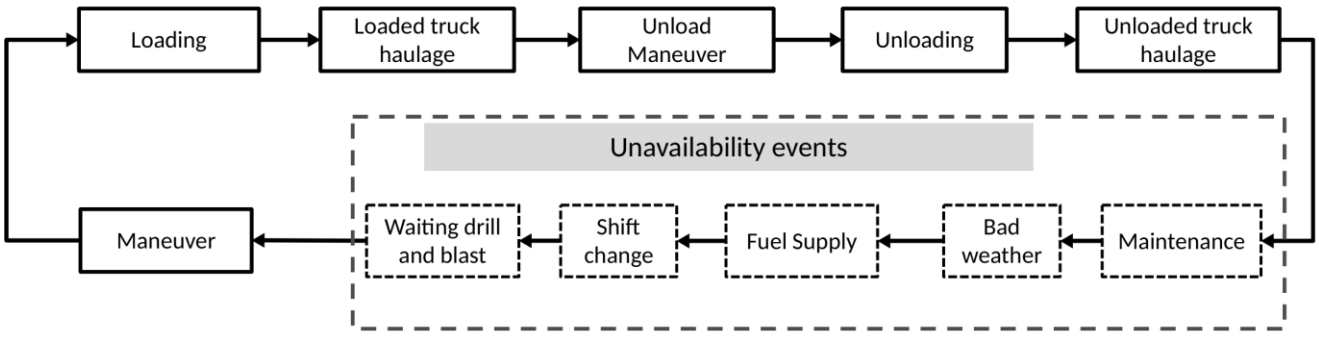


Figure 13. Original truck event logic as represented in the historical Arena implementation.

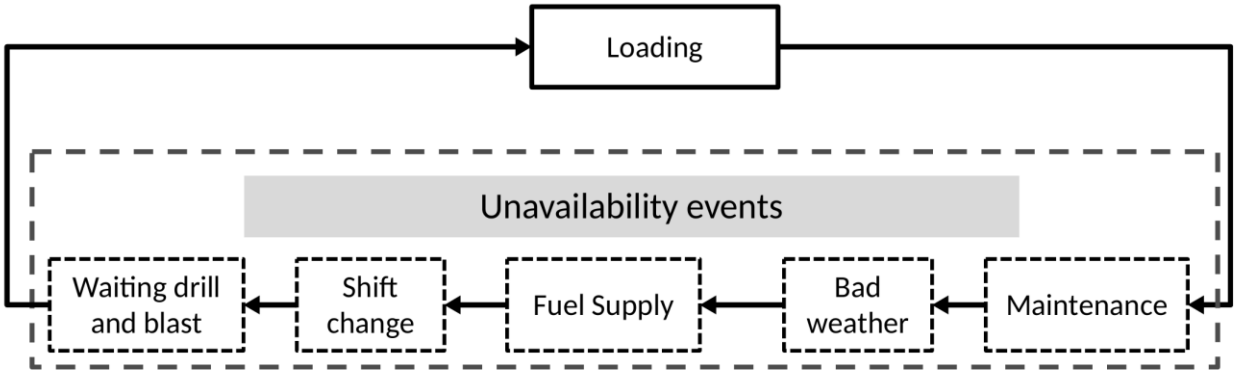


Figure 14. Original loading equipment event logic in the historical implementation.

Correlated interruptions require particular care. Rain can affect several roads and fleets simultaneously, while a crusher outage diverts material to stockpiles and changes future blending opportunities. Drawing independent delays for each resource can understate such system effects. The archive is insufficient to determine how these dependencies were treated, and the revised discussion does not assume that they were modelled.

5.2 Reoptimization and state conservation

The historical reoptimization triggers are completion of 75% of scheduled trips, 60 minutes without a truck trip, or 60% of trucks waiting for loader allocation. They are reported heuristic settings, not calibrated universal thresholds. A practical controller should also detect a newly inaccessible face, route closure or infeasible quality forecast. Event priorities and a minimum decision interval should prevent multiple simultaneous triggers from issuing inconsistent plans.

At a trigger, the simulator must export an atomic state snapshot: remaining tonnes by portion, accessible MMUs, loader positions and outages, truck locations and trips in progress, destination inventories, credited deliveries and residual time. A trip that has already loaded material must retain ownership of that material while it is in transit. Reoptimization must not return those tonnes to the available face inventory or count their delivery twice. Likewise, completed trips are removed from the residual demand rather than requested again.

The historical stopping conditions are exhaustion of planned mass, completion of the simulation horizon, or all available faces requiring drilling and blasting. The third condition can be a temporary operational blockage rather than permanent exhaustion. The controller should report the cause of

termination and whether a future release event could resume production. A physically impossible optimization request should lead to a diagnostic and a documented fallback allocation, rather than uncontrolled resubmission of the same problem.

5.3 Verification and validation design

Verification tests whether the implementation follows its stated logic; validation concerns adequacy for a defined real use [9]. The audit below verifies arithmetic relationships in archived outputs. Field validation would additionally compare observed and simulated throughput, cycle time distributions, queues, equipment availability and delivered chemistry over independent periods. Aggregate monthly agreement can conceal daily misallocation or grade failures.

For a future comparative experiment, independent replications should use recorded seeds and common random numbers across scenarios where event coupling permits. Differences in total movement, operating cost and off specification tonnes should be reported with confidence intervals, together with the replication count and the handling of terminated runs. The 29 day mine plan is a finite horizon depletion experiment, so any warm up removal must be justified against the actual initial inventories and equipment state. It should not be copied from a steady state queueing study.

6 Historical case and audit method

6.1 Scenario definition

The original computer was a Windows 10 system with an i7 processor at 2.3 GHz and 16 GB RAM. These details describe the historical environment, not a contemporary performance benchmark. The source states one complete 29 day simulation per scenario. It supplies neither replications nor runtimes, random seeds, confidence intervals or optimization gaps. The horizon was shortened from 30 days because the available monthly mass would be exhausted near the end. This choice must be considered when interpreting terminal production and remaining inventory.

