How Reliably Can Low-Flow Rates of the Danube to Prescribed Frequencies Be Estimated for Nuclear Power Station Capacity Expansion at Paks?

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Abstract

Nuclear power stations must be designed with consideration to extreme events having return periods (T_r) of up to 10^5 (sometimes even 10^6 for the largest plants) years. This is a daunting task for occurrences influenced by hydro-meteorological variables, such as floods and stream baseflow rates in the light of climate variability and change. Here we demonstrate the unavoidable large uncertainty encountered in such estimates, further illustrated by the simple task of deriving the T_r value for the latest annual minimum (Q_{min}) observed at Budapest during the recent Europe-wide drought of 2026 that resulted in record breaking low water levels (but not necessarily in discharge rates) at most gauging stations in Hungary due to a general incision of the river bed into its alluvium. Depending on the homogenization method or absence thereof, the latest Q_{min} of 662 m^3s^{-1} observed on October 8, 2026 in Budapest, has an associated T_r of 89 years with a 68% confidence interval of [47; 168] years before homogenization, a T_r of 126 [63; 254] years after homogenization, and 516 [180; 1,477] years when the observed increase in annual minima is also accounted for. As a consequence, any percentile estimate (and especially its inverse task of finding T_r for a given Q_{min}) based on hypothetical future trajectories of the hydro-meteorological variables must be taken with extreme caution as such estimates must necessarily have a very wide combined confidence interval thus making them highly uncertain.

Keywords

generalized extreme value distribution, percentile confidence interval, de-trending, climate change, Paks nuclear power station

1 Introduction

In the summer and fall of 2026 the stage of the Danube reached record-breaking low levels at almost all gauging stations in Hungary, including Budapest and Paks (Fig. 1), and prompted the rapid construction of a bottom sill near Paks to increase the water level in the vicinity of a diversion canal that provides cooling water for the nuclear reactor. Since an extension of the power plant is under way in order to boost its energy production capacity, safety guidelines require the specification of critical values to given recurrence intervals (or return periods, T_r) of man-made and natural hazards, such as flood peak and low flow (i.e., baseflow) rates, among other threats to ensure the safety and undisturbed functioning of the critical infrastructure. Nuclear power stations typically use T_r values up to 10^5 (occasionally up to 10^6) years or when expressed as frequency, f (= T_r^{-1}), 10^{-5} (10^{-6}) yr^{-1} [1].

There are two immediate problems with the definition of such rare events. First, hydrological record in Hungary is much shorter than 10^5 years, thus such rare events could not be observed and recorded

in the past. For daily mean flow rates (Q) of the Danube, Budapest has the longest available continuous record of 103 years, starting in 1924 and further employed in this study. Second, even if they could have been observed in the form of annual extrema, homogeneity of those values remains questionable as hydro-meteorological variables follow natural long-term variability even without climate change [2, 3]. Such climate variability combined with potential climate change effects make reliable estimation of any percentile value or the frequency of even an observed value laden with high uncertainty as it is demonstrated in this study for annual minima of the daily mean discharges. Annual minima are chosen for the present study due to the high costs involved when the energy output of the power plant must be reduced due to critically low water levels of the Danube, as happened in the summer and fall of 2026. Also, discharge rates rather than water levels are analyzed because water levels heavily depend on local river geometry and hydraulic properties yielding highly inhomogeneous time series due to long-term (and often sudden) natural and man-made changes in those properties.

