

Incorporating Variable Demand Assignment Into Discrete Network Design Problem

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ABSTRACT

The network design problem (NDP) is a bi-level problem with integer and decimal variables that aims to minimize the users' total cost under the budget constraints. Although utilizing variable demand models will theoretically change the NDP's result, the demand was assumed to be fixed and known in the literature. In this paper, a mathematical analysis will be presented to justify the importance of using variable demand in the discrete network design problem (DNDP). The DNDP for Sioux-falls network will be solved in both variable and fixed demand conditions by using total enumeration (in the upper-level) and Frank-Wolf (in the lower-level) method. The result shows that DNDP findings for variable and fix demand conditions have significant differences, especially in the mid-budget level.

Keywords: network design problem, variable demand, Frank-Wolf method

1. INTRODUCTION

Making decisions about how to allocate budget to potential projects is a major concern in transportation studies. NDP defines as selecting a subset of feasible options (projects), like building a new road, expanding an existing road, which minimizes the users' total cost and does not exceed a certain budget level. Theoretically, NDP can be formulated as a two-level problem with integer and decimal variables. In the upper-level problem, the number of integer variables is equal to the number of projects, and in the lower-level problem, the number of decimal variables is equal to the number of flow variables in the network's links. NDP had better study with realistic assumptions, or the system managers will be misled. A fundamental parameter that significantly influences the NDP result is users' travel demand, which can be deemed either fix or variable. The trip rate between origin and destination pairs typically assumes to be fixed and known. In reality, however, these trip rates may be influenced by the level of service on the network. For example, as congestion increases, motorists may decide to use a different mode of travel (e.g., subway), shift the time of travel (outside the design period), or forego some trips altogether. In order to take this phenomenon into account, the trip rate between every O-D pair in the network can be considered variable and assumed to be a function of the travel time. Although scientists believe that considering variable demand can better formulate the users' behavior, this realistic assumption has never been used in the NDP yet. In this paper, static and definite traffic demand assignment with variable demand is utilized in the lower-level problem, and the total enumeration method is used to find a feasible set of projects in the upper-level problem. In addition, the effect of using variable demand assumptions on DNDP's result will be investigated, and by comparing the results in the fix and variable demand conditions, the importance or unimportance of using variable demand assumptions will be discussed. This paper is organized as follows: First, a literature review on NDP is provided in Section 2. In the subsequent sections, DNDP formulation is extended in Section 3 to integrate the problem. The proposed approaches to model the variable demand function is presented and discussed in Section 4. A mathematical analysis is provided in section 5 to discuss the importance of using variable demand in DNDP and find the condition in which DNDP with variable and fix demand have different results. Numerical examples to ascertain the performance of the proposed approach is given in Section 6. Finally, Section 7 concludes the paper.

2. LITERATURE REVIEW

NDP has been continuously studied during the last five decades, and it is regarded as one of the most complicated concepts in transportation study because of computational difficulties in solving the mixed-integer nonlinear and bi-level program formulation. Dantzig et al. (1979) asserted that NDP is concerned with building new streets or expanding the capacity of existing streets. NDP can be divided into three different categories; (1) discrete network design problem

(DNDP), which includes discrete design decisions such as constructing new roads, adding new lanes, determining the directions of one-way streets, and determining the turning restrictions at intersections, (2) continuous network design problem (CNDP), which deals with continuous design decisions such as expanding the capacity of streets, scheduling traffic lights, and determining toll for some specific streets, and (3) mixed network design problem (MNDP), which concentrate on a combination of continuous and discrete decisions (Farahani et al., 2013). DNDP was first investigated by Steenbrink (1974). He utilized a single optimization method to minimize total societal costs. System optimization (SO) method was used as the traffic assignment model, the demand was deemed to be fixed and known, decision variables were new street constructing and street capacity expansion, and iterative decomposition algorithm was used to solve the problem. Leblanc (1975) used a single optimization method to minimize total travel time by considering definite user equilibrium (DUE) with fix demand as traffic assignment model, street capacity expansion as a decision variable, and Branch and Bound algorithm as a solution method. Poorzahedy and Turnquist (1982) solved DNDP with a similar assumption to Leblanc (1975). The differences were considering new street constructing as well as street capacity expansion and using Branch and Backtrack heuristics algorithm instead of the Branch and Bound algorithm. Chen and Alfa (1991) solved DNDP with a single optimization method that minimizes travel and construction cost. Stochastic user equilibrium (SUE) with fix demand was utilized as a traffic assignment model. Branch and Bound algorithm applied to solve the upper-level problem that includes new street constructing as a decision variable, and stochastic increment assignment applied for solving the lower-level problem. Gao et al. (2005) solved DNDP with a similar assumption to Leblanc (1975), and the differences were considering new street constructing as well as street capacity expansion and using generalized Benders decomposition method with support function instead of Branch and Bound algorithm. Poorzahedy and Abulghasemi (2005) solved DNDP with a similar assumption to Poorzahedy and Turnquist (1982); however, the Ant Systems algorithm was applied as a solution method. Poorzahedy and Rouhani (2007) adopted Poorzahedy and Turnquist's (1982) assumptions to solve DNDP; nonetheless, hybrids of ant systems, ant colony with a genetic algorithm, simulated annealing and Tabu search were utilized as a solution method. Miandoabchi and Farahani (2010) used a single optimization method to maximize reserve capacity by considering (DUE) with fix demand as traffic assignment model, street capacity expansion, lane allocation in two-way streets, and making some streets one-way as decision variables. Hybrid of simulated annealing, genetic algorithm, and evolutionary simulated annealing algorithm were used as solution methods. Babazadeh et al. (2011) solved DNDP with a similar assumption to Poorzahedy and Turnquist (1982); however, the particle swarm optimization algorithm was applied as a solution method. Wang et al. (2013) adopted Poorzahedy and Turnquist's (1982) assumptions to solve DNDP, and the differences were utilizing two global optimization algorithms as a solution method. These global optimization algorithms use the relation between SO and UE. Finally, Wang et al. (2015) solved DNDP with a similar assumption to Poorzahedy and Turnquist (1982); however, a global optimization method proposed as a solution method that not only does find the

optimal subset of the project but also calculates their ideal capacity. It is worth mentioning that, variable demand assumption has never been utilized in NDP yet (Farahani et al., 2013).

3. DNDP FORMULATION

Let $N(I, A)$ be a graph representing a transportation network with node-set I and arc set A . Consider \bar{A} as a set of arc projects in which $\bar{A} \cap A$ includes link improvement projects (e.g., expanding an existing road) and $\bar{A} - A$ consists of new project links. y_a is a decision variable for link $a \in \bar{A}$, and $y = (y_a)$ is a decision vector that consists of decision variables y_a . The decision variable y_a defines whether the problem is CNDP or DNDP. In DNDP y_a is a binary variable, selection or not selection a project, and in the CNDP y_a is an integer variable. For instance, if the maximum and minimum link's capacity are supposed to be C_{\max} and C_{\min} , the constraint $C_{\min} < y < C_{\max}$ defines a continuous range for decision y . In DNDP's decision about selection or not selection a project, expanding or not expanding a project defines with the constraint $y \in \{0, 1\}$. Since this paper adopts DNDP, its formulation will be discussed in this section. Assume that $\bar{A}(y) \subseteq \bar{A}$ is a subset of selected projects in the decision y , which is shown as $\bar{A}(y) = \{a \in \bar{A} \mid y_a = 1\}$. DNDP for finding a decision variable y in the budget level B is presented as follows.

$$DNDP \left\{ \begin{array}{l} \text{Min}T(y) = \sum_{a \in A - \bar{A}(y)} x_a t_a(x_a) + \sum_{a \in \bar{A}(y)} x_a \bar{t}_a(x_a) \\ \text{s.t.} \\ \sum_{a \in \bar{A}} c_a y_a \leq B \\ y_a \in \{0, 1\} \quad \forall a \in \bar{A}(y) \\ x(y) = \text{Assign}(y) \end{array} \right. \quad (1)$$

$T(y)$: Network's total time for decision y

x_a : Link's flow for $a \in \bar{A} \cup A$

$t_a(x)$: Travel time function for link $a \in A$ in the base network

$\bar{t}_a(x)$: Travel time function for link $a \in \bar{A}$ after link improvement

y_a : Decision variable for link $a \in \bar{A}$

$x(y)$: Vector of links' flow after the decision y , $x(y) = (x_a)_{a \in A \cup \bar{A}(y)}$

c_a : Cost of implementing a project $a \in \bar{A}$

B : Budget level

Assign: An assignment algorithm that finds vector $x(y)$ for decision y

In this problem, the objective function searches for finding a decision variable $y = (y_a)$ that minimizes the networks total time $T(y)$. The first and second constraints are feasibility constraints for the decision variable y . The first one is a budget constraint, and the second one is a feasible area constraint. The third constraint is a traffic assignment model that finds the traffic flow for each network configuration and demand condition.

3.1. TRAFFIC ASSIGNMENT PROBLEM

Wardrop (1952) first discussed the equilibrium condition in the transportation network. According to Wardrop's first rule, for each O-D pair, at user equilibrium, the travel time on all used paths are equal, and less than or equal to the travel time that would be experienced by a single vehicle on any unused path. The traffic condition that satisfies this situation is called UE (User Equilibrium). The UE problem with two fundamental assumptions would be changed to an optimization problem; (1) each link's travel time function is only dependent on its own flow, (2) travel time function should be both positive and strictly increasing. This problem was first developed by Beckmann et al. (1956) and is presented as follows.

$$\left\{ \begin{array}{l} \min z(x) = \sum_a \int_0^{x_a} t_a(\omega) d\omega \\ \text{s.t.} \\ \sum_k f_k^{rs} = q_{rs} \quad \forall r, s \\ f_k^{rs} \geq 0 \quad \forall r, s, k \\ x_a = \sum_{rs} \sum_k f_k^{rs} \delta_{ak}^{rs} \quad \forall a \in A \end{array} \right. \quad (2)$$

In this model, x_a is the flow of link a , $t_a(x)$ is the travel time function of link a , f_k^{rs} is flow in path k for origin-destination rs and q_{rs} is fix and known demand between origin-destination rs . The parameter δ_{ak}^{rs} is equal to 1 if the links a lies on the path k that is related to origin-destination rs and is equal to zero otherwise. In the Beckman model, the objective function is

equal to the sum of travel time functions' integral and has neither economic nor behavioral meaning. The first constraint indicates that the sum of path flow, which is related to an origin-destination pair should be equal to that origin-destination pair's demand. The second constraint was mentioned to assure the feasibility of the solution, and the third constraint shows the relation between flow in the link a and the flow in the path k which is related to origin-destination rs .