Table 5. Original loader quantities and derived nominal aggregate capacity. The three scenarios contain 12, 11 and 10 individual loaders, respectively, across five equipment types.

Loader type	Scenario 1	Scenario 2	Scenario 3
CAT390	4	4	3
CAT992	4	4	3
CAT7295	1	0	1
L1350	2	2	2
RH170	1	1	1
Total loaders	12	11	10
Nominal total t/h	9,282	7,832	8,199

Scenario 2 removes the CAT7295 from Scenario 1. Scenario 3 restores that large excavator and removes one CAT390 and one CAT992 relative to Scenario 1. Thus Scenario 3 is not Scenario 2 with one additional loader removed. Its nominal capacity is higher than Scenario 2 despite its lower equipment count. Any explanation based solely on fewer machines is consequently incomplete.

6.2 Recoverable numerical evidence

The audit extracts cached series from the seven native Word charts underlying Figures 15-21. It preserves point indices and missing observations. For the first three scatter charts, the source package contains y values but no explicit x values; the revised charts assign the documented day indices 1-29 so that the original series display correctly. No y value is changed. The supplement includes the extracted data, calculations and verification script.

For cumulative ore, daily production is computed by first differences with zero production credited before day 1. The coefficient of variation uses the sample standard deviation divided by the 29 day mean. Grade compliance counts only the numerical records present in the cache and uses inclusive plotted bounds. It is an unweighted record proportion, since stockpile tonnes, duration and complete laboratory records are unavailable. Missing grade records are not replaced by zero and are not counted as compliant.

7 Results and discussion

7.1 Movement and daily variability

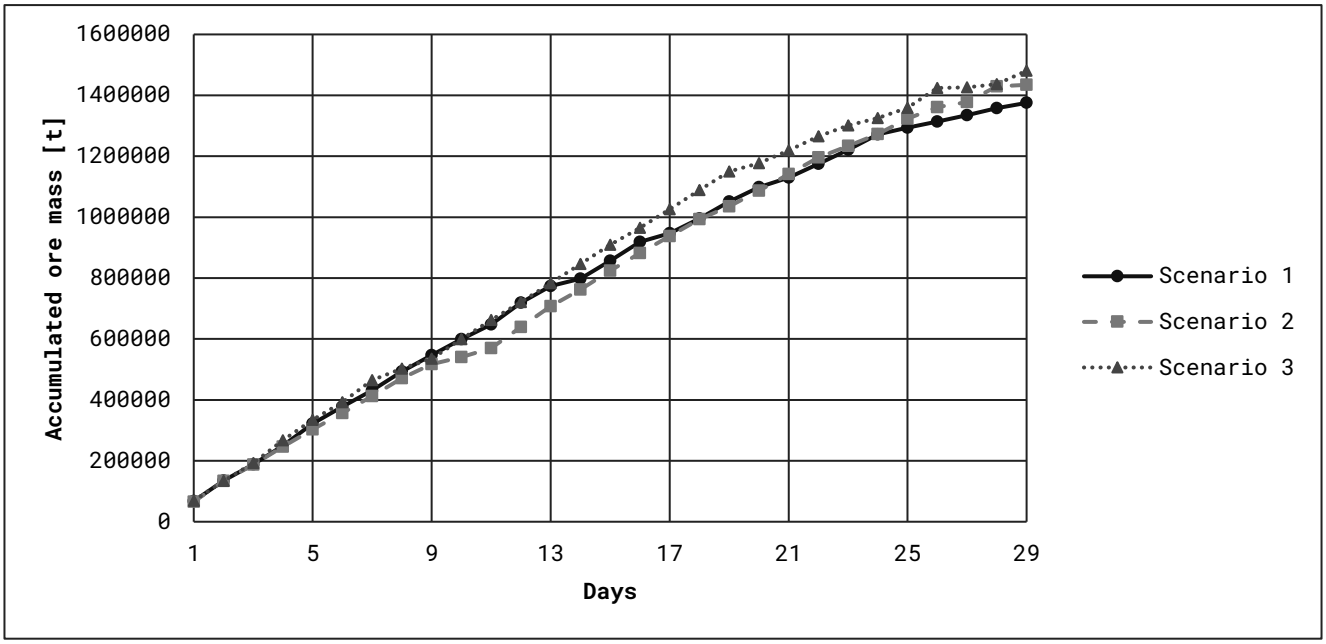


Figure 15. Original accumulated ore series for all three scenarios over 29 days. Explicit day indices restore the display of the original cached values; the series values are unchanged.