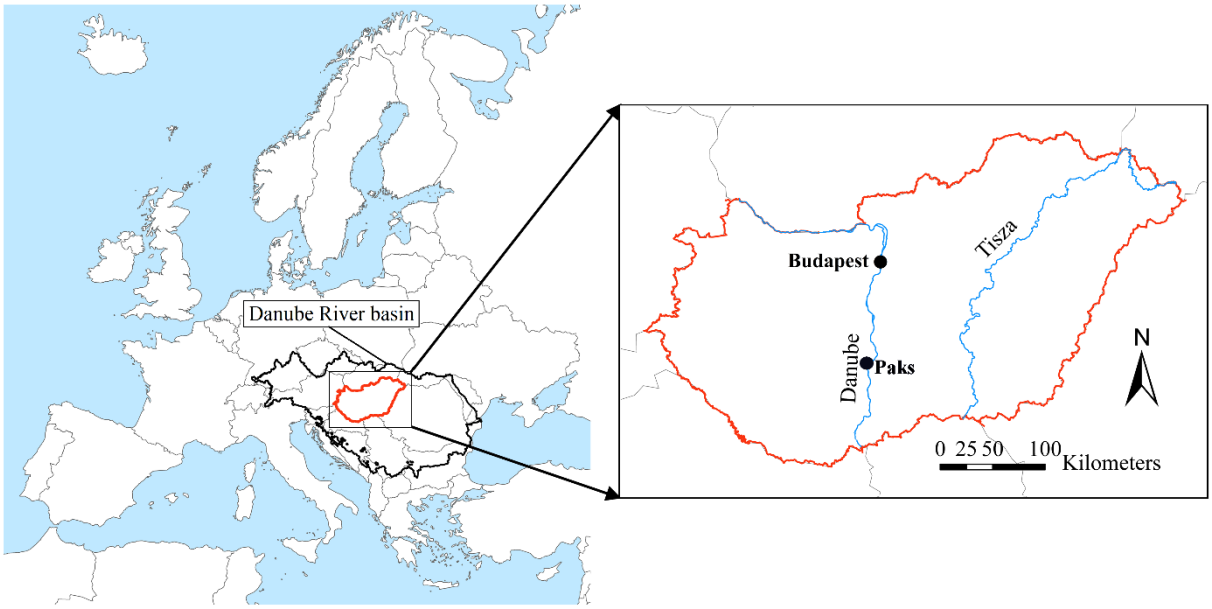


Fig. 1 Location of the Danube Basin within Europe and the gauging stations at Budapest and Paks in Hungary.

2 Methods

Quasi-hourly instantaneous discharge values of the Danube at Budapest and Paks were downloaded from the Hungarian Water Authority's website (https://data.vizugy.hu) from which daily averages were calculated for the available periods of 1924-2026 for Budapest and 1981-2026 for Paks. The three lowest discharge values of 568, 537 and 538 m^3s^{-1} at Budapest on January 13-15, 1954 had to be corrected because it was caused by ice jamming (that cleared by the 16th due to a rapid, small flood-wave) upstream of the gauging station [4]. Correction was achieved by plotting the preceding twelve days' daily mean discharge values on a semi-logarithmic graph and extending the best fit line to the 15th and reading off the so-derived Q = 622, 615 and 610 m^3s^{-1} values for the three affected days. Fig. 2 displays the annual maxima, means, and minima (the explained correction already included) of the daily mean discharges for Budapest.

Runs test for randomness of the Q_{min} values yielded a p value of 0.016 suggesting that the annual values are not fully random. This finding was corroborated by the modified Mann-Kendall test [5] for monotonic trend (p = 0.025). While the annual minima display a statistically significant upward trend [the trend line obtained by ordinary least squares (OLS)], the annual mean values (Q_{mean}) follow a statistically significant (p = 0.049) downward trend (Fig. 2). An upward trend can be seen in Fig. 2 for the annual maxima as well, but that trend is not significant (p = 0.84). The opposing significant trends in annual mean flow and minima are interesting because it suggests that the water regime of the Danube at Budapest has narrowed contrary to what climate model based hydrological predictions generally claim [6 – 8].

