Offline System Identification of DC motor model using Recursive Least Squares through MATLAB

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Abstract

This paper aims to obtain the model of DC motor using a system identification technique. System identification is the backbone of an adaptive control and the best means of obtaining mathematical model of most physical systems, so that it can be used to obtain a DC motor model. Offline (input – output) measured data of speed control of the motor was taken with help of MATLAB simulation. The measured data is used with recursive least squares RLS Algorithm to identify the motor model. The results indicated that the second order model provided the best estimation performance achieved quadratic cost function value of (J=0.7365) which showed best fit with 84.5% and that was very good validation of the model

Keywords: DC motor; System identification; recursive least squares RLS Algorithm.

1. Introduction

In control systems, a mathematical model of a process or system is in most cases necessary for the design of the controller in adaptive and robust control systems [Paraskevopoulos, 2012]. System identification is the best means of obtaining mathematical models of most physical systems, because most systems are complex. The system identification can be applied for DC motor model identification which will give more concise estimation of DC motor model ~~than~~ compare to mathematical modeling [Bature et al, 2013]. The two aspects taken into account in DC motor is speed and position control from these aspects two models can be identified [Othman et al, 2009]. An identification based on measuring system data is called an experimental identification. The experimental identification has two

methods namely Offline and Online identifications. In offline identification, the measuring of the data is recorded and the mathematical model is created after the measuring process is over. While in the online identification the computational of the mathematical model parameters occur during the measuring process. Some authors have used impulse response data to identify an Auto Regressive Moving Average (ARMA) model of DC motor position [Michal, Rudolf, 2010 and T.A. Tutuji, 2005]. RLS method was used for estimation of unknown parameters for online identification of an Electro-hydraulic system [Rozaimi, et al, 2009]. Bature et al have designed regulator state feedback controller using the System identification least square method [Bature et al, 2013], while Peter et al have showed the identification results measured on real linear discrete system [Peter et al, 2017]. In fore mentioned researches work, little attention has been paid to the use of programming codes for offline parameter identification.

This work aims to perform offline identification of DC motor model using recursive least square Algorithm applying to (input -output) measured data via MATLAB commands. It is known that the recursive least-square algorithm could provide improved performance as compared to the least mean square (LMS) – based algorithms, especially in terms of convergence rate [Camelia, et al, 2020]. The RLS using the Newton search which leads to a faster convergence compare to the steepest- decent based algorithms [Mohammad et al, 2021].