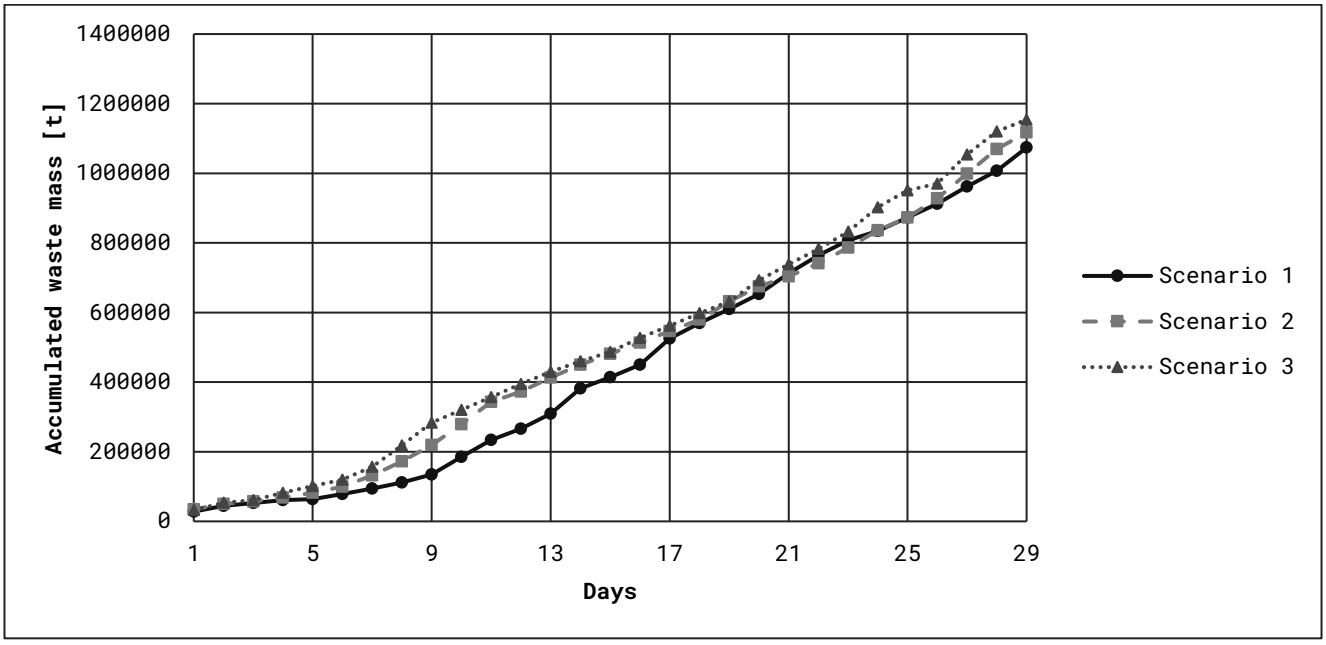


Figure 16. Original accumulated waste series over the same 29 days, with the same display repair and unchanged y values.

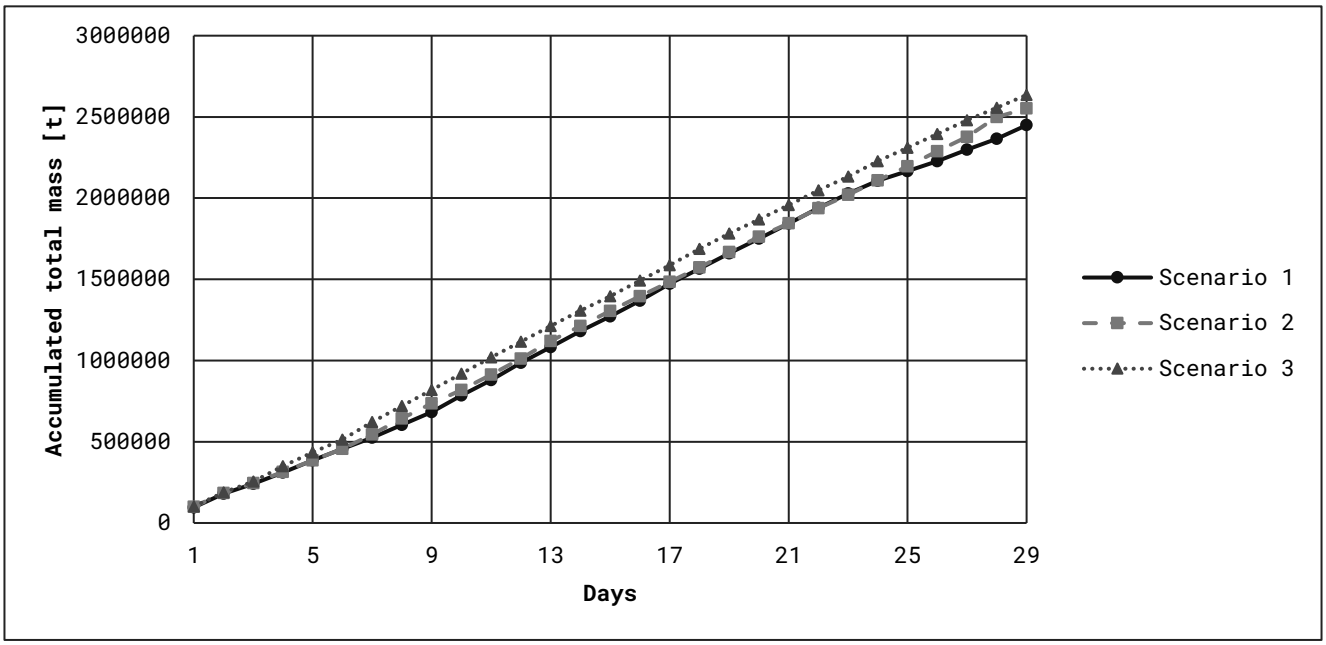


Figure 17. Original accumulated total movement. At every cached day, total tonnes equal ore plus waste tonnes for each scenario.

Table 6. Independently calculated terminal movement and daily ore variability from the embedded chart caches. Changes are relative to Scenario 1; no new simulation was performed.

Metric	Scenario 1	Scenario 2	Scenario 3
Ore t	1,375,228	1,434,521	1,480,683
Waste t	1,073,720	1,116,952	1,154,306
Total t	2,448,948	2,551,473	2,634,989
Total change %	0.00	4.19	7.60
Mean daily ore t	47,421.7	49,466.2	51,058.0
Min daily ore t	16,862	5,246	1,542
Max daily ore t	72,403	69,230	74,860
Daily ore CV	0.353	0.310	0.365

Scenario 3 has the greatest historical movement: 1,480,683 t of ore and 1,154,306 t of waste, giving 2,634,989 t in total. Relative to Scenario 1, the differences are 7.67% for ore, 7.51% for waste and 7.60% for total movement. Scenario 2 increases total movement by 4.19%. These are descriptive differences between the archived runs. Without paired replications or controlled relocation and dispatch settings, they do not establish a statistically supported gain or prove that the removed loaders were redundant.

Near linear cumulative curves do not imply stable daily feed. The daily ore coefficient of variation ranges from 0.310 to 0.365, and the daily minimum in Scenario 3 is only 1,542 t despite its highest terminal output. Interpreting a cumulative slope as a measure of daily stability would obscure this variation. Plant feed reliability should be assessed with daily shortfall, interruption hours and quality compliant tonnes as well as the terminal cumulative total.

The apparent benefit of some configurations is consistent with several mechanisms: avoiding long routes, changing access to favourable ore, altering the match between trucks and loaders, or modifying queue contention. The archive does not identify their individual contributions. A factorial experiment separating loader count, type, location and dispatch policy is needed before making a fleet investment or reduction decision. Nominal loading capacity alone cannot predict the throughput of the coupled system.