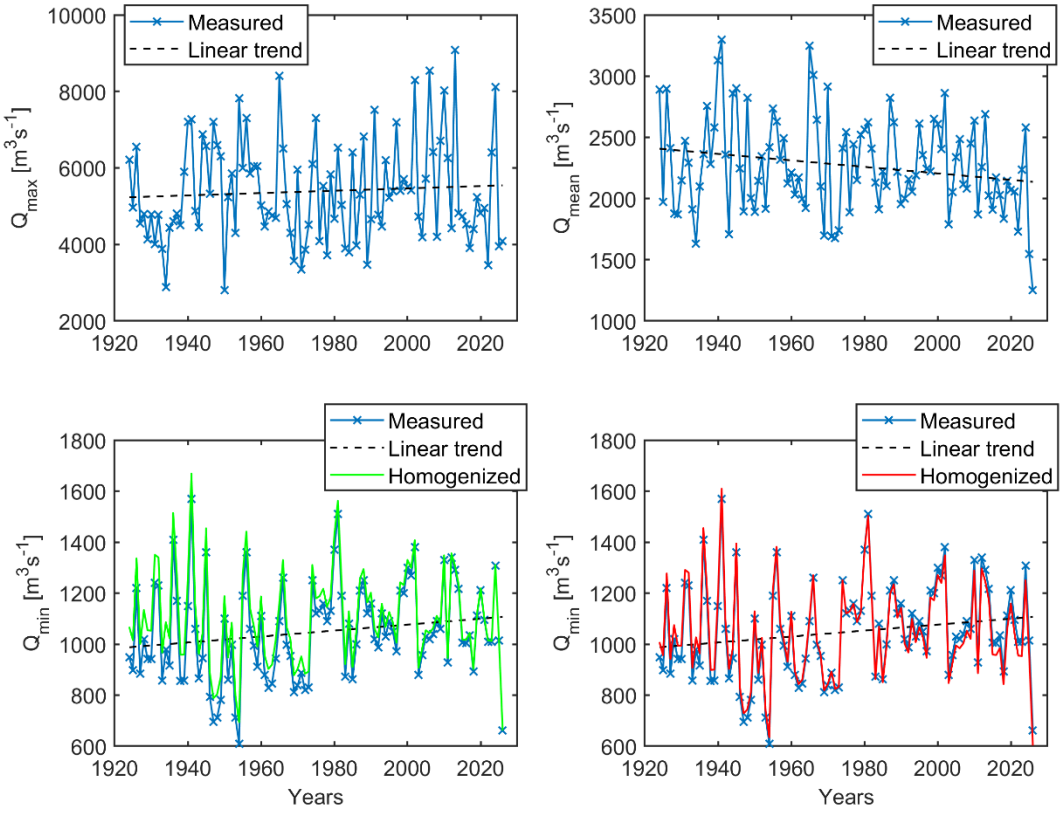


Fig. 2. Annual maxima (Q_{max}), means (Q_{mean}), and minima (Q_{min}) of the daily mean discharge values (1924 – 2026) of the Danube at Budapest. The straight lines (trends) are the OLS-fitted first-order polynomials. For homogenization, the de-trended values are shifted upwards by either the last value of the linear trend (green) or by the mean of the Q_{min} values.

Homogenization of the Q_{min} values was achieved by the classical approach [9] of adding the change in the linear trend backward in time from its 2026 ending value to the corresponding Q_{min} values (Fig. 2). It is a de-trending approach [10] where the de-trended time series (i.e., the original minus the best-fit linear trend) is shifted upward by a constant equaling the trend line value at the time of the last measurement. This approach assumes that future fluctuations will be fully random occurring around the last value (i.e., obtained in 2026) of the linear trend. This is a reasonable assumption when no information is available on the general direction of any possible future long-term tendencies, may it be increasing, non-existent or a declining one. Note that future climate change scenario derived water regimes of the Danube do not expect [6, 7] increased baseflow rates.

The runs test for the homogenized values yields p = 0.195 thus the derived values can be considered fully random and the obtained values are ready for percentile estimations.

A certain 'homogenization', however, can also be achieved by simply reshuffling the values to make the perceived trend disappear and thus making the annual values appear fully random. Naturally, the two treatments would not result in the same moments as the former would have higher mean and lower variance than the latter which of course preserves the moments of the original values as it indeed consists of them but in a different order. The question then becomes as to which homogenization approach is better suited for deriving the 'true' percentiles. One may argue that it depends on whether the annual minima will indeed be higher in the future than what were observed at the beginning of the available record. Some clue to an answer may be obtained from fitting a higher order polynomial to the annual values (Fig. 3).

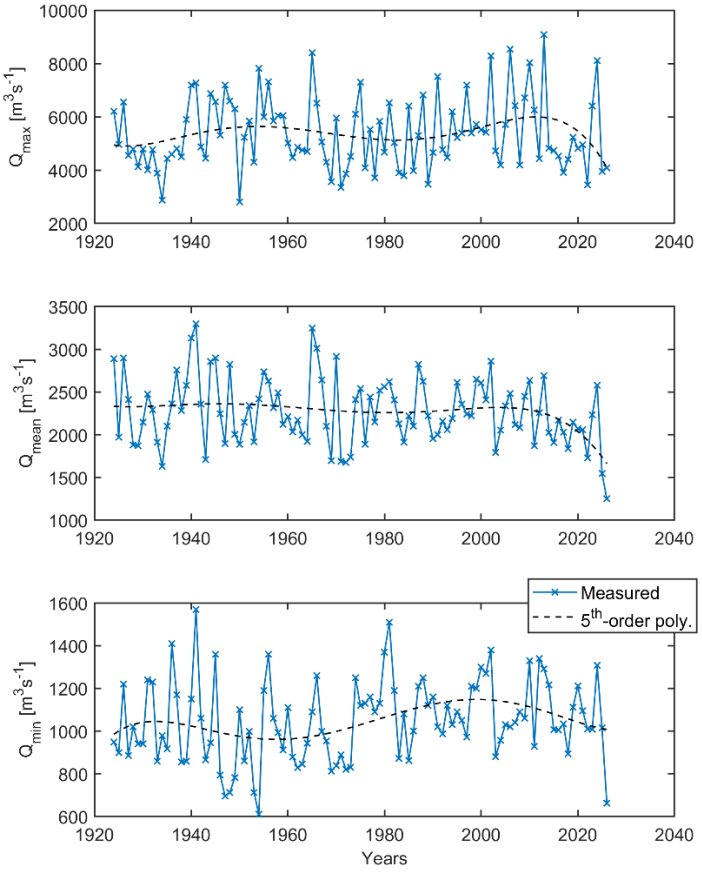


Fig. 3. Annual maxima (Q_{max}), means (Q_{mean}), and minima (Q_{min}) of the daily mean discharge values (1924 – 2026) of the Danube at Budapest and the OLS-fitted 5th-order polynomials.