The optimization UE model is formulated by assuming fix and known demand between every origin-destination pairs. However, in the real condition, the demand may be influenced by the level of service. For instance, as congestion increases, the traveler is more likely to choose other modes of transportation. To formulate UE with variable demand consider q_{rs} as the variable demand between origin-destination rs , which is a function of travel time ($q_{rs} = D_{rs}(u_{rs})$). u_{rs} is the minimum travel time between origin-destination rs and $D_{rs}(\cdot)$ is a demand function between origin-destination rs . The UE with variable demand is as follows.

$$\left\{ \begin{array}{l} \min z(x, q) = \sum_a \int_0^{x_a} t_a(\omega) d\omega - \sum_{rs} \int_0^{q_{rs}} D_{rs}^{-1}(\omega) d\omega \\ \text{st.} \\ \sum_k f_k^{rs} = q_{rs} \quad \forall r, s \\ f_k^{rs} \geq 0 \quad \forall r, s, k \\ q_{rs} \geq 0 \quad \forall r, s \end{array} \right. \quad (3)$$

In this model, $D_{rs}^{-1}(\cdot)$ is the inverse of demand function between origin-destination rs and the equation $x_a = \sum_{rs} \sum_k f_k^{rs} \delta_{ak}^{rs}$ shows the relation between flow in link a and path k . The additional term in the objective function comparing to the Beckman model is the sum of the inverse of demand functions' integrals. The first and second constraints were discussed previously and the third one mentioned to assure the feasibility of demand between each origin-destination.

3.2. BI-LEVEL DNDP

DNDP can be expressed by using either SO or UE as an assignment model. Utilizing the SO model would result in creating a one level problem that is easier to solve; nonetheless, a precise result will not be obtained since the assumption is not realistic. On the other hand, using the UE model would create a bi-level formulation. Regardless of difficulties in solving the problem, this formulation was widely used since it was based on more realistic assumptions than the one-level formulation. In this model, the upper-level is devoted to optimizing policymakers' objective(s)

with regard to budget constraints, and the lower-level could be UE model with either fixed or variable demand. This paper adopts bi-level programming to solve DNDP. This problem for variable demand conditions can be shown as follows.

$$\begin{cases}
 \text{Min} T(y) = \sum_{a \in A - \bar{A}(y)} x_a t_a(x_a) + \sum_{a \in \bar{A}(y)} x_a \bar{t}_a(x_a) \\
 \text{s.t.} \\
 \sum_{a \in \bar{A}} c_a y_a \leq B \\
 y_a \in \{0, 1\} \quad \forall a \in \bar{A}(y) \\
 x(y) \text{ is solution of } LLP(y)
 \end{cases} \quad (4)$$

$$\begin{cases}
 \text{min } z(x, q) = \sum_{a \in A - \bar{A}(y)} \int_0^{x_a} t_a(w) dw + \sum_{a \in \bar{A}(y)} \int_0^{x_a} \bar{t}_a(w) dw - \sum_{rs} \int_0^{q_{rs}} D_{rs}^{-1}(\omega) d\omega \\
 \text{s.t.} \\
 \sum_k f_k^{rs} = q_{rs} \quad \forall r, s \\
 f_k^{rs} \geq 0 \quad \forall r, s, k \\
 q_{rs} \geq 0 \quad \forall r, s \\
 x_a = \sum_{(r,s)} \sum_k f_k^{rs} \delta_{ak}^{rs} \quad \forall a \in A \cup \bar{A}(y)
 \end{cases} \quad (5)$$

In this model, the upper-level problem searches for finding the decision variable $y = (y_a)$ that minimizes the network's total time $T(y)$, and the lower-level problem is a traffic assignment model that yields a flow vector $x(y)$ for decision y .

3.3. SOLUTION ALGORITHM

In this paper, a convex combination based algorithm is used to solve the TAP with variable demand. This algorithm can be summarized as follows (Sheffi, 1985).

Step 0: Initialization. Find an initial feasible flow pattern for x_a^n and q_{rs}^n . Set $n = 1$.

Step 1: Update. Set $t_a^n = t_a(x_a^n) \quad \forall a$. Compute $D_{rs}^{-1}(q_{rs}^n) \quad \forall r, s$.

Step 2: Direction finding. Compute the shortest path, m , between each O_D pair r, s based on t_a^n . C_k^{rs} is the time of path k related to origin-destination rs . Execute the assignment rule as follows:

$$\begin{cases} \text{if } C_m^{rs^n} < D_{rs}^{-1}(q_{rs}) \Rightarrow g_k^{rs} = \bar{q}^{rs} \ \& \ g_k^{rs} = 0 \quad \forall k \neq m \\ \text{if } C_m^{rs^n} > D_{rs}^{-1}(q_{rs}) \Rightarrow g_k^{rs} = 0 \quad \forall k \end{cases}$$

This assignment yields an auxiliary flow pattern y_a^n, v_{rs}^n . With these variables, the descent direction is a vector d^n with elements $(y_a^n - x_a^n)$ and $(v_{rs}^n - q_{rs}^n)$. g_k^{rs} is a temporary path variable.

$$\begin{cases} y_a^n = \sum_{rs} \sum_k g_k^{rs} \cdot \delta_{ak}^{rs} & ; \forall a \\ v_{rs}^n = \sum_k g_k^{rs} & ; \forall r, s \end{cases}$$

Step 3: Move size. Find α_n by solving the following minimization program.

$$\begin{cases} \min Z(\alpha) = \sum_a \int_0^{x_a^n + \alpha(y_a^n - x_a^n)} t_a(\omega) d\omega - \sum_{rs} \int_0^{q_{rs}^n + \alpha(v_{rs}^n - q_{rs}^n)} D_{rs}^{-1}(\omega) d\omega \\ \text{s.t.} \\ 0 \leq \alpha \leq 1 \end{cases}$$

Step 4: Flow update. Find x_a^{n+1} and q_{rs}^{n+1} by using equations $x_a^{n+1} = x_a^n + \alpha(y_a^n - x_a^n)$ and $q_{rs}^{n+1} = q_{rs}^n + \alpha(v_{rs}^n - q_{rs}^n)$.

Step 5: Convergence criterion. If the following inequality holds, terminate. Otherwise, set $n = n+1$ and go to step 1.

$$RG = \frac{\sum_a f_a t_a - \sum_{rs} q_{rs} u_{rs}}{\sum_a f_a t_a} \leq \kappa$$

4. VARIABLE DEMAND

Travel demand refers to the amount and type of travel that people would choose in particular situations. Various demographic factors like employment rate and preferences, geographic factors including roadway design and walkability, and economic factors consist of road toll, and fuel prices can affect travel demands. Models that reflect these relationships can predict how various trends, policies and projects will affect future travel activity, and therefore evaluate potential problems and transport system improvement strategies (Litman 2013). Travel demand is mostly dependent on travel costs, including financial cost, time cost, travel risk, etc. The relation between these parameters could be formulated as a decreasing function (Fig. 1) since as

the travel cost augments, the travel demand diminishes and vice versa. As shown in Figure 1, the vertical axis approaches a constant and limited amount which means if travel cost reduces drastically, users will increase their travel in the network. Travel demand's sensitivity is measured by its elasticity. As shown in Figure 2, in the demand function with higher elasticity, an infinitesimal change in travel cost lead to a conspicuous shift in travel demand, but in travel demand with lower elasticity changes in travel cost has far less effect on travel demand.

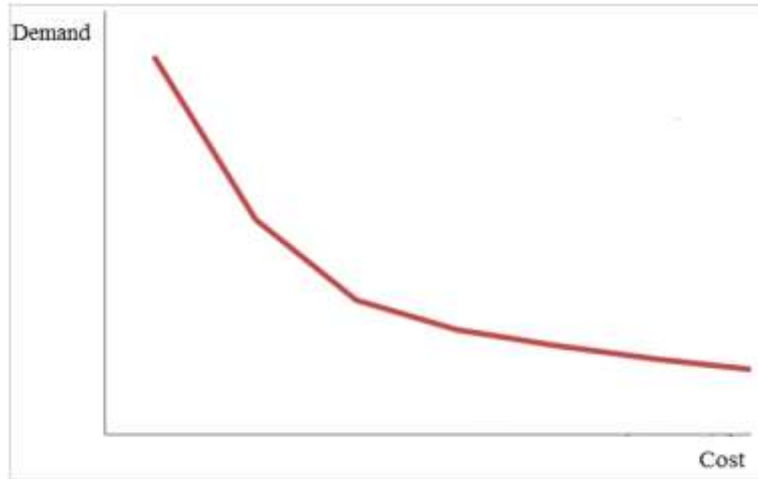


Figure 1. Travel demand function

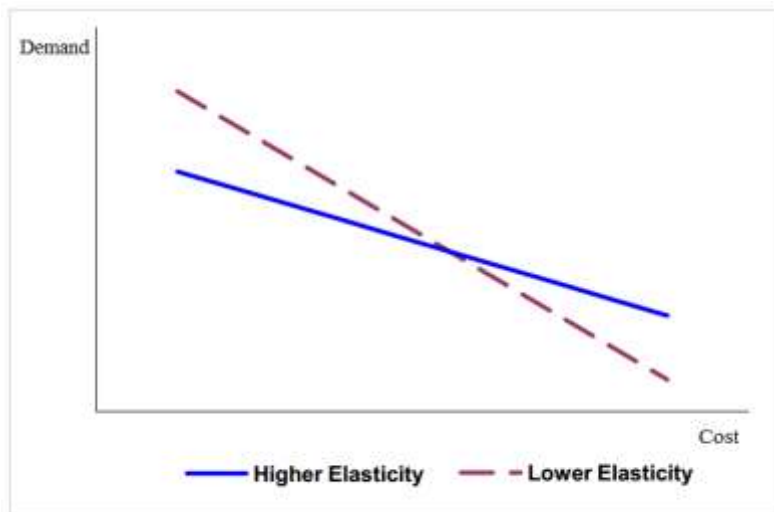


Figure 2. Travel demand sensitivity

Travel demand elasticity with respect to travel time is defined as the amount of change in travel demand caused by one unit change in travel time. This elasticity was reported for several models

including literature EU, the Netherlands' national model, Italian national model and model for Brussels (De Jong et al. 2001). Table 1 shows travel demand elasticity with respect to travel time for these four models in the short term and long term conditions. According to this Table, the elasticity changes in the range of $(-0.62, -0.12)$ for the literature EU, $(-0.58, -0.04)$ for the Netherlands' national model, $(-0.70, -0.09)$ for the Italian national model and $(-0.36, -0.23)$ for the Brussels model. Since travel demand elasticity with respect to travel time in all models can be restricted to the range of $(-1, 0)$, the valid range for elasticity is assumed to be $(-1, 0)$ in this paper.