7.2 Depletion and stripping obligations

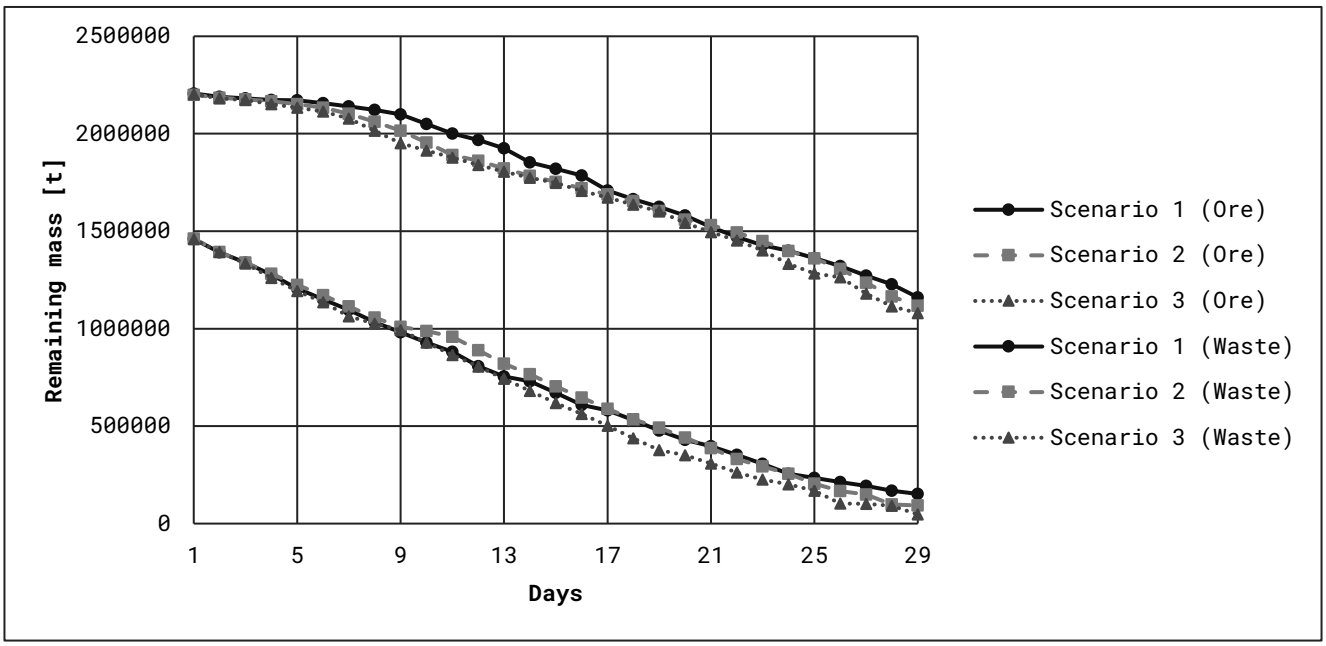


Figure 18. Original remaining ore and waste series. All scenarios begin from the same recoverable inventory inferred by adding cumulative extraction to remaining mass.

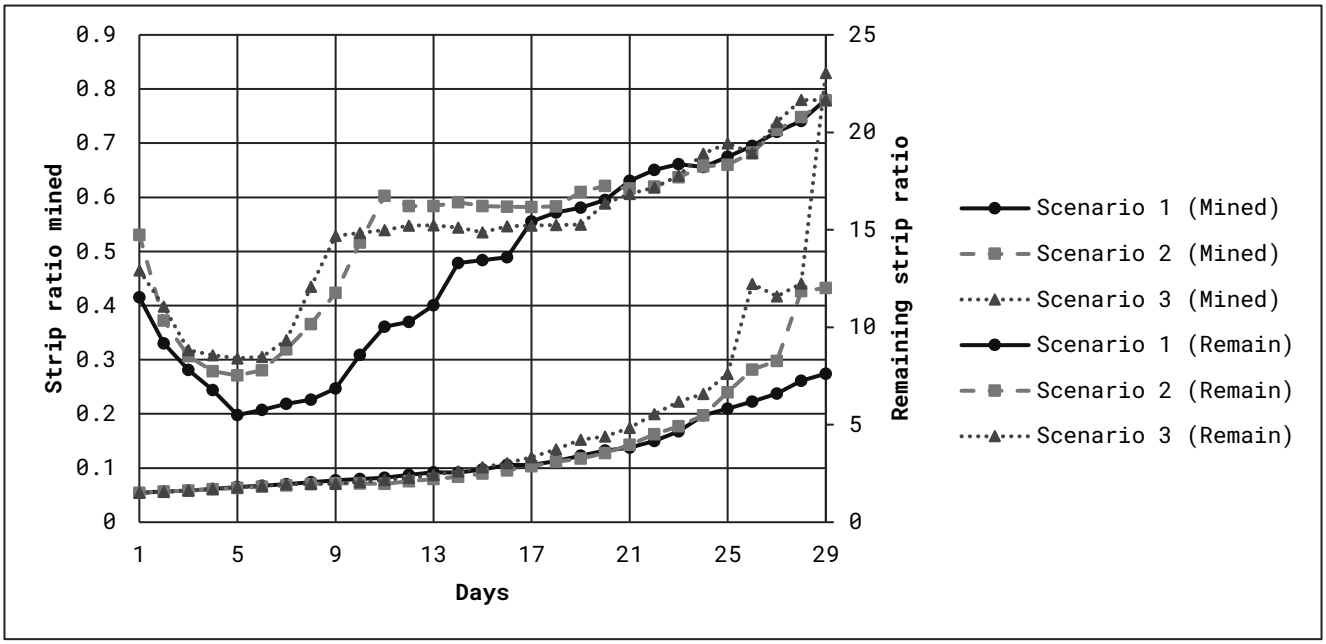


Figure 19. Original mined waste to ore ratio on the primary axis and remaining waste to ore ratio on the secondary axis. The ratios refer to different inventories and must not be compared as if they were the same indicator.