As seen, the annual maxima overall kept increasing slightly up until the 2010's after which they dropped sharply. At the same time the annual mean values dropped monotonically albeit not at the same rate, with a steepening also starting around the 2010's. The annual minima display more variations: reaching their minimum right before 1960, a maximum around 2000 and returning to almost the same level it started at in 1924. Therefore, one may argue that the increased Q_{min} rate obtained from the linear trend of Fig. 2 cannot be expected maintained in the future.

As it is not known for certain how the Q_{min} tendencies will behave in the future, three scenarios must be considered for percentile estimations. First, due to the general warming occurring over Europe and within the Danube basin in particular, the baseflow regime must have shifted. The annual minima no longer occur predominantly in the wintertime as before at the beginning of the records but shift back frequently to the fall (Fig. 4) due to elevated flowrates in the cold season (Fig. 5) as a result of weaker snow accumulations caused by a higher proportion of the winter precipitation falling

as rain and less persistent snow cover owing to periodic snow melting triggered by the observed significant warming, especially in the wintertime [6, 7]. Note that the decreased snow accumulations in the Alps also lead to declining spring and summer flow rates (Fig. 5) while the more frequent rain and snow-melting events cause the noticeable increase in the winter flow at the same time when the fall discharges remain unchanged. Only the summer trend is statistically significant (p = 0.003) at the 5% level while the winter trend is borderline (p = 0.053). The spring trend is not significant only because of the large inter-period variability of the spring discharge values. As a consequence, percentiles must be estimated from the Q_{min} values homogenized around the last linear trend value in this scenario.



Fig. 4. Season of occurrence of the annual Q_{min} values (01-January-1924 – 31-December-2025) of the Danube at Budapest.

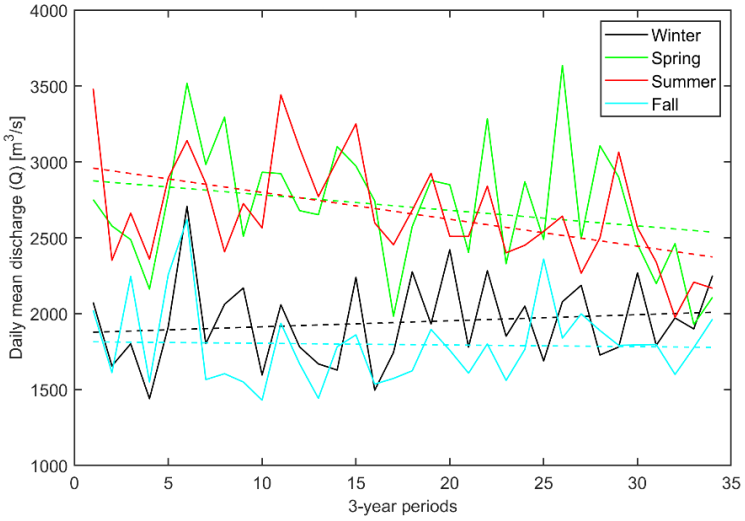


Fig. 5. Consecutive three-year averages of the seasonal daily mean flowrates (01-January-1924 – 31-December-2025) of the Danube at Budapest and their OLS-fitted linear trend lines.

Second, because the Q_{min} values express a declining trend after 2000 (Fig. 3) and return to their initial values a century before, no homogenization is necessary as it is possible that the Q_{min} values on the long term cannot maintain their increased level but instead maintain their already observed average under an expected generally declining future (and already observed) flowrate regime caused mainly

by increases in actual evapotranspiration rates in a warming climate and an overall unchanged annual precipitation sum [7].

Third, the original inhomogeneous (or 'homogenized' by reshuffling only) Q_{min} values yield enhanced annual variability due to a possible trend present in the data. Therefore, de-trending is necessary, but the de-trended values are shifted upward by the mean of the observed minima, thus reducing the observed trend-enhanced variability while preserving the original mean of the Q_{min} values. This approach inherently assumes that the future Q_{min} values will keep fluctuating around the observed mean, just as in the second scenario. The consequences of the three possible scenarios on percentile estimations are detailed below.

3 Results and discussion

The empirical cumulative distribution function (ECDF) of the original annual minima at Budapest (1924 – 2026) is displayed in Fig. 6 together with the Maximum Likelihood (ML) fitted Generalized Extreme Value (GEV) distribution. The fit is strong with a Kolmogorov-Smirnov test p value of 0.033. The 68% confidence interval (CI) of the CDF values was obtained by the Delta method [11] employing a logit transformation [12].