Table 1. Travel demand elasticity with respect to travel time

Term / Purpose	Model For Brussels	Italian National Model	The Netherlands' National Model	Literature EU
<i>Short Term:</i>				
Commuting	-0.23	-0.54	-0.39	-0.62
Home-based business		-0.29	-0.07	
Non-home-based business			-0.04	
Education		-0.66	-0.06	
Other		-0.09	-0.11	-0.52
Total			-0.20	-0.60
<i>Long Term:</i>				
Commuting	-0.36	-0.56	-0.58	-0.41
Home-based business		-0.29	-0.12	-0.30
Non-home-based business			-0.10	-0.12
Education		-0.70	-0.19	-0.57
Other		-0.09	-0.21	-0.52
Total			-0.33	-0.29

4.1 VARIABLE DEMAND MODELING

Variable demand functions that were used in traffic assignment problems can be categorized in two different models, including linear and exponential functions. Assume that for each origin-destination pair, rs the variable demand q_{rs} is shown as a function of travel time u_{rs} between this origin-destination pair. Florian et al. (1974) and Ashtiani (1976) used the linear variable demand function $q_{rs}(u_{rs}) = \phi_{rs} - \theta_{rs}u_{rs}$ for every origin-destination pair rs , and the parameters ϕ_{rs} and θ_{rs} were selected randomly. Han et al. (2015) adopted the same linear function with parameters $\phi_{rs} = 800$ and $\theta_{rs} = 500$. Yang et al. (1997), Yang et al. (2004), Zhou et al. (2014) utilized an exponential variable demand function $q_{rs}(u_{rs}) = \alpha_{rs}e^{(-\beta_{rs}u_{rs})}$ with random parameters α_{rs} and β_{rs} .

Assume that $q = q(u)$ as a variable demand function and q_0 as a fixed demand for a specific origin-destination pair. If the minimum travel time between this pair for the fixed demand q_0 was shown by u_0 , the variable demand function $q(u)$ could be defined for this pair under two fundamental conditions.

Condition 1. The demand for the u_0 position should be q_0 , or $q_0 = q(u_0)$.

Condition 2. Travel demand elasticity with respect to travel time (ε) for all feasible value u should be in the range of $(-1, 0)$.

Consider the linear demand function $q_u = q_0 - b(u - u_0)$ with fix variable b and suppose ε_0 as elasticity in the point (q_0, u_0) . Therefore, this elasticity can be shown as:

$$\varepsilon_0 = \frac{dq(u_0)}{du} \times \frac{u_0}{q_0} = \frac{-bu_0}{q_0} \quad (6)$$

The parameter b can be calculated as follows:

$$b = -\varepsilon_0 \frac{q_0}{u_0} \quad (7)$$

As a result, the linear demand function can be shown as $q_u = q_0 + \frac{\varepsilon_0 q_0}{u_0}(u - u_0)$ and is depicted in

Figure 3.

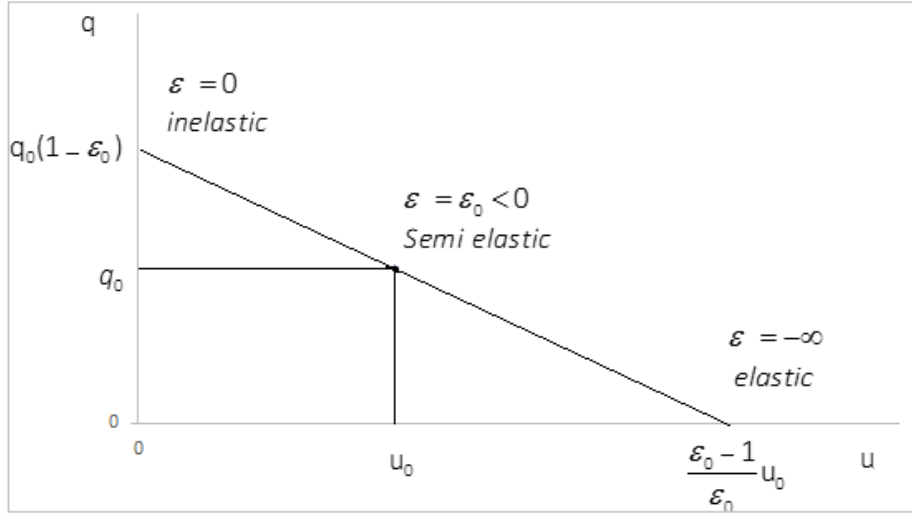


Figure 3. Linear demand function

In the same way, to hold the two fundamental conditions, the exponential demand function should be defined as $q = q_0 e^{-b(u-u_0)}$. In this function, b is a fixed variable and demand elasticity with respect to u in the point (q_0, u_0) would be calculated as follows.

$$\epsilon_0 = \frac{dq(u_0)}{du} \times \frac{u_0}{q} = -bq_0 e^{-b(u_0-u_0)} \times \frac{u_0}{q_0} = -bu_0 \quad (8)$$

Therefore, the exponential demand function can be shown as $q = q_0 e^{\frac{-\epsilon_0}{u_0}(u - u_0)}$ and is depicted in Figure 4.

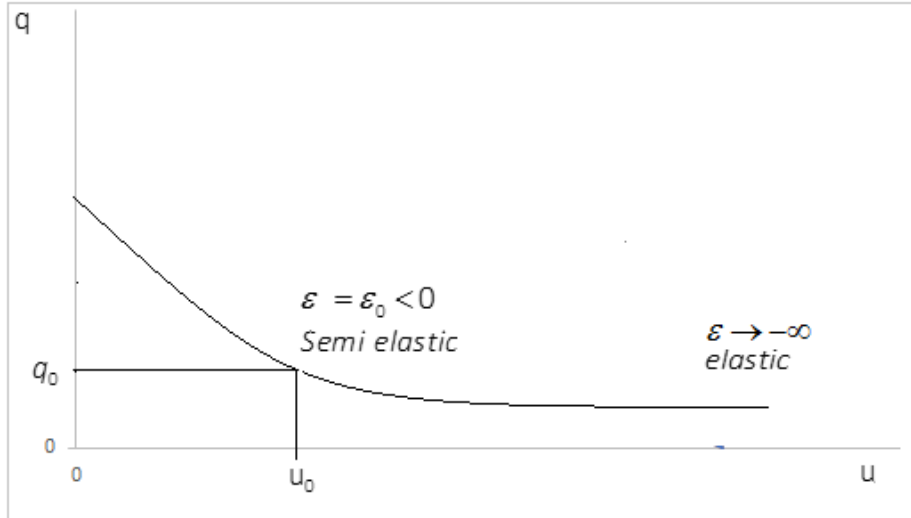


Figure 4. Exponential demand function

5. MATHEMATICAL ANALYSIS

This section aims to find a relation between DNDP results with fix and variable demand with respect to travel time and variable demand functions' parameters. Consider a base network with 2 nodes and n parallel arcs like the one used in Sheffi (1985) as depicted in Figure 5, and also assume that the arc $n+1$ as a project link. DNDP's objective is to decide about selecting or not selecting the project link; therefore, two networks exist for the problem, which are the networks with n and $n+1$ parallel links. The objective functions for these two networks are TC_0 and TC_1 respectively.

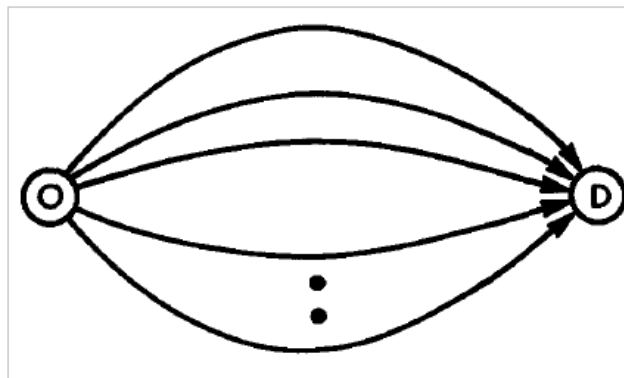


Figure 5. Base network with 2 nodes

The travel time function for each link i is $t_i = \alpha_i + \beta_i x_i$ with fix variables α_i and β_i , and x_i is the flow in link i . In the base network, for fix demand condition $x_{i,fix}$ and $t_{i,fix}$ indicate flow and travel time for link i respectively, u_0 and q_0 denote minimum travel time and fix demand between origin-destination pair in the same order as already mentioned and TC_1^{fix} shows the network's total time. By assuming links' flow to be positive, the minimum travel time between origin-destination pair is equal to links' travel time.

$$t_{i,fix} = u_0 \quad (9)$$

Considering link's travel time function, the link's flow can be shown as:

$$x_{i,fix} = \frac{t_{i,fix} - \alpha_i}{\beta_i} \quad (10)$$

Substituting (9), the link's flow would be:

$$x_{i,fix} = \frac{u_0 - \alpha_i}{\beta_i} \quad (11)$$

The sum flow in all links is equal to the fixed demand since links are parallel.

$$\sum_{i=1}^n x_{i,fix} = q_0 \quad (12)$$

If the link's flow from (11) is substituted in the (12), the following relation will hold.

$$\sum_{i=1}^n \frac{u_0 - \alpha_i}{\beta_i} = q_0 \quad (13)$$

Therefore, the minimum travel time between the origin-destination pair can be calculated from (13) and is:

$$u_0 = \frac{q_0 + \sum_{i=1}^n \frac{\alpha_i}{\beta_i}}{\sum_{i=1}^n \frac{1}{\beta_i}} \quad (14)$$

Network's total time defines as the sum of products of links' flow and link's travel time and is shown in (15).

$$TC_0^{fix} = u_0 q_0 = \frac{q_0 + \sum_{i=1}^n \frac{\alpha_i}{\beta_i}}{\sum_{i=1}^n \frac{1}{\beta_i}} \times q_0 \quad (15)$$

If the project link is added to the base network, the candidate network will be obtained. By assuming $x'_{i,fix}$ and $t'_{i,fix}$ flow and travel time of link i respectively and u_1 as the minimum travel time between origin-destination pair, the network's total time in the candidate network for fix demand condition can be calculated in the same way.

$$u_1 = \frac{q_0 + \sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i}}{\sum_{i=1}^{n+1} \frac{1}{\beta_i}} \quad (16)$$

$$TC_1^{fix} = u_1 q_0 = \frac{q_0 + \sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i}}{\sum_{i=1}^{n+1} \frac{1}{\beta_i}} \times q_0 \quad (17)$$

A linear variable demand function with the form of $q = q_0 - b(u - u_0)$ is used to investigate the network in variable demand conditions. In this function, fix variable b assumes to be positive ($b > 0$), and u denotes the minimum travel time between origin-destination pair in variable demand condition. Obviously, the demand in the base network would become q_0 because $q_0 = q(u_0)$. Consequently, the network's total time in the base network for variable demand would be equal to TC_0^{fix} as shown in the subsequent equation.

$$TC_0^{els} = u_0 [q_0 - b(u_0 - u_0)] = u_0 q_0 = TC_0^{fix} \quad (18)$$

In the candidate network, $x'_{i,els}$ and $t'_{i,els}$ indicate flow and travel time of link i respectively, u' shows the minimum travel time between the origin-destination pair and the equation $q' = q(u')$ holds consequently. In this network, the minimum travel time between origin-destination pair is equal to links' travel time.