Table 7. Terminal inventory and stripping indicators calculated from the original caches.

Metric	Scenario 1	Scenario 2	Scenario 3
Remaining ore t	152,329.7	93,036.7	46,874.7
Remaining waste t	1,160,806.1	1,117,574.1	1,080,220.1
Mined waste to ore t/t	0.781	0.779	0.780
Remaining waste to ore t/t	7.62	12.01	23.04

The reconstructed initial inventory is approximately 1,527,557.7 t of ore and 2,234,526.1 t of waste in every scenario. Across the 29 recorded days, cumulative extraction plus remaining inventory is constant within numerical rounding. This is a useful conservation check of the archived output, but it does not verify the geological tonnes, dispatch measurements or event implementation independently.

The mined waste to ore ratio ends close to 0.78 t/t in all scenarios. The remaining ratio is much larger: 7.62, 12.01 and 23.04 t/t. In Scenario 3, remaining ore falls to about 46,875 t while more than 1.08 million tonnes of waste remain. The large ratio is driven partly by the small denominator. It signals a substantial residual stripping burden within the represented inventory; it does not by itself quantify next month’s feasible schedule or a reserve loss. Access dependencies, available waste destinations and a longer planning horizon must be examined before drawing that conclusion.

Shortening the experiment before final depletion also limits the interpretation of the ranking. A scenario that consumes accessible ore faster can achieve a better 29 day total while leaving a more difficult continuation problem. Evaluation over a common longer horizon, with comparable initial and terminal inventory obligations, is necessary to distinguish accelerated extraction from a sustainable improvement.

7.3 Quality compliance

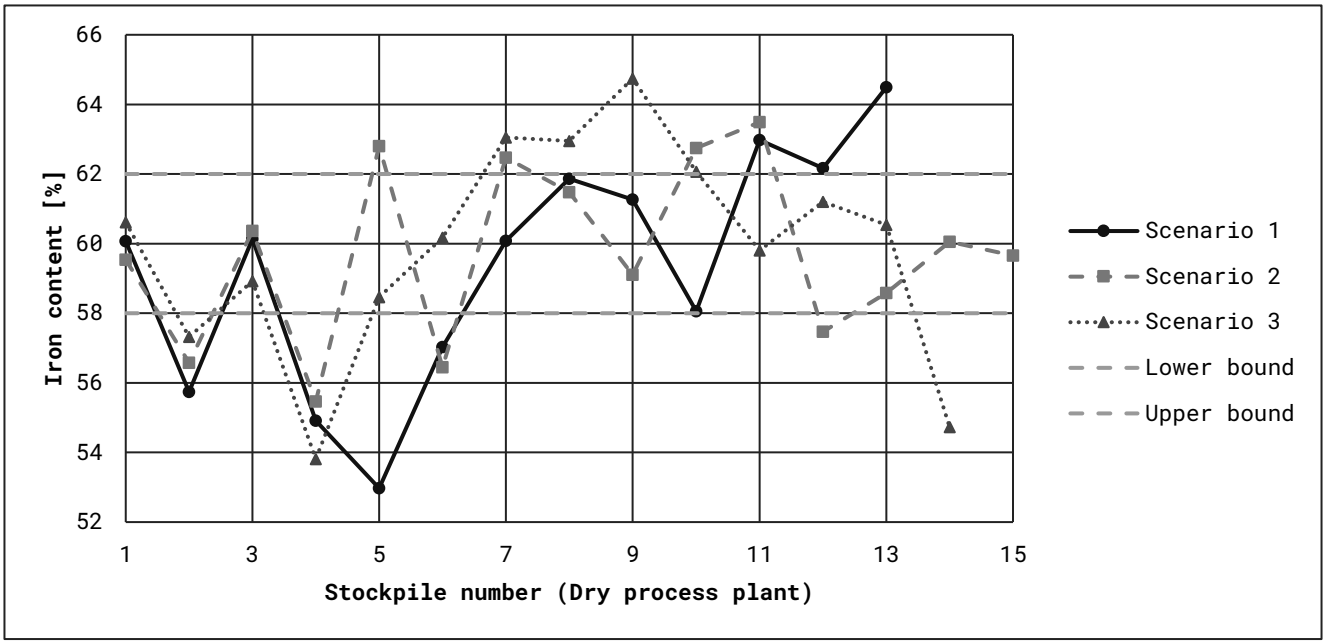


Figure 20. Original dry route stockpile iron grades, with plotted target bounds of 58-62% Fe. Missing source records remain missing; they are not zero grade observations.