2. Methodology

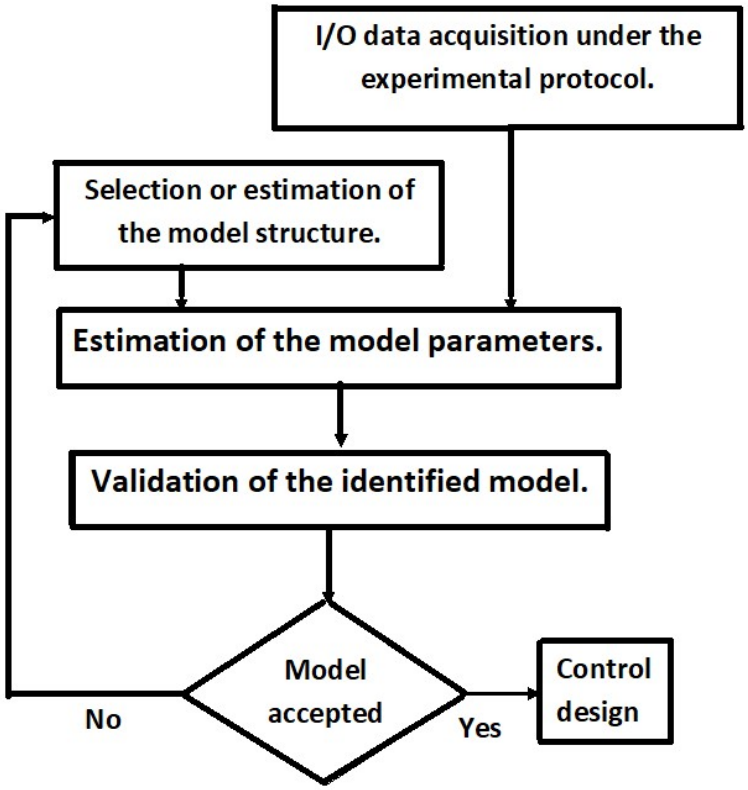
2.1 Data selection

In order to use the algorithm to identify the system, MATLAB command (>>load DCM data) was written to export the M-files of input - output data into workspace. Then the second column of output file was selected to measure speed values.

2.2 System Identification Loop

The system identification procedure has the flowing steps and is shown in Figure (1):

1. Collection input-output data acquisition under an experimental protocol.
2. Selection or estimation of the model structure (comlexity).
3. Estimation of the model parameters.
4. Validation of the identified model (structure and values of the parameters).
5. The validation is the mandatory step to decide if the identified model is acceptable or not. The same procedure was used elswhere [Wellstead, Zarrop, 1991].



6.

7. **Figure (1) System identification loop**.

2.3 Recursive Least Squares Algorithm

The Recursive Least Square method assumes that the object system conform a linear discrete relationship between the measurement (y) and the input (u) [Ljung, 2002] as:

$$y(k) + a_1 y(k\text{-}1) + a_2 y(k\text{-}2) + \ldots + a_n y(k\text{-}n) = b_1 u(k\text{-}1) + b_2 u(k\text{-}2) + \ldots + b_n u(k\text{-}n) + \varepsilon \quad \ldots\ldots(1)$$

Where the coefficients $a_{1\ldots}a_n$, $b_{1\ldots}b_n$ are system characteristics constants, n is positive integer and ε is the modeling error [Liangtao, 2008].

Let us assume that the system model is given in the following form:

$$A(Z^{-1})y(k) = B(Z^{-1})u(k-1) \text{ .....(2)}$$

Where: $A(Z^{-1})$ and $B(Z^{-1})$ are polynomials of the form:

$$A(Z^{-1}) = 1 + a_1 Z^{-1} + a_2 Z^{-2} + ... + a_n Z^{-na} \text{ .....(3)}$$

$$B(Z^{-1}) = 1 + b_1 Z^{-1} + b_2 Z^{-2} + ... + b_n Z^{-na} \text{ .....(4)}$$

Here: Z^{-1} is the backward shift operator defined by:

$$Z^{-1}y(k) = y(k-i) \text{ .....(5)}$$

The (RLS) is the result of minimization of quadratic cost function of the prediction error $\varepsilon(k)$ of the following form:

$$J = \sum_{i=0}^{n} (\varepsilon(k))^2 (6) \text{ .....(6)}$$

Where:

$$\varepsilon(k) = y(k) - \bar{y}(k) \text{ .....(7)}$$

$y(k)$ is the actual value of the process output, n is the number of data point and $\bar{y}(k)$ is the predicted value of the process output.

To formulate the RLS identification algorithm, the process model is formulated in terms of the parameters to be estimated. A convenient expression is:

$$y(k) = \Theta^T(k)\Psi(k) + e(k) \text{.....(8)}$$

Where *Θ(k)* is the vector of the estimated model parameters, and *Ψ(k)* is the regression vector which is made up of the past measured inputs and outputs of the process.

More specifically:

$$\Theta(k) = [-a_1, -a_2, ..., a_n, b_1, b_2, ..., b_n] \text{ .....(9)}$$

$$\Psi(k) = [y(k-1),...,y(k-n),u(k-1),...,u(k-n)] \text{.....(10)}$$

The standard RLS algorithm provides a means of interactively updating the estimated parameters, $\Theta(k)$ and works by minimizing the sum of the squares of the modeling error. The algorithm can be summarized as follows:

At each sample number k:

Step 1: Read system input $u(k)$, and output, $y(k)$, and form $\Psi(k)$ using past input and output values.