$$t'_{i,els} = u' \quad (19)$$

Considering linear travel time function, link's flow can be shown as:

$$x'_{i,els} = \frac{t'_{i,els} - \alpha_i}{\beta_i} \quad (20)$$

By substituting (19) into the later equation, link's flow would be:

$$x'_{i,els} = \frac{u' - \alpha_i}{\beta_i} \quad (21)$$

The sum of all links' flow equals the variable demand between the origin-destination pair since they are parallel.

$$\sum_{i=1}^{n+1} x'_{i,els} = q' = q_0 - b(u' - u_0) \quad (22)$$

Substituting link's flow from (21) into (22) would result in:

$$\sum_{i=1}^{n+1} \frac{u' - \alpha_i}{\beta_i} = q' \quad (23)$$

Therefore, the minimum travel time between the origin-destination pair is equal to:

$$u' = \frac{q_0 + bu_0 + \sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i}}{b + \sum_{i=1}^{n+1} \frac{1}{\beta_i}} \quad (24)$$

Finally, the network's total time for the candidate network and variable demand condition would be shown as follows:

$$TC_1^{els} = u'q' = \frac{q_0 + bu_0 + \sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i}}{b + \sum_{i=1}^{n+1} \frac{1}{\beta_i}} \times \left[q_0 - b \left(\frac{q_0 + bu_0 + \sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i}}{b + \sum_{i=1}^{n+1} \frac{1}{\beta_i}} - u_0 \right) \right] \quad (25)$$

The strategies that DNDP's result differs in fix and variable demand conditions can be found by comparing the equations (15), (17) and (25). These strategies are:

$$\begin{cases} TC_1^{els} > TC_0^{els} & \text{if } TC_1^{fix} < TC_0^{fix} \\ TC_1^{els} < TC_0^{els} & \text{if } TC_1^{fix} > TC_0^{fix} \end{cases} \quad (26)$$

Considering (18) the previous strategies would result in one of the following inequalities:

$$TC_1^{els} > TC_0^{fix} > TC_1^{fix} \quad (27)$$

$$TC_1^{els} < TC_0^{fix} < TC_1^{fix} \quad (28)$$

By expanding the first part of (27), the following term will hold:

$$TC_1^{els} > TC_0^{fix} \rightarrow u' [q_0 - b(u' - u_0)] > q_0 u_0 \rightarrow b > \frac{q_0}{u'} = \frac{q_0 \left[b + \sum_{i=1}^{n+1} \frac{1}{\beta_i} \right]}{[q_0 + b u_0] + \sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i}} \quad (29)$$

Simplification of (29) gives a quadratic inequality as follows:

$$b^2 u_0 + b \left[\sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i} \right] - q_0 \left[\sum_{i=1}^{n+1} \frac{1}{\beta_i} \right] > 0 \quad (30)$$

Since $b > 0$ inequality's result can be confined in the following range:

$$b > \frac{- \left[\sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i} \right] + \left[\left(\sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i} \right)^2 + \left(\sum_{i=1}^{n+1} \frac{1}{\beta_i} \right) \times 4 u_0 q_0 \right]^{0.5}}{2 u_0} \quad (31)$$

In addition, expanding the other side of (27) will lead to:

$$TC_1^{fix} < TC_0^{fix} \rightarrow u_1 q_0 < u_0 q_0 \rightarrow \frac{q_0 + \sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i}}{\sum_{i=1}^{n+1} \frac{1}{\beta_i}} \times q_0 < \frac{q_0 + \sum_{i=1}^n \frac{\alpha_i}{\beta_i}}{\sum_{i=1}^n \frac{1}{\beta_i}} \times q_0 \quad (32)$$

Simplification of (32) gives the following inequality for q_0 :

$$q_0 > \alpha_{n+1} \times \left(\sum_{i=1}^n \frac{1}{\beta_i} \right) - \sum_{i=1}^n \frac{\alpha_i}{\beta_i} \quad (33)$$

Furthermore, elaboration of (28) results in the subsequent inequality:

$$TC_1^{fix} > TC_0^{fix} \rightarrow u_1 q_0 > u_0 q_0 \rightarrow \frac{q_0 + \sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i}}{\sum_{i=1}^{n+1} \frac{1}{\beta_i}} \times q_0 > \frac{q_0 + \sum_{i=1}^n \frac{\alpha_i}{\beta_i}}{\sum_{i=1}^n \frac{1}{\beta_i}} \times q_0 \quad (34)$$

By simplifying (34) the following term will be found:

$$q_0 + \sum_{i=1}^n \frac{\alpha_i}{\beta_i} - \alpha_{n+1} \times \left(\sum_{i=1}^n \frac{1}{\beta_i} \right) < 0 \quad (35)$$

If (35) be divided by the positive term $\left(\sum_{i=1}^n \frac{1}{\beta_i} \right)$, the next inequality will holds:

$$\frac{q_0 + \sum_{i=1}^n \frac{\alpha_i}{\beta_i}}{\sum_{i=1}^n \frac{1}{\beta_i}} - \alpha_{n+1} < 0 \quad (36)$$

Substituting (14) the previous term results in:

$$u_0 - \alpha_{n+1} < 0 \quad (37)$$

If (37) be divided by positive term β_{n+1} , the link's flow will be obtained as it shown in the next term.

$$x_{n+1} = \frac{u_0 - \alpha_{n+1}}{\beta_{n+1}} < 0 \quad (38)$$

Since the link's flow can never be negative, the former term and consequently (28) will never hold. As a result, just when the following inequalities hold, DNDP's result in fix and variable demand conditions differs.

$$\left\{ \begin{array}{l} b > \frac{-\left[\sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i} \right] + \left[\left(\sum_{i=1}^{n+1} \frac{\alpha_i}{\beta_i} \right)^2 + \left(\sum_{i=1}^{n+1} \frac{1}{\beta_i} \right) \times 4u_0 q_0 \right]^{0.5}}{2u_0} \\ q_0 > \alpha_{n+1} \times \left(\sum_{i=1}^n \frac{1}{\beta_i} \right) - \sum_{i=1}^n \frac{\alpha_i}{\beta_i} \end{array} \right. \quad (39)$$

5.1. EXAMPLE

Assume a network like Figure 5 with $n=2$ parallel arcs and link number 3 to be the project link. The parameters α_i and β_i for links are shown in Table 2, the variable demand function is $q = q_0 - b(u - u_0)$ and $q_0 = 25$.

Table 2. Links' parameters

Arc Number	Origin Node	End Node	α_i	β_i
1	1	2	0.13	0.62
2	1	2	0.7	0.72
3 (Project link)	1	2	0.4	0.25

Table 3 shows the networks' total time for base and candidate networks in the fix demand condition (TC_0^{fix} and TC_1^{fix} respectively). As it was shown in the Table $TC_1^{fix} < TC_0^{fix}$ and therefore DNDP's result in this situation will be selection of project link (candidate network). To finds DNDP's result in the variable demand condition, (39) will be utilized. By using the problem's input data, the second term in (39) will be $q_0 > 0.019$ that holds for $q_0 = 25$, and the first term of (39) can be summarized as $b > 4.3$. Table 3 shows the networks' total time for base and candidate networks in the variable demand condition for $b = 4.2$ and $b = 4.4$. According to the condition $b = 4.4 > 4.3$ lead to $TC_1^{els} > TC_0^{fix}$ and therefore, DNDP's result will be not selecting the project link (base network). As a result, utilizing variable demand in DNDP can drastically affect the result comparing to the fix demand condition.

Table 3. Networks' total time

Networks' Total Time		Value
TC_0^{fix}		218.05
TC_1^{fix}		99.20
TC_1^{els}	$b = 4.2$	215.53
	$b = 4.4$	219.60

6. NUMERICAL EXAMPLE

To show the importance of using variable demand function in DNDP, the problem for both fix and variable demand conditions will be solved for the Sioux Falls network and the result will be compared. The Sioux Falls network is a widely known network in transportation engineering and consists of 24 nodes, 76 arcs and 552 origin-destination pairs. The travel time for link (i, j) is assumed to be the BPR function ($t_{ij} = a_{ij} + b_{ij}x_{ij}^4$) with the same parameters that were used in Leblanc et al. (1975). Moreover, 10 projects propose to be deemed for network improvement with condition and order mentioned in Babazadeh et al. (2011), and as a result, 2^{10} different networks should be assessed. As it was shown in section 5, utilizing variable demand in DNDP can significantly change the result with regard to the fix demand condition. In this section, DNDP for both fix and variable demand condition will be solved by using total enumeration method for upper-level problem. To solve the lower-level problem (traffic assignment problem),

a Frank-Wolf algorithm (Frank et al. 1956) is used for fix demand condition and a convex combination based algorithm is adopted for variable demand conditions (Sheffi 1985) as it was presented in section 3.3.

Assume q_{rs}^0 as the fixed demand between origin-destination pair rs with the same value that were used in Leblanc et al. (1975), u_{rs}^0 as the minimum travel time between origin-destination pair rs which is found by solving TAP for fix demand q_{rs}^0 , $q_{rs}(u_{rs})$ as the variable demand function, and u_{rs} as the minimum travel time between origin-destination pair rs in the network. An exponential variable demand function is used as follows.

$$q_{rs} = q_{rs} + \frac{1}{n+1}[q_{rs}(u_{rs}) - q_{rs}] \quad \forall rs \quad (40)$$

This function satisfies the first fundamental condition mentioned in the section 4.1 since $q_{rs}^0 = q_{rs}(u_{rs}^0)$. To meet the second fundamental condition, the elasticity is assumed to be $\varepsilon_0 = -0.7$, because the results from TAP with variable demand for 1024 networks show that by adding project links to the base network u_{rs} usually decreases (or remain constant) and q_{rs} increases consequently. Therefore, the elasticity increases according to the (41).

$$\varepsilon = \frac{-b_{rs} u_{rs}}{q_{rs}} \quad (41)$$

Adopting different values for ε_0 in the range of $(-1,0)$ shows that using $\varepsilon_0 = -0.7$ would keep the elasticity of all candidate networks in the range of $(-1,0)$. According to this assumption, the parameter b_{rs} can be defined as follows.

$$b_{rs} = \frac{-\varepsilon_0}{u_{rs}^0} = \frac{-0.7}{u_{rs}^0} \quad (42)$$

Thereby, the variable demand function can be described as follows.

$$q_{rs} = q_{rs}^0 e^{\frac{-0.7}{u_{rs}^0}(u_{rs} - u_{rs}^0)} \quad \forall rs \quad (43)$$

First, DNDP will be solved for fix demand conditions. The upper-level problem is solved using the total enumeration method that is coded by MATLAB software, and the lower-level problem is implemented by the Frank-Wolfe algorithm which is coded with Visual Studio C++ software. The problem is solved on a computer with Intel Core i7-CPU @ 2.26 GHz and 2 GB RAM in 243.57 second. The result for different budget levels is shown in Table 4. In this Table, the

selected projects are indicated by 1, and not selected project are shown by zero. Every budget level is defined by a ratio (B/C), the parameters B and C are the available budget and total cost of all 10 projects, respectively.