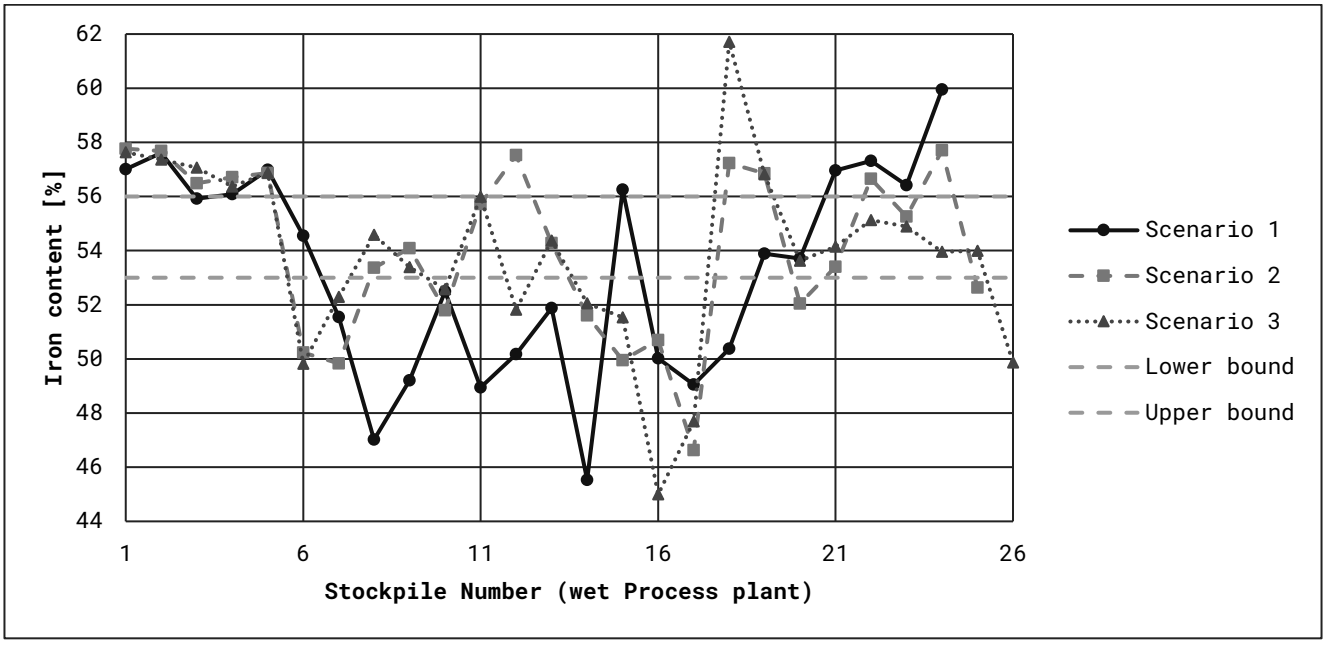


Figure 21. Original wet route stockpile iron grades, with plotted target bounds of 53-56% Fe. The horizontal index is the generated stockpile record, not necessarily a calendar day.

Table 8. Unweighted compliance of available grade records with the plotted bands. Means and ranges are in percent Fe; proportions describe records, not delivered tonnes.

Route and scenario	Records in band / n	In band %	Mean Fe %	Range Fe %
Dry S1	6 / 13	46.2	59.37	52.97 to 64.49
Dry S2	7 / 15	46.7	59.75	55.47 to 63.49
Dry S3	7 / 14	50.0	59.88	53.81 to 64.74
Wet S1	4 / 24	16.7	53.29	45.54 to 59.95
Wet S2	6 / 25	24.0	54.12	46.63 to 57.76
Wet S3	10 / 26	38.5	53.87	45.00 to 61.72

The dry route has 13, 15 and 14 available grade records in Scenarios 1-3; only 6, 7 and 7 are in band. The corresponding wet route counts are 4 of 24, 6 of 25 and 10 of 26. The means can lie within a target band even when most individual records lie outside it. For example, the wet route mean in Scenario 1 is 53.29% Fe, but only 16.7% of its available records are within 53-56%. This distinction changes the interpretation of the historical implementation: the displayed outputs do not demonstrate consistent quality compliance.

The plotted bands are described as production targets in the source. Departures could reflect soft penalties, insufficient accessible blending options, stockpile accumulation rules or a mismatch between the optimizer’s quality aggregation interval and the reported stockpile record. The archive does not permit a causal diagnosis. If the same bands were intended as hard acceptance limits on each represented stockpile, the violations would require correction of the model or its interface before use. Reporting the exact quality accounting unit is therefore essential.

A more informative future objective would penalize off specification delivered tonnes, constituent shortfall and plant interruption, with hard rejection limits kept separate from operating targets.

Stockpile level record compliance should be reported alongside mass weighted compliance and assay uncertainty. The current data support the first measure only.

8 Conclusions

The integrated architecture connects a transparent allocation model to a simulator capable of representing queues and interruptions. The revised reference formulation clarifies the relationship between trip counts, tonnes, operating hours, size dependent chemistry and ordered planning priorities. Its state exchange requirements make material conservation during replanning explicit.

The historical case supports a narrower conclusion than a general claim that fewer loaders improve productivity. Scenario 3 records 7.60% more total movement than Scenario 1, but the equipment compositions differ and the archive contains one run per scenario. Daily output varies substantially, and the available grade records show frequent departures from the plotted target bands. These findings make quality compliant throughput and residual stripping obligations necessary evaluation criteria.

The numerical supplement makes the recoverable arithmetic reproducible. Operational conclusions require the original executable models, dispatch data, replicated experiments and independent validation periods. The present evidence establishes internal consistency of the archived mass series and identifies specific modelling and evaluation changes; it does not establish a validated operational optimum.

Data provenance and acknowledgements

The full dispatch database, mine block model and original executable Arena and LINGO models were not supplied for this revision. The accompanying supplement contains the recoverable chart caches and the code used for the numerical audit. Funding and competing interest information were not documented in the supplied files; no declaration of absence is inferred.

References

[1] Newman A M, Rubio E, Caro R, Weintraub A, Eurek K (2010). A review of operations research in mine planning. Interfaces 40(3), 222-245. https://doi.org/10.1287/inte.1090.0492

[2] Fioroni M M, Franzese L A G, Bianchi T J, Ezawa L, Pinto L R, Miranda G Jr (2008). Concurrent simulation and optimization models for mining planning. Proceedings of the Winter Simulation Conference, 759-767. https://doi.org/10.1109/WSC.2008.4736138

[3] Coelho G F, Oliveira B R P, Pinto L R, Martins A G (2014). Análise do congestionamento do sistema produtivo de uma mina a céu aberto comparativo entre soluções propostas por modelos de otimização e simulação. XLVI Simpósio Brasileiro de Pesquisa Operacional. https://www.din.uem.br/sbpo/sbpo2014/pdf/arq0404.pdf

[4] Both C, Dimitrakopoulos R (2020). Joint stochastic short-term production scheduling and fleet management optimization for mining complexes. Optimization and Engineering 21, 1717-1743. https://doi.org/10.1007/s11081-020-09495-x

[5] Manríquez F, González H, Morales N (2023). Short-term open-pit production scheduling optimizing multiple objectives accounting for shovel allocation in stockpiles. Optimization and Engineering 24, 681-707. https://doi.org/10.1007/s11081-021-09701-4

[6] Eustace C, Hynard K (2024). Application of simulation and optimization to support mine plan execution. Mining Metallurgy and Exploration 41, 3789-3800. https://doi.org/10.1007/s42461-024-01118-8