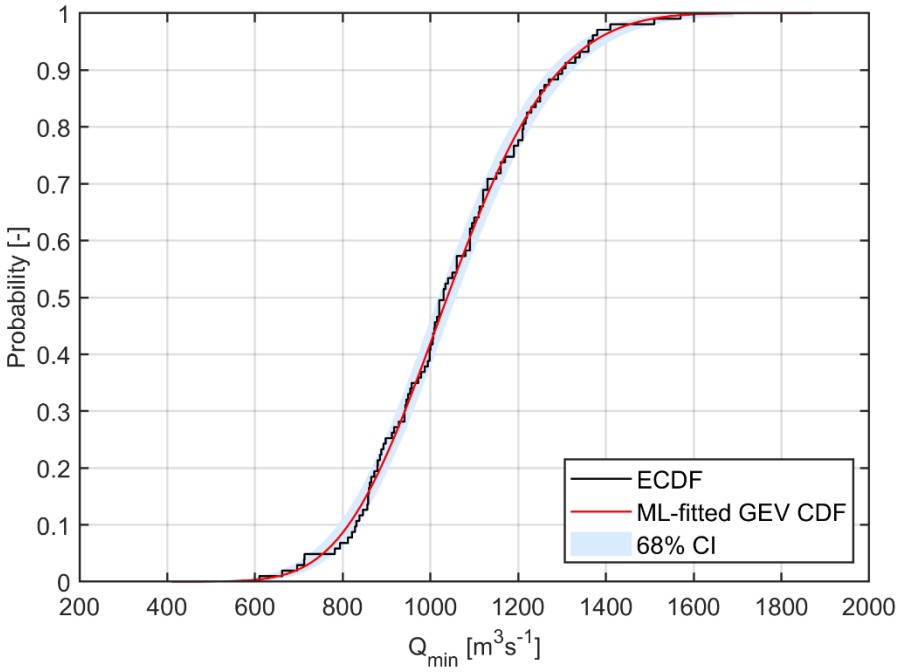


Fig. 6. Empirical (ECDF) and Maximum-Likelihood-fitted Generalized Extreme Value (GEV) cumulative distribution functions (CDF) of the original (before any homogenization) Q_{min} values at Budapest.

Fig. 7 displays the bottom part of Fig. 6 together with the prescribed values of frequencies (T_r^{-1}): 0.2, 0.1, 0.05, 0.033, 0.02, 0.01, 0.003, 0.002, 0.001, 0.0001, 0.00001, corresponding to T_r values of 5, 10, 20, 30, 50, 100, 300, 500, 10^3, 10^4, and 10^5 years, arbitrarily chosen for this study. Note that the numerical value of any frequency equals that of probability for annual minima. The 68% confidence intervals (green horizontal line sections) for the percentiles belonging to the different T_r values were obtained by the profile likelihood method [13].

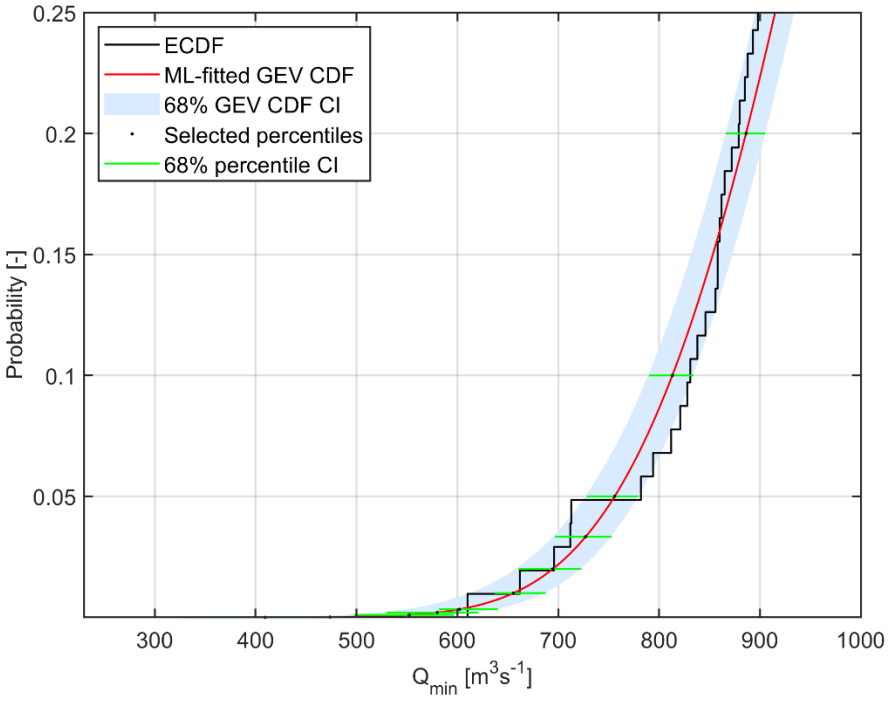


Fig. 7. The lower portion of Fig. 6 with the selected percentiles (black dots) and their confidence intervals (green lines).