Table 4. DNDP result with fix demand condition

$\frac{B}{C}$	PROJECT										NETWORK TOTAL COST
	1	2	3	4	5	6	7	8	9	10	
0.00	0	0	0	0	0	0	0	0	0	0	74.80
0.10	1	1	0	0	0	0	0	0	0	0	69.06
0.20	0	1	0	0	0	0	0	1	0	0	59.40
0.30	1	1	1	0	0	0	0	1	0	0	56.18
0.40	1	1	1	0	1	0	0	1	0	0	53.78
0.50	0	1	1	0	1	0	0	1	0	1	51.21
0.60	0	1	0	0	1	0	1	1	0	1	49.33
0.70	1	1	0	1	1	0	1	1	0	1	47.44
0.80	0	1	0	1	1	0	1	1	1	1	46.70
0.90	1	1	1	1	1	0	1	1	1	1	45.67
1.00	1	1	1	1	1	1	1	1	1	1	45.40

The result shows that by adding projects to the base network, u_{rs} for most origin-destination pairs decreases. The variation of maximum, minimum and average of u_{rs} for all origin-destination pairs in different budget levels is depicted in Figure6. As it was expected, increases in budget level lead to a decrease in maximum, minimum and average of u_{rs} .

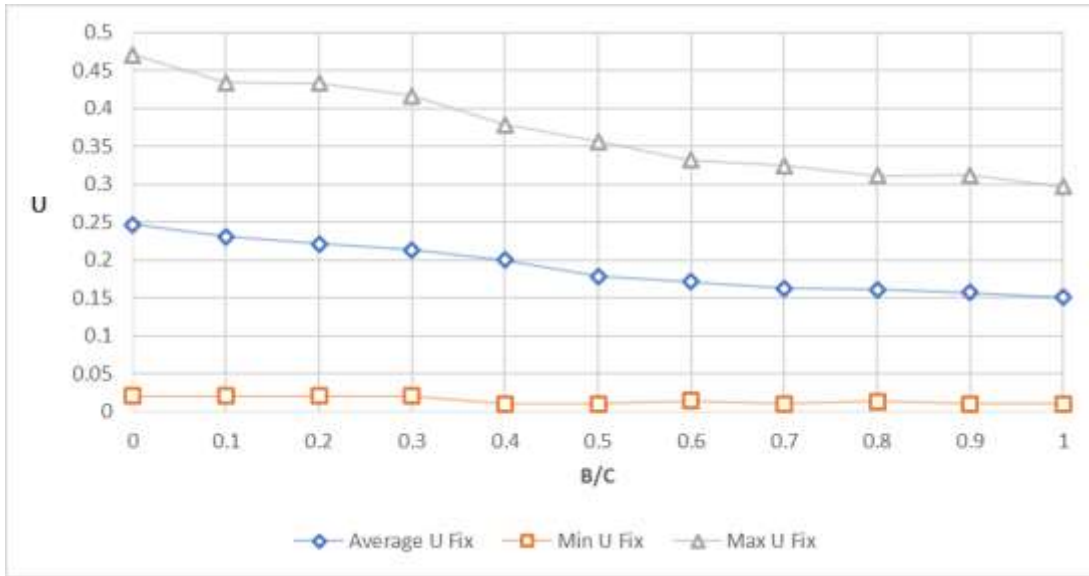


Figure 6. Variation of maximum, minimum and average of u_{rs} for all origin-destination pairs in fix demand condition

Moreover, the variation of maximum, minimum and average of links' flow and links' time for different budget levels in fix demand condition are depicted in Figure 7 and 8, respectively. Although in some cases augmentation of budget level leads to an increase of maximum and minimum of links' flow and links', these parameters show a decreasing trend.

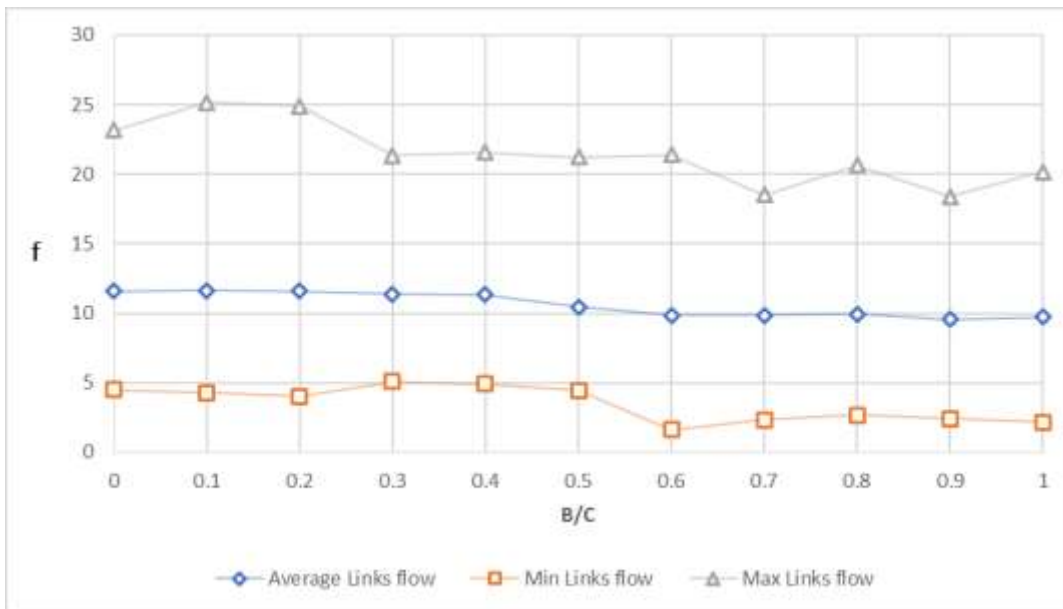


Figure 7. Variation of maximum, minimum and average of links' flow in fix demand condition

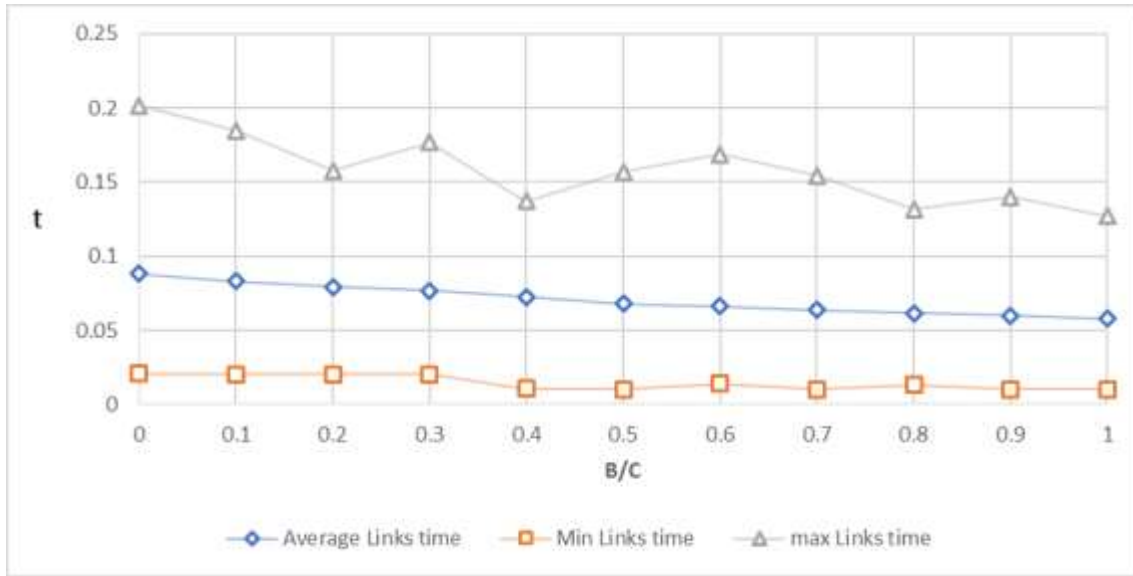


Figure 8. Variation of maximum, minimum and average of links' time in fix demand condition

By finding parameters b for all origin-destination according to (42), DNDP is solved for variable demand conditions. The convex combination based algorithm is adopted for solving the lower-level problem which implemented by Visual Studio C++ software, and the upper-level problem is solved by the total enumeration method like the fix demand condition. The solution time is found to be 295.35 seconds, which is 51.78 seconds more than the fix demand condition. The best project in different budget levels and their relative network total time is shown in Table 5.

Table 5. DNDP result for variable demand condition

$\frac{B}{C}$	PROJECT										NETWORK TOTAL COST
	1	2	3	4	5	6	7	8	9	10	
0.00	0	0	0	0	0	0	0	0	0	0	74.80
0.10	1	1	0	0	0	0	0	0	0	0	74.48
0.20	0	1	0	0	0	0	0	1	0	0	70.67
0.30	0	0	0	0	0	0	0	1	0	1	69.78
0.40	1	1	0	0	0	0	0	1	0	1	68.83

0.50	0	1	0	0	0	0	1	1	0	1	67.78
0.60	0	1	0	0	1	0	1	1	0	1	67.11
0.70	1	0	0	0	1	0	1	1	1	1	65.99
0.80	0	1	1	0	1	0	1	1	1	1	65.44
0.90	1	1	1	1	1	0	1	1	1	1	64.92
1.00	1	1	1	1	1	1	1	1	1	1	64.71

The variation of maximum, minimum and average of u_{rs} for all origin-destination pairs in different budget levels is depicted in Figure 9. Similar to the fix demand condition, increases in budget level lead to a decrease in maximum, minimum, and average of u_{rs} .