[7] Pathan S M, Pathan A G, Memon M S (2025). Simulation optimization of shovel-truck system in open-pit mines considering rockmass parameters. Journal of Advanced Transportation 2025, 7939037. https://doi.org/10.1155/atr/7939037

[8] Tang H, Li C, Chen G, Han S, Mavrovouniotis M, Li M, Yuan L (2026). Robust scheduling policy learning for ore-flow blending in dynamic and uncertain environments. Computers and Operations Research 194, 107526. https://doi.org/10.1016/j.cor.2026.107526

[9] Sargent R G (2013). An introduction to verification and validation of simulation models. Proceedings of the Winter Simulation Conference. https://informs-sim.org/wsc13papers/includes/files/027.pdf

[10] Martins A G (2013). Simulação das operações de lavra da mina de Brucutu utilizando um modelo de programação linear para alocar os equipamentos de carga. Master's dissertation, Universidade Federal de Ouro Preto. Historical implementation antecedent cited in the supplied manuscript.

[11] Quevedo J M G (2009). Modelo de simulação para o sistema de carregamento e transporte em mina a céu aberto. Master's dissertation, Pontifícia Universidade Católica do Rio de Janeiro. https://www.ppgep.ind.puc-rio.br/aluno/johanna-mirelle-gomez-quevedo/

[12] Gurobi Optimization (2026). Multiple objectives. Optimizer Reference Manual. Accessed 29 October 2026. https://docs.gurobi.com/projects/optimizer/en/current/features/multiobjective.html

Appendix A Original mathematical record

This appendix preserves every native equation and legacy equation object present in the supplied manuscript, including inline notation and the full original formulation. They document the historical model and retain its original symbols. Their preservation is not an endorsement of dimensional ambiguities, missing domain statements or the aggregation of unlike deviations. Equations C1-C12 in the main text provide the clarified reference model; they were not used to regenerate the historical simulation results.

A1 Original compact notation and objective

uml: Minimum mining units de : Ore destinations

jm: Loading equipment kc : Haulage equipment

te: Controlled grades gr : Particle size controls

bl: Mining benches $type$: Material types

$$min\left(eprod + \sum_d \sum_t \left(emax_{d,t} + emin_{d,t}\right) + erem\right) \qquad (1)$$

The aggregate historical objective combines production deviations ($eprod$), grade controls ($t \in te$) at destinations ($d \in de$) and a stripping deviation ($erem$). The main text explains why this sum does not by itself implement the reported hierarchy.

MMU uml_i can receive loading equipment jm_j only when that equipment is available.

Assignments of loader jm_j to MMU uml_i must respect the face allocation capacity.

At bench b, loading equipment jm_j must be available rather than under repair.

A loader assigned to MMU uml_i is also assigned to its parent bench b, to which MMU uml_i belongs.

Outgoing mass from bench b must respect the assigned loading capacity over the planning interval.

Mass leaving bench b equals the sum leaving its constituent MMUs uml_i.

Outgoing mass from MMU uml_i cannot exceed its available material inventory.

The original fleet time account for trips between MMU uml_i and destination d is bounded by a 60 minute planning interval.

Trip counts assigned to truck class k between MMU uml_i and destination d must remain within the fleet trip capacity.

Control grade t at destination d is subject to its upper specification.

Control grade t at destination d is subject to its lower specification.

Ore delivered from MMU $i \in uml_i$ to destination d must meet the destination production requirement.

A2 Original full formulation

The following set definitions, parameters, decision variables, objective and constraints are retained in their original order. Minor prose formatting is standardized; mathematical expressions and embedded equation objects are unchanged. The source uses UML and MMU for the same minimum mining unit concept.

Sets

uml Minimum mining units from which loading equipment extracts ore or waste.

de Ore receiving destinations, including crushers and stockpiles.

jm Loading equipment.

kc Haulage equipment or truck classes.

te Controlled chemical grades.

gr Controlled particle size ranges.

bl Mining benches.

$tipo$ Material types.

Parameters

Parameter vectors

$cava_i$, $i \in uml$ Pit containing MMU i.

$regiao_i$, $i \in uml$ Mining region containing MMU i.

$altitude_i$, $i \in uml$ Bench elevation of MMU i.

$ntipo_i$, $i \in uml$ Material type in MMU i.

$capacidade_i$, $i \in uml$ Maximum number of loaders simultaneously permitted at MMU i.

$numbanco_i$, $i \in uml$ Mining bench containing MMU i.

atv_i , $i \in uml$ Ore indicator equal to one for ore and zero for waste.

mlc_d , $d \in uml$ Mass already credited to destination d.

$produt_d$, $d \in uml$ Required production mass at destination d.

$britador_d$, $d \in uml$ Indicator identifying a crusher or stockpile destination.

$p\min_j$, $j \in jm$ Minimum productivity of loader j.

$p\max_j$, $j \in jm$ Maximum productivity of loader j.

eqe_j , $j \in jm$ Loader mobility class, wheeled or tracked.

dj_j , $j \in jm$ Availability of loader j.

$aclocatual_j$, $j \in jm$ Current location of loader j.

cm_k , $k \in kc$ Ore payload per truck trip in class k.

ce_k , $k \in kc$ Waste payload per truck trip in class k.

F_k , $k \in kc$ Number of trucks in class k.

$tciclo_k$, $k \in kc$ Average cycle time per trip for truck class k.

Parameter matrices

$t_{i,t}$, $i \in uml$ e $t \in teores$ Grade of controlled attribute t in MMU i.

$Part_{i,t}$, $i \in uml$ e $t \in teores$ Size partition grade information for attribute t in MMU i, as named in the historical model.

$y_{d,c,j}$, $d \in de$ e $c \in kc$ e $j \in jm$ Compatibility between destination, truck class and loading equipment.

$l\inf_{d,t}$, $d \in de$ e $t \in te$ Lower quality bound at destination d.

$l\sup_{d,t}$, $d \in de$ e $t \in te$ Upper quality bound at destination d.