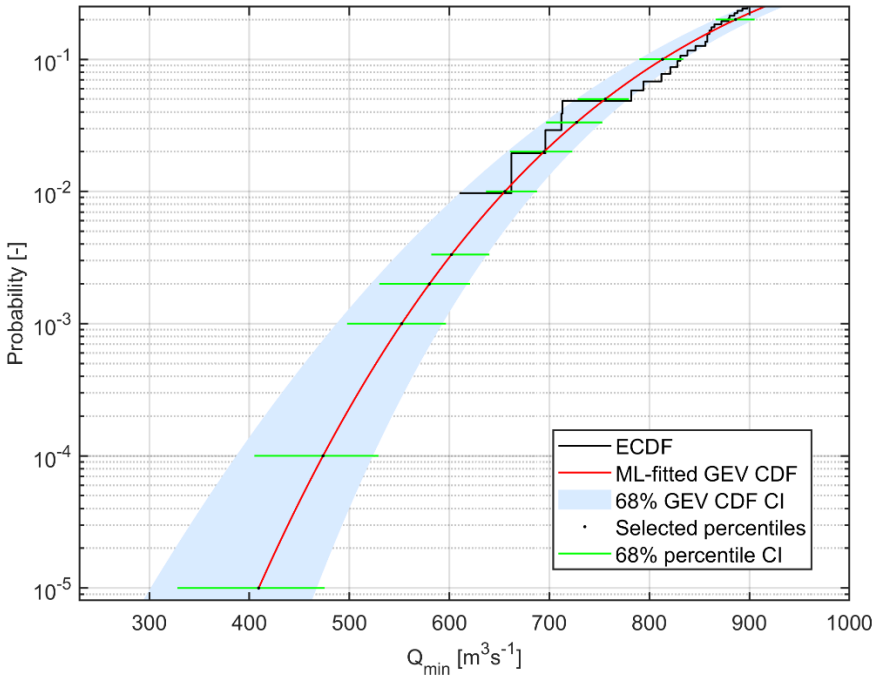


Fig. 8. The semilogarithmic version of Fig. 7.

As seen in Fig. 8, the high-ends of the horizontal percentile confidence intervals are closer to the edge of the shaded band (representing the 68% CI for the CDF values) than the low-ends, for yet unknown reasons. The largest differences are found for T_r = 100 and 300 years (i.e., probability of 0.01, 0.003). In order to check if there was an error in the profile likelihood method, percentile intervals were also estimated by a Monte-Carlo (MC) simulation procedure.

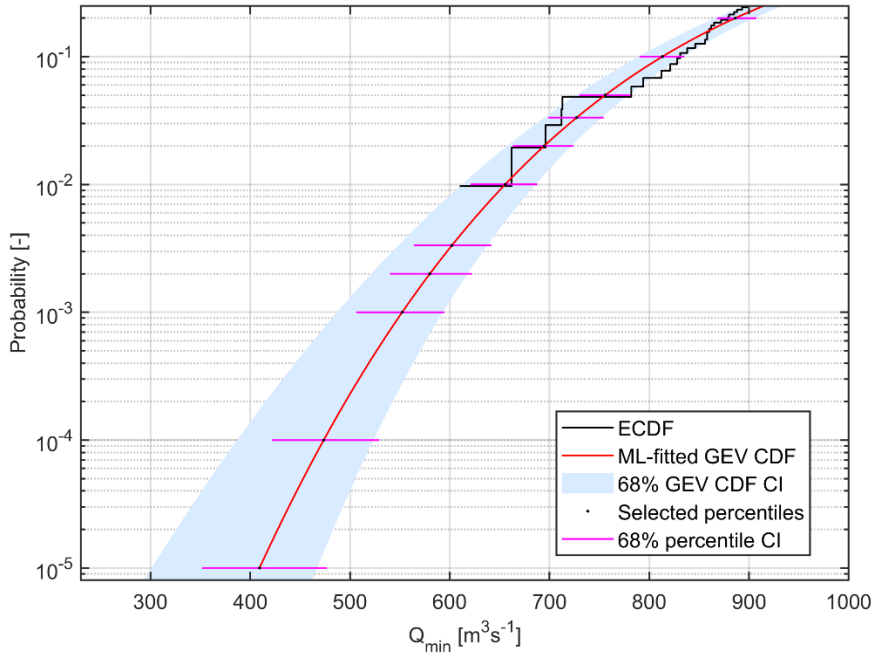


Fig. 9. The semilogarithmic version of Fig. 7. The 68% CI for each selected percentile here was obtained from MC-type simulations.