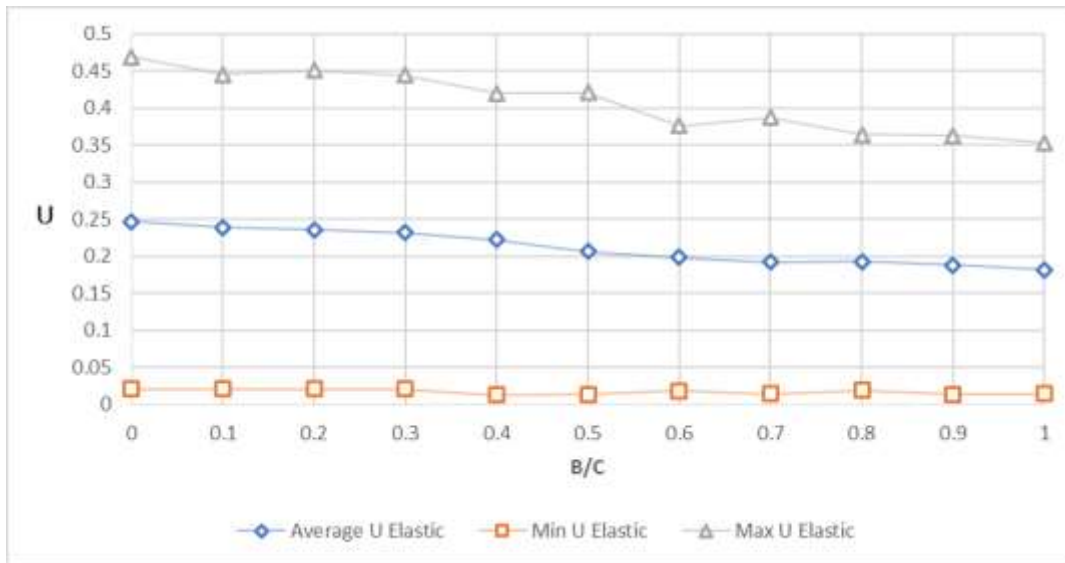


Figure 9. Variation of maximum, minimum and average of u_{rs} for all origin-destination pairs in variable demand condition

Figure 10 compares the variation of average u_{rs} for all origin-destination pairs in fixed and variable demand conditions. Although the average of u_{rs} for fixed and variable demand condition begin from 0.24, in the fixed demand condition, it reaches 0.15 in the full network ($B/C=1$) and

in the variable demand condition, the reduction is milder, and the average of u_{rs} meets the point 0.18.

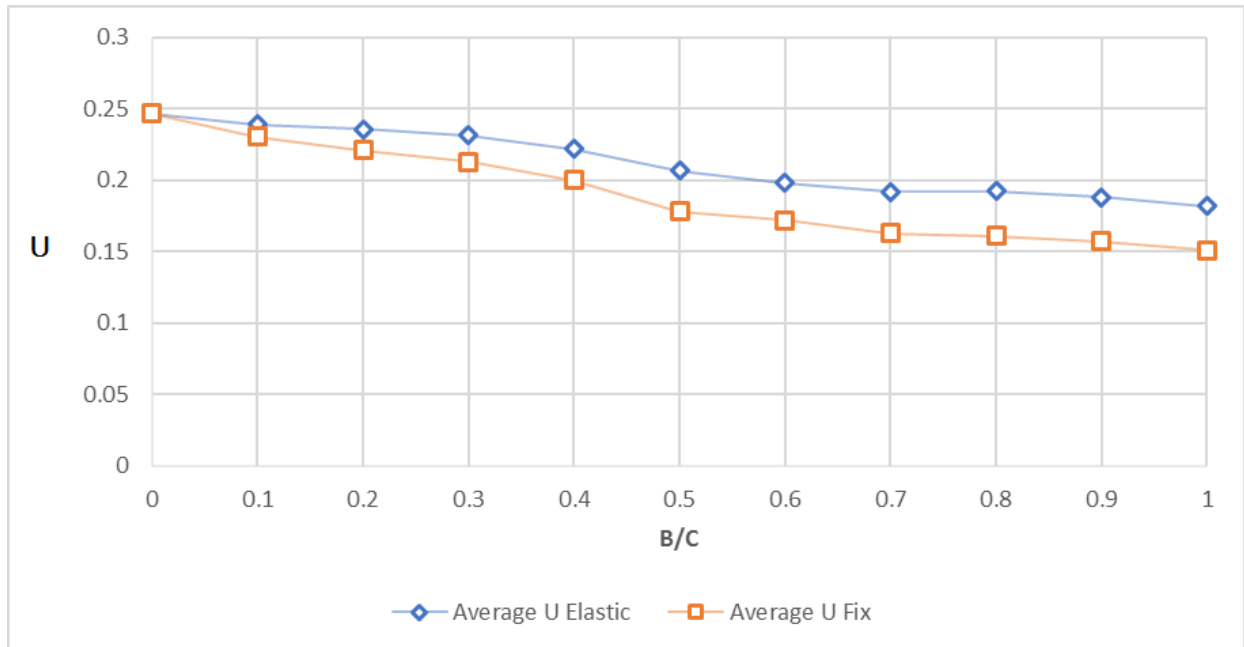


Figure 10. Comparing the average of u_{rs} for all origin-destination pairs in a fix and variable demand condition

Figure 11 shows the variation in the sum of demand between all origin-destination pairs in fix and variable demand conditions. Since u_{rs} and q_{rs} have inverse relations, it is expected that sum of q_{rs} increases by adding a new project to the base network.

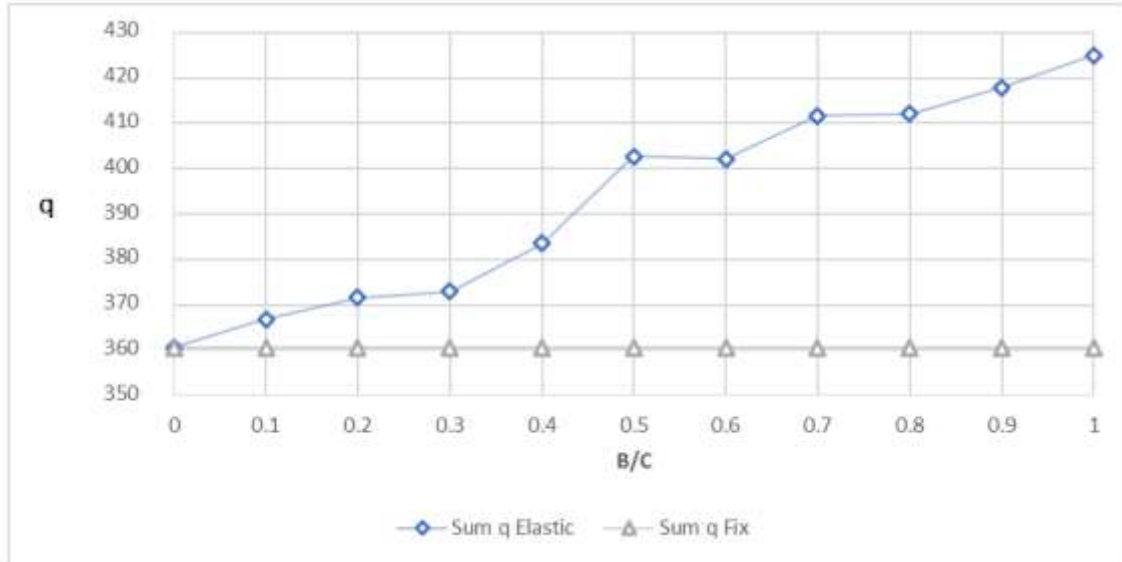


Figure 11. Variation in sum of demand between all origin-destination pairs in a fix and variable demand condition

The variation of maximum, minimum and average of links' flow and links' time for different budget level in variable demand condition are depicted in Figure 12 and 13 respectively. Similar to the fix demand condition in some cases adding new project links to the base network results in increases of maximum and minimum of links' flow and links'; however, these parameters shows a decreasing trend.

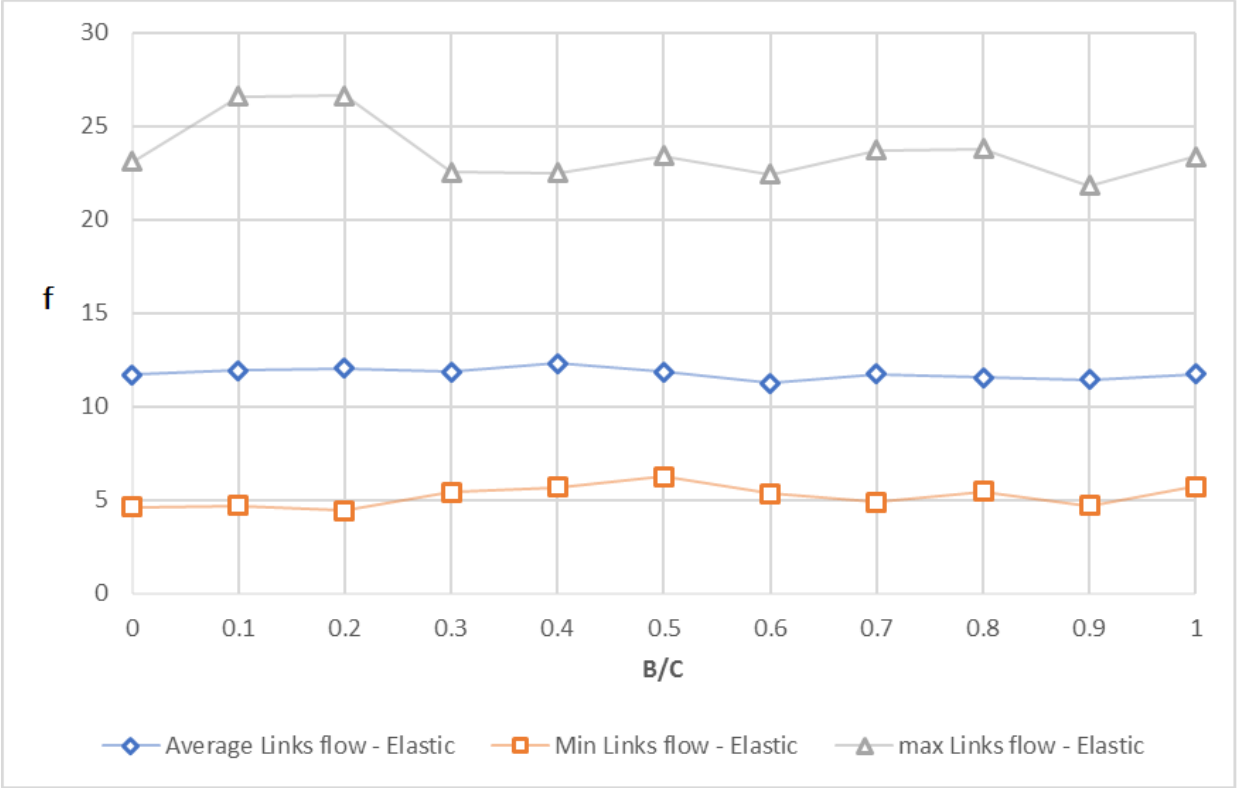


Figure 12. Variation of maximum, minimum and average of links' flow in variable demand condition