Step 2: Calculate the a priori prediction error, $\varepsilon(k)$, as the difference between the actual output and the predicted model output, using parameter estimates from the previous sampled, $\Theta(k-1)$ *as*:

$$\varepsilon(k) = y(k) - \Psi(k)\Theta^T(k-1) \text{ .....(11)}$$

Step 3: Form $L(k)$, the Kalman gain vector, as:

$$L(k) = \frac{\Psi(k)P(k-1)}{\left[1 + \Psi(k)P(k-1)\Psi^T(k)\right]} \text{.....(12)}$$

And, calculate the symmetric covariance matrix, $P(k)$, using:

$$P(k) = P(k-1)\left[I - L(k)\Psi^T(k) \text{ .....(13)}\right]$$

Step4: Update the estimate parameter $\Theta(k)$:

$$\Theta(k) = \Theta(k-1) + L(k)\varepsilon(k) \text{ .....(14)}$$

Step5: Wait for next sample, and then loop back to step 1 [Wellstead, Zarrop, 1991].

2.4 The Block diagram of DC Motor

The block diagram of DC Motor control is illustrated in Figure (2)

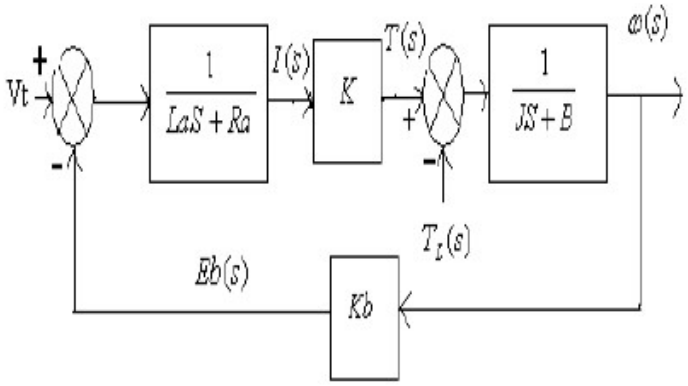


Figure (2): The block diagram of DC Motor.

The transfer function of the DC Motor with respect to the input voltage can be written as follows [Aditya, et al, 2013]:

$$T(S)=\frac{\omega(S)}{V(S)}=\frac{K_T}{(R_a+SL_a)+(J_S+B)+K_bK_T} \quad \text{.....(15)}$$

Where: Let T= torque developed by the motor, R_a = armature resistance, L_a = armature self-inductance, I_a =armature current, E_B = back EMF in armature, V_t = applied voltage, K_T = motor torque constant, K_b = feedback constant, B =equivalent coefficient of friction of the motor shaft, J = moment of inertia of motor shaft and $\omega(S)$ = angular velocity.

2.5 DC Motor model structure types

A mathematical model of DC motor either to be first order or second order they have following structures [Mohammed A., 2016]:

First order structures:

$$1.\ G(z)=\frac{b}{z+a} \quad \text{.....(16)}$$

$$2.\ G(z)=c+\frac{b}{z+a} \quad \text{.....(17)}$$

Second order structures:

$$3.\ G(z)=\frac{cz+d}{z^2+az+b} \quad \text{.....(18)}$$

4. $G(z)=\frac{c}{z^2+az+b}$ .....(19)

5. $G(z)=\frac{cz}{z^2+az+b}$ .....(20)

5. $G(z)=d+\frac{cz+e}{z^2+az+b}$ .....(21)

3 Simulation results:

The previous model structures (equations from 15 to 20) were applied into RLS algorithm to choose the valid model, the codes were run and the following graphs have been obtained:-