The MC approach involved generating N (= 10,000) percentiles (for each prescribed T_r) by randomly picking values from the 68% confidence interval of the three ML-derived GEV parameters that were provided by the 'gevfit' MATLAB function [13] followed by calculating the percentiles for the eleven T_r values via the CDF the randomly chosen three GEV parameters define and repeating these steps N times. It was assumed that the ML-fitted GEV parameters are normally distributed (which is true asymptotically [14]). The sought-after CI is the shortest interval (enclosing the mode) that contains 68% of the sorted N percentile values.

As seen in Fig. 9, the resulting percentile confidence intervals behave similarly to those derived by the profile likelihood method, except that the match is better for probabilities of 0.01, 0.003, but somewhat worse for probability of 0.002 and smaller. The 68% (and 95%) CI values of the profile likelihood method are displayed in Table 1, except for T_r = 100 and 300 yrs, where those of the MC-method are included for the 68% CI bounds. For the homogenized Q_{min} values (first scenario) the MC-type simulation is better (in capturing the horizontal width of the CDF CI band at the prescribed T_r values) at T_r = 1,000 yrs for the 68% CI, and for the mean-preserving homogenized Q_{min} values (third scenario) the MC-approach is better at T_r = 300 and 1,000 yrs for the 68% CI therefore Table 1 displays the MC-derived confidence intervals for these affected T_r values. For the 95% CI bounds the profile likelihood method was always better than the MC-type approach, so in this case no replacements were necessary for the CI bounds.

The obtained percentile values to prescribed return periods differ significantly in Table 1 among the three scenarios. The lowest percentiles and the widest confidence intervals come from the original (second scenario) Q_{min} values as these express the largest variance, while the highest, with the narrowest CI bounds, from the first scenario (the CI widths are rather similar for the two homogenized cases). It is seen that the 100-year critical level can be 656, 673, or even 732 m^3s^{-1}, while the 1000-year level becomes 552, 578 and 638 m^3s^{-1}, depending on the scenario applied. For the largest, 10^5-year critical level considered, one obtains 409, 449, and 508 m^3s^{-1}, representing an

increase of up to 25% in the lowest estimate, with a combined 68% confidence interval of [328; 565] m^3s^{-1}. Naturally, the combined 95% CI is significantly wider (i.e. [231; 610] m^3s^{-1}).

Table 1. Percentiles and their confidence intervals (CI) to prescribed return periods (T_r) by the three scenarios considered.

T_r (yr)	T_r^{-1} (yr^{-1})	Original Q_{min} values (2nd scenario) Percentile	68% CI bounds (m^3s^{-1})	95% CI bounds (m^3s^{-1})	Homogenized Q_{min} (1st scenario) Percentile	68% CI bounds (m^3s^{-1})	95% CI bounds (m^3s^{-1})	Mean-preserving homogenized Q_{min} (3rd scenario) Percentile	68% CI bounds (m^3s^{-1})	95% CI bounds (m^3s^{-1})
5	0.2	**886**	866-905	845-923	**949**	930-967	910-984	**889**	870-907	850-925
10	0.1	**813**	790-834	764-853	**879**	858-899	834-917	**820**	798-840	774-857
20	0.05	**756**	728-780	696-800	**826**	800-848	771-867	**766**	741-788	712-807
30	0.033	**727**	697-753	661-774	**799**	771-823	739-843	**740**	712-763	679-783
50	0.02	**695**	661-723	620-746	**769**	738-795	702-816	**709**	678-735	642-757
100	0.01	**656**	621-688	570-713	**732**	698-761	656-785	**673**	638-702	597-726
300	0.003	**602**	564-642	500-670	**683**	642-717	593-744	**624**	588-659	533-685
500	0.002	**580**	530-621	470-653	**663**	619-699	566-728	**604**	559-640	507-668
10^3	10^{-3}	**552**	498-597	433-631	**638**	596-677	532-708	**578**	538-619	473-648
10^4	10^{-4}	**474**	405-529	323-573	**566**	507-615	436-653	**507**	447-555	376-593
10^5	10^{-5}	**409**	328-475	231-527	**508**	438-565	355-610	**449**	379-506	295-550

The inverse problem of finding the T_r value for the last Q_{min} of 662 m^3s^{-1} observed on October 8th, 2026, highlights the wide differences among the results even clearer (Table 2). Here it is assumed that the October 8th discharge value will remain indeed the lowest value in 2026 for which there is a high likelihood. The original Q_{min} values yield a return period of 89 years with a corresponding 68% CI (given by the Delta method of [11]) of [47; 168] years, while the mean-preserving homogenization provides a T_r of 126 years and a 68% CI of [63; 254] years, and finally the homogenization taking into account the statistically significant long-term trend in Q_{min} yields T_r = 516 years and 68% CI bounds of [180; 1477] years. This last T_r is almost six-fold of the T_r estimate obtainable by the original Q_{min} values.