$tlc_{d,t}$, $d \in de$ e $t \in te$ Current controlled grade at destination d.

$plc_{d,t}$, $d \in de$ e $t \in te$ Current size partition quality at destination d, using the historical parameter definition.

$l\inf g_{d,g}$, $d \in de$ e $g \in gr$ Lower particle size control bound at destination d.

$l\sup g_{d,g}$, $d \in de$ e $g \in gr$ Upper particle size control bound at destination d.

$material_{d,p}$, $d \in de$ e $p \in tipos$ Compatibility between material type p and destination d.

Decision variables

$x_{i,j}$, $i \in uml$ e $j \in c\arg a$ Binary assignment of loader j to MMU i.

$xbanco_{b,j}$, $b \in bl$ e $j \in jm$ Binary assignment of loader j to bench b.

$w_{d,i,c}$, $d \in de$ e $i \in uml$ $c \in K$ Number of trips from MMU i to destination d assigned to truck class c.

$P_{i,d}$, $i \in uml$ e $d \in de$ Mass from MMU i sent to destination d.

$Pb_{b,d}$, $b \in banco$ e $d \in de$ Mass from bench b sent to destination d.

$e\max_{d,t}$, $d \in de$ e $t \in te$ Upper quality deviation for attribute t at destination d.

$e\min_{d,t}$, $d \in de$ e $t \in te$ Lower quality deviation for attribute t at destination d.

eprod Sum of production target deviations at ore destinations.

Original objective function

$$\min\left(eprod + \sum_{d\ tal\ que\ britador_d=1} \sum_j \left(e\max d,j + e\min d,j \right) + erem \right)$$

Original constraints

Allocation constraints

$$x_{i,j} \le d_j \quad \forall i \in uml \quad \forall \in jm$$

$$\sum_{j \in jm} \left(x_{i,j} \right) \le capacidade_i \quad \forall i \in uml$$

$$xbanco_{i,j} \le d_j \quad \forall i \in uml \quad \forall \in jm$$

$$x_{i,j} = xbanco_{p,j} \quad \forall i \in uml \quad \forall p \in banco \quad tal \quad que \quad numbanco_i = banco_p \qquad \forall j \in uml$$

$$\sum_{d \in de} \left(Pb_{p,d} \right) \le \sum_{j \in jm} \left(xbanco_p \times p\max_j \times tempo \right) \quad \forall p \in banco$$

$$\sum_{d \in de} \left(Pb_{p,d} \right) = \sum_{i \in uml \quad tal\ que \quad numbancoi = bancop} \left(P_{i,d} \right) \qquad \forall p \in banco$$

$$\sum_{d \in de} \left(P_{i,d} \right) \le massa_i \times \sum_{j \in jm} x_{i,j} \quad \forall i \in uml$$

$$w_{d,i,k} \le \sum_{j \in jm} y_{d,k,j} \times 60 \times x_{i,j} \quad \forall d \in de \quad \forall i \in uml \quad \forall k \in kc$$

$$\sum_{i \in uml} \sum_{d \in de} w_{d,i,k} \le \left(\left(60 \div tciclo_k \right) \times F_k \times tempo \right) \quad \forall k \in kc$$

Quality constraints

$$\sum_{i \in uml} (t_{i,j} \times atv_i \times P_{i,d} \times part_{i,t} \div 10000) + plc_{d,t} \times tlc_{d,t} \times mlc_d \div 10000 - e\max_{d,t} \le l\sup_{d,t} \times (plc_{d,t} \times mlc_d +$$
$$\sum_{i \in uml} atv_i \times P_{i,d} \times part_{i,t}) \div 10000 \quad \forall d \in de, tal\ que\ britador_d = 1 \quad \forall t \in te$$

$$\sum_{i \in uml} (t_{i,j} \times atv_i \times P_{i,d} \times part_{i,t} \div 10000) + plc_{d,t} \times tlc_{d,t} \times mlc_d \div 10000 + e\min_{d,t} \ge l\inf_{d,t} \times (plc_{d,t} \times mlc_d +$$

$$\sum_{i\in uml} atv_i \times P_{i,d} \times part_{i,t}) \div 10000 \quad \forall d \in de, tal\ que\ britador_d = 1 \quad \forall t \in te$$

Mass constraints

$$\sum_{i\in uml} (P_{i,d}) + eprod \geq produt_d \quad \forall d \in de, tal\ que\ britador_d = 1 \quad ou\ britador_d = 5 \quad e \quad \forall t \in te$$

Strip ratio constraints

$$\sum_{i\in uml} \sum_{d\in de} ((1 - atv_i) \times P_{i,d}) - \sum_{d\in de} (rem \times \sum_{i\in uml} (atv_i \times P_{i,d})) + erem \geq 0$$

Appendix B Historical input example

Mina	Regiao	Altitude	Ordem	Material	Perfurar	Massa	Densidade	G1	G2	G3
3	regiao5	1000	1	M	0	6898.054	1.842	19.38114	26.951	16.48126
3	regiao5	1000	1	R	0	6653.176	1.842	17.52459	28.94586	18.21166
3	regiao5	1000	1	P	0	36263.75	1.842	20.0431	22.81354	11.24464
3	regiao5	1000	2	es	0	13089.91	1.842	0.01	0.01	0.01
3	regiao5	1000	2	M	0	3096.514	1.842	17.68651	27.62167	16.10979
3	regiao5	1000	2	R	0	13214.31	1.842	13.77144	28.09033	18.61745
3	regiao5	1000	2	P	0	20991.24	1.842	18.34567	22.52187	9.675101
3	regiao5	1000	3	es	0	17796.41	1.842	0.01	0.01	0.01
3	regiao5	1000	3	P	0	32613.71	1.842	21.04688	19.96398	8.318575
3	regiao5	1000	4	es	0	36208.22	1.842	0.01	0.01	0.01
3	regiao5	1000	4	M	0	2048.087	1.842	6.7961	18.1207	15.937

Figure 22. Original excerpt of the minimum mining unit input file. Portuguese field names and all displayed values are retained. The excerpt is insufficient to reconstruct the complete geological inventory or executable case.