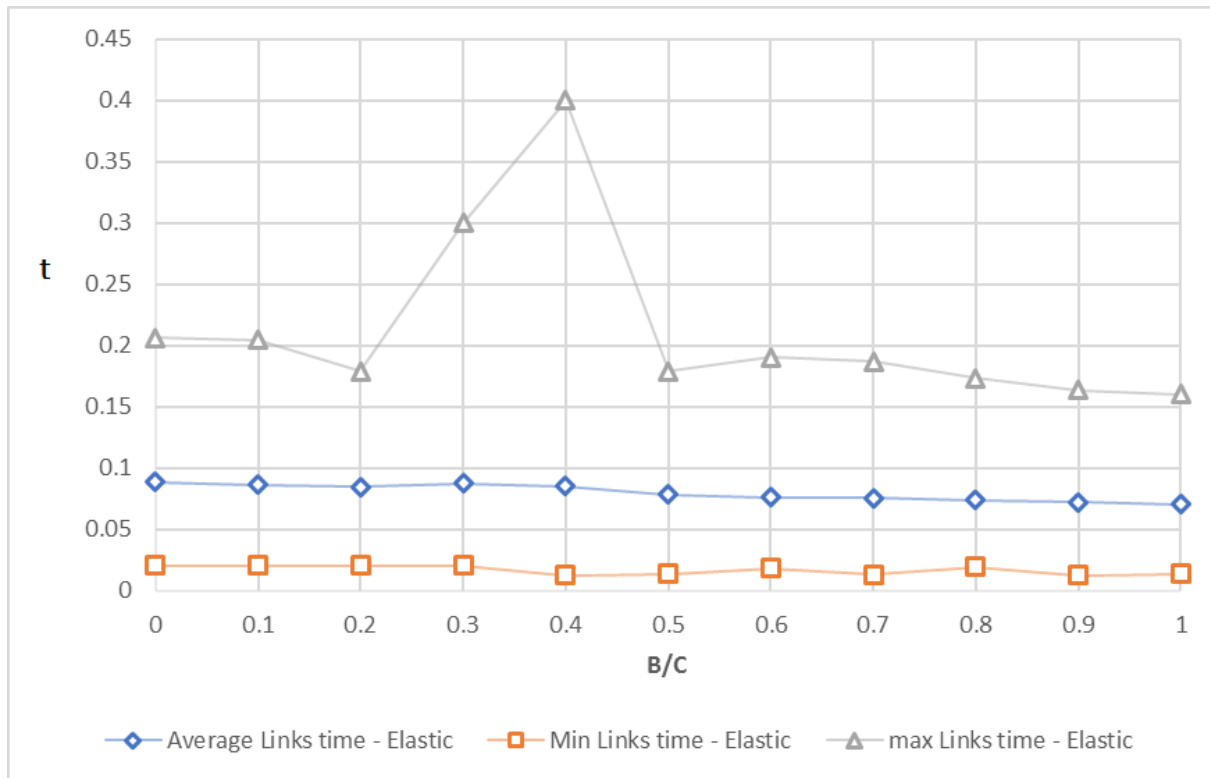


Figure 13. Variation of maximum, minimum and average of links' time in variable demand condition

The average of links' flow and link's time in fix demand and variable demand conditions are depicted in Figure 14 and 15 respectively. In both demand condition the average of links' flow and link's time have a decreasing trend; nonetheless, the fix demand condition experiences more significant reduction in both parameters.

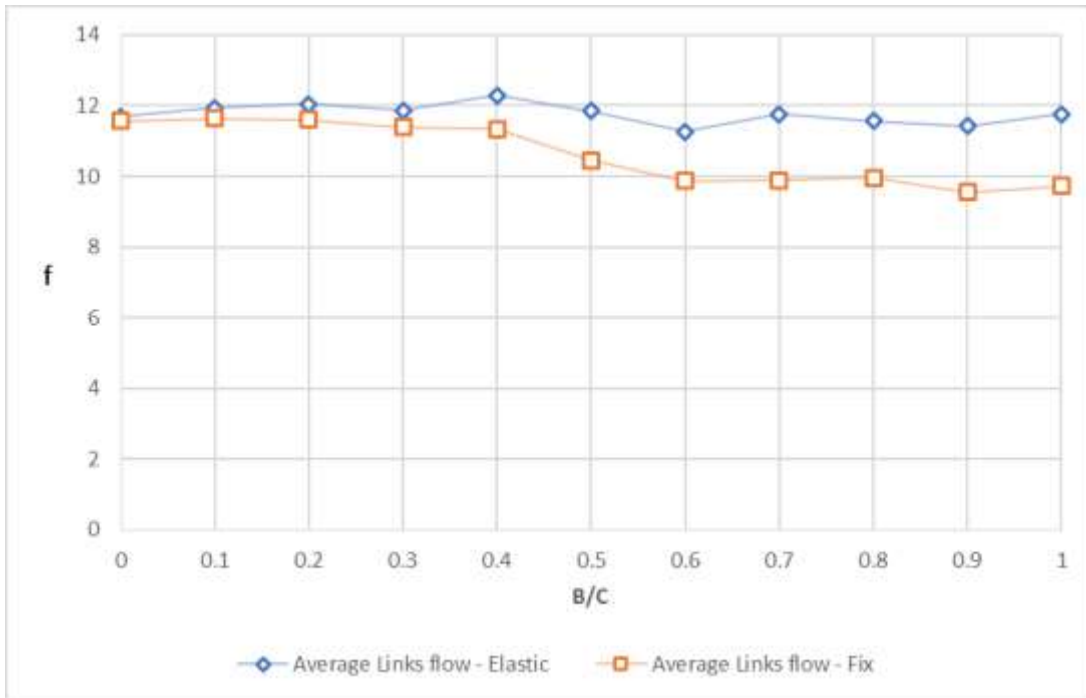


Figure 14. Comparing the average of links' flow in a fix and variable demand condition

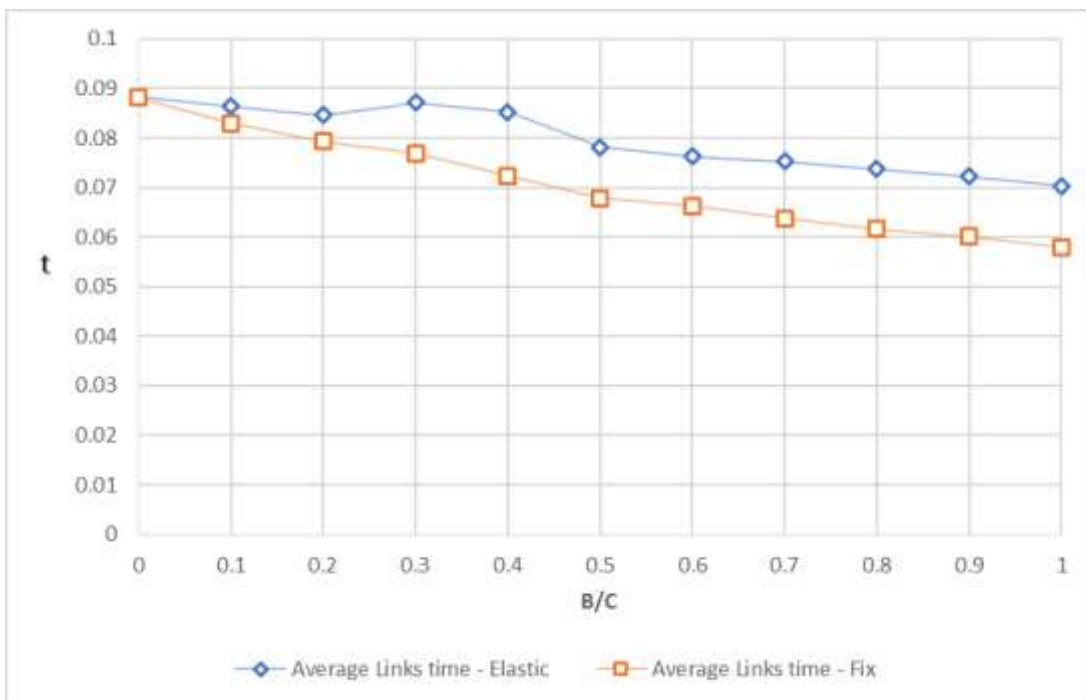


Figure 15. Comparing the average of links' time in a fix and variable demand condition

As it was mentioned previously, the initial elasticity was selected to be -0.7 in order to keep elasticity in the range of (-1,0) for all network conditions. The average, maximum and minimum of elasticity for different budget levels are mentioned in Table 6, and the variation of these parameters is depicted in Figure 16.

Table 6. The average, maximum and minimum of elasticity

Elasticity	$\frac{B}{C}$										
	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Average	-0.70	-0.68	-0.68	-0.67	-0.64	-0.61	-0.59	-0.58	-0.58	-0.56	-0.55
Minimum	-0.78	-0.83	-0.84	-0.88	-1.01	-1.05	-0.90	-0.94	-0.85	-0.92	-0.89
Maximum	-0.62	-0.48	-0.48	-0.44	-0.06	-0.07	-0.04	-0.05	-0.04	-0.04	-0.04

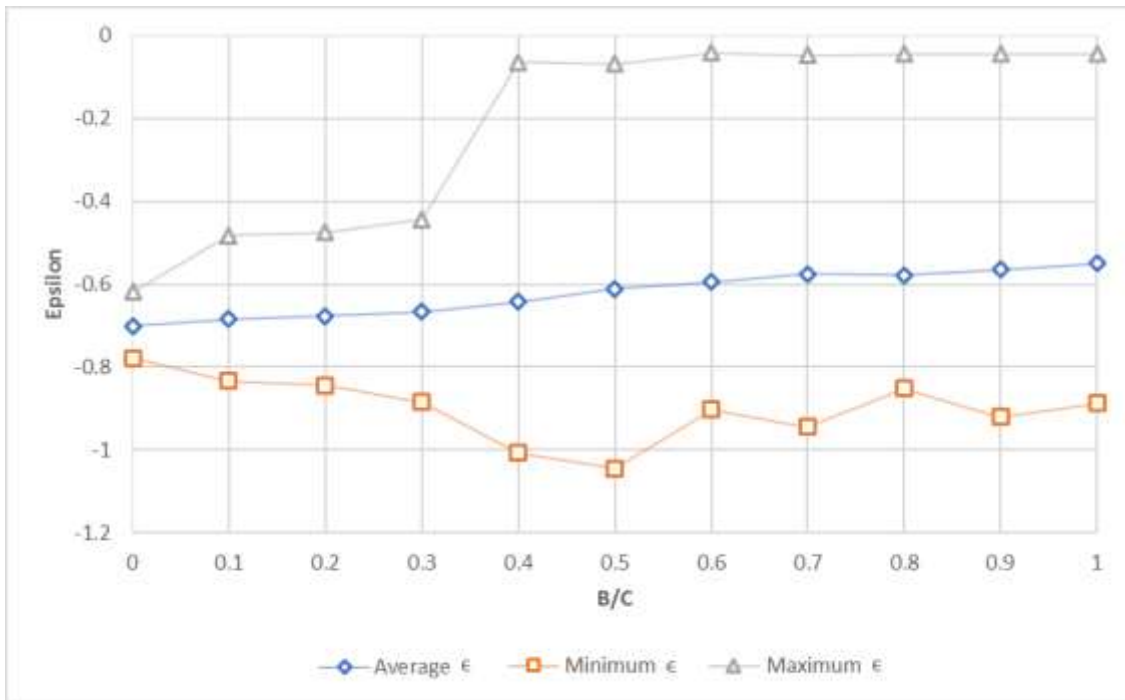


Figure 16. The average, maximum and minimum of elasticity

Although the selection or not selection of projects in each budget level was shown with 1 or zero, respectively in the first part of the section, they can be shown by using a decimal number. For instance, a candidate network that includes project link 1 and 2, can be represented by 3 ($2^0 + 2^1 = 3$). Figure 17 shows DNDP result by using decimal number for both fix and variable demand condition in different budget level. According to this Figure, the results of DNDP for fix and variable demand conditions differ significantly in the budget levels $0.3 \leq B/C \leq 0.5$ and $B/C = 0.70$. This result shows that using variable demand conditions in solving DNDP can drastically affect the result.