Table 2. Return period estimates with CI bounds by the Delta method for the latest Q_{min} value of 662 m^3s^{-1} observed in 2026.

Latest observed Q_{min} value (m^3s^{-1})	Original Q_{min} (2nd scenario) T_r (yr)	68% CI bounds (yr)	95% CI bounds (yr)	Homogenized Q_{min} (1st scenario) T_r (yr)	68% CI bounds (yr)	95% CI bounds (yr)	Mean-preserving homogenized Q_{min} (3rd scenario) T_r (yr)	68% CI bounds (yr)	95% CI bounds (yr)
662	89	47 - 168	26 - 313	516	180 - 1477	66 - 4105	126	63 - 254	32 - 503

While the percentiles and the inverse problem of specifying T_r for a given Q_{min} value were derived for Budapest having the longest record of discharge rates, the results can directly be applied for Paks as well. This is so because the Danube does not receive any significant tributary or suffer from water diversions between the two cities and lateral groundwater contribution is also negligible along the river stretch [15]. For these reasons, the regression line of the low daily mean discharge (i.e., baseflow) rates of Budapest and Paks (with a lag of 1.5 days) becomes an identical 1:1 linear relationship, as displayed in Fig. 10. Note that such a 1:1 regression line would not necessarily be

valid for the Q_{max} values due to natural flood attenuation and bank storage processes along the river reach.

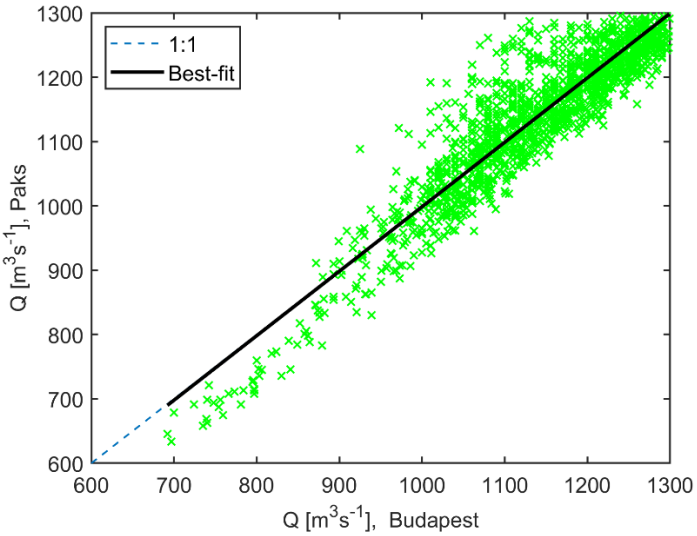


Fig. 10. Regression plot of the 1.5-day shifted daily mean discharge (Q < 1300 m^3s^{-1}) rates of Budapest and Paks for their common periods of 1981 – 2026 with the OLS-fitted regression line displayed.

As a summary, it was demonstrated that any future percentile estimates of the low-flow values of the Danube at Budapest (and Paks), mandated by regulatory agencies for the maintenance of crucial infrastructure safety and undisturbed functioning are inherently laden with large uncertainties due to the general inhomogeneity (and possible large occasional measurement errors) found in the available discharge record plus the largely unknown future behavior of the hydro-meteorological processes involved. For example, the extremely low discharge values of January 13-15, 1954 have never been officially corrected in available public records because the circumstances leading to the erroneous readings must have remained unknown to the relevant professionals. It took several days of exploratory work and the use of AI to discover the root source of the error, buried in an archive that was published eight years after the ice-jam occurrence that took place upstream of the gauging station involved. The observer at the time did not report ice, because the water was ice-free downstream of the ice-jam, yet the stage reading was heavily affected.

Based on the above, it is recommended that any attempt at future low-flow percentile estimation at Paks (or at any other gauging station of the Danube in Hungary) ought to be compared to the findings of the present study. In case it supplies confidence intervals significantly narrower than [328; 565] m^3s^{-1} for the 68% CI (or [231; 610] m^3s^{-1} for the 95% CI) when T_r is 10^5 years, then its conclusions ought to be considered with reservations, similarly when it supplies no CI at all and/or employs an uncorrected Q_{min} for 1954 at Budapest or uncorrected discharge values on January 13-15, 1954. The benchmark values of the combined CI boundaries are obtainable from Table 1 for the remaining T_r values employed in this study.

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