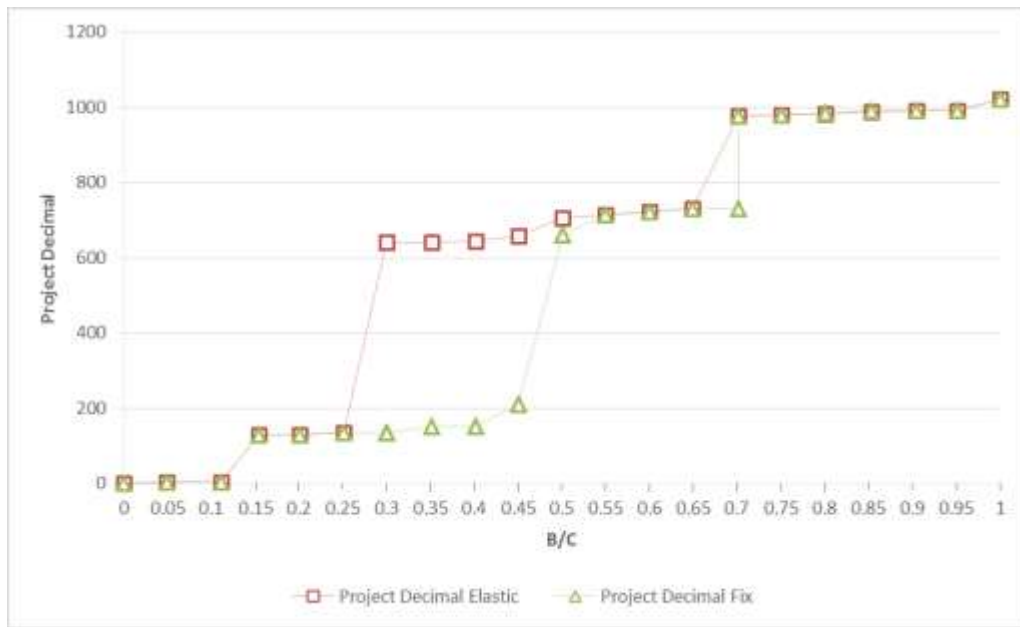


Figure 17. DNDP result for fix and variable demand condition

Figure 18 shows DNDP objective functions (the optimum network's total time) for both fixed and variable demand conditions. Both demand conditions have the same initial objective functions and show a decreasing trend by adding new project links to the base network; however, the objective function in fix demand conditions reduces more significantly. The difference between objective function value in the budget level $B/C = 0.4$ is more than 0.25 percent, and this difference increases by adding new project links to the base network. Since most real DNDPs are solved for the budget level $B/C \geq 0.4$, utilizing variable demand conditions seems to be necessary.

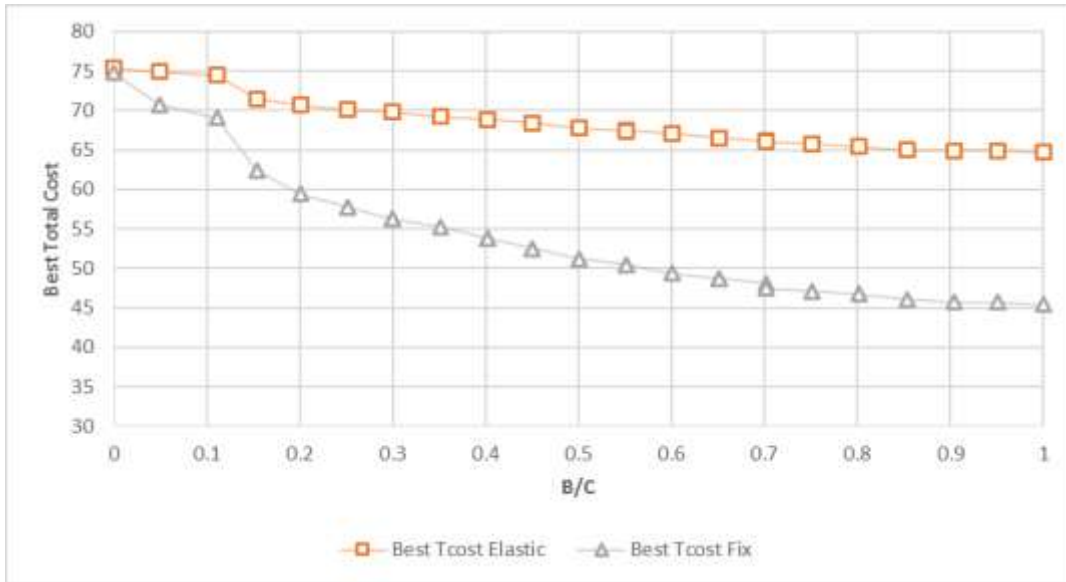


Figure 18. DNDP objective for fix and variable demand condition

Finally, the selected projects for different budget levels in a fixed and variable demand conditions are shown in Figure 19. This Figure also shows that the results differ more significantly in the mid budget level and further support the importance of utilizing variable demand in DNDP.

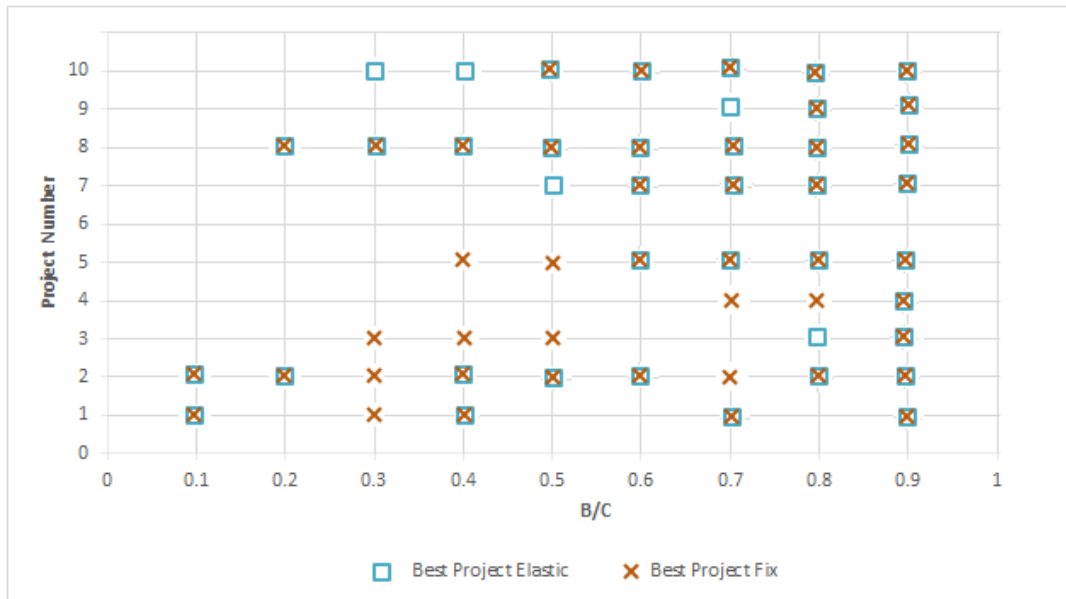


Figure 19. The selected projects in a fix and variable demand conditions

7. SUMMARY AND CONCLUSIONS

This research aims to find the importance of using variable demand conditions in DNDP. Although using variable demand conditions might theoretically change DNDP results, this condition has never been discussed yet. First, DNDP formulation was presented for both fixed and variable demand conditions. In the second step, a variable demand function was proposed by considering two fundamental conditions that make the assumptions as realistic as possible. In the next step, the DNDP result for fix and variable demand conditions were mathematically analyzed to find the situation that changes the DNDP result in a fixed and variable demand condition. A set of mathematical inequalities was found as a boundary that causes DNDP result to be different in a fixed and variable demand condition. After that, the DNDP problem was solved for a network with two arcs and two nodes in order to verify the utility and integrity of the proposed mathematical inequalities. Finally, DNDP with fix and variable demand conditions was solved for Sioux Falls network to more profoundly investigate the importance of using variable demand instead of fix demand.

The results show that utilizing variable demand conditions will significantly change the DNDP result comparing to the fix demand condition. The differences were much more significant in the mid-budget level, including $0.3 \leq B/C \leq 0.5$ and $B/C=0.7$. In addition, although DNDP with fix and variable demand conditions have the same objective functions (the optimum network's total time) in the base network, adding the project to the base network decreases the objective function in the fix demand condition far more than variable demand condition. For example, the differences were 25 percent in the budget level $B/C=0.4$, and it increases as the budget level goes to 1. Moreover, the average of links' flow and links' time diminish more in the fix demand condition in comparison to the variable demand condition when the budget level increases. The variable demand function elasticity was set -0.7 for all origin-destination pairs in the base network. The results show that the variable demand function elasticity increased when more projects were added to the base network. The average elasticity was found to be -0.61 in the budget level $B/C=0.5$ and -0.55 in the full network ($B/C=1$). The average of U in both fixed and variable demand conditions have the same amount in the base network and shows a decreasing trend when new projects were added to the base network; nonetheless, the fix demand condition experiences much more reduction. q and U has an inverse relationship so it is expected that increasing the budget levels q increases significantly.

According to the DNDP result for variable demand condition, the sum of q is 360.6 in the base network, 402.7 in the $B/C=0.5$ and 424.9 in the full network. To recapitulate, using variable demand in DNDP does make the assumption more realistic and has significantly different results compared to the fix demand condition. For instance, in the Sioux Falls network and budget level $B/C=0.5$, the optimum result (selected project links) in the fix and variable demand conditions

would be {2,3,5,8,10} and {2,7,8,10} respectively. Therefore, it is strongly suggested that the variable demand condition be considered in DNDP. Future studies may apply advanced machine learning (Rahimi et al., 2019; Jin et al., 2019) and statistical models (Azimi et al., 2019, 2020) to investigate the impact of variable demand conditions in DNDP.

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