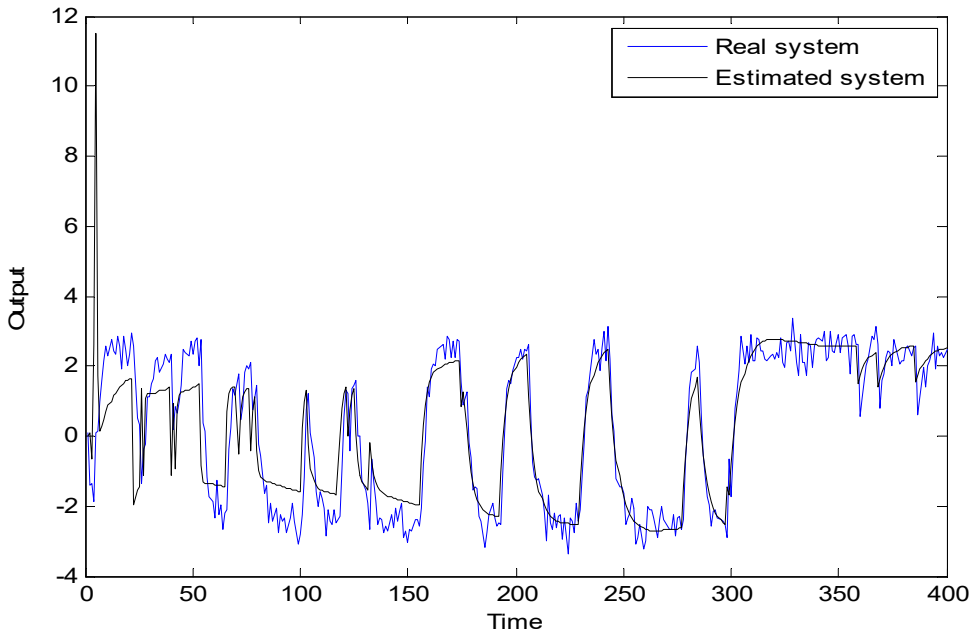


Figure (3) Real system responses versus estimated for the first order model 1.

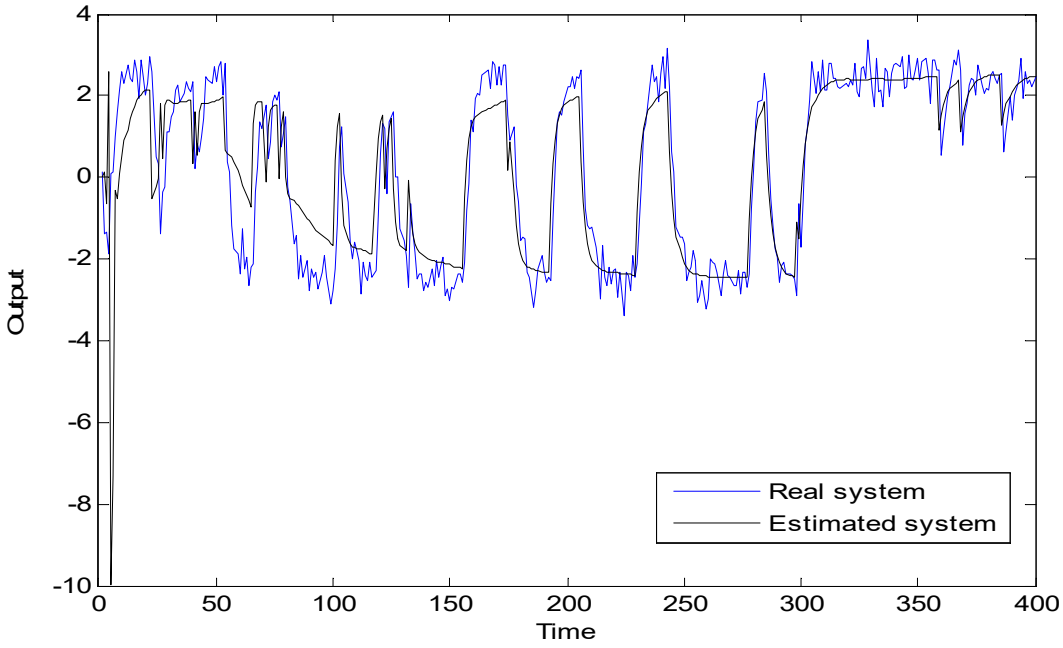


Figure (4): Real system responses versus estimated for the first order model 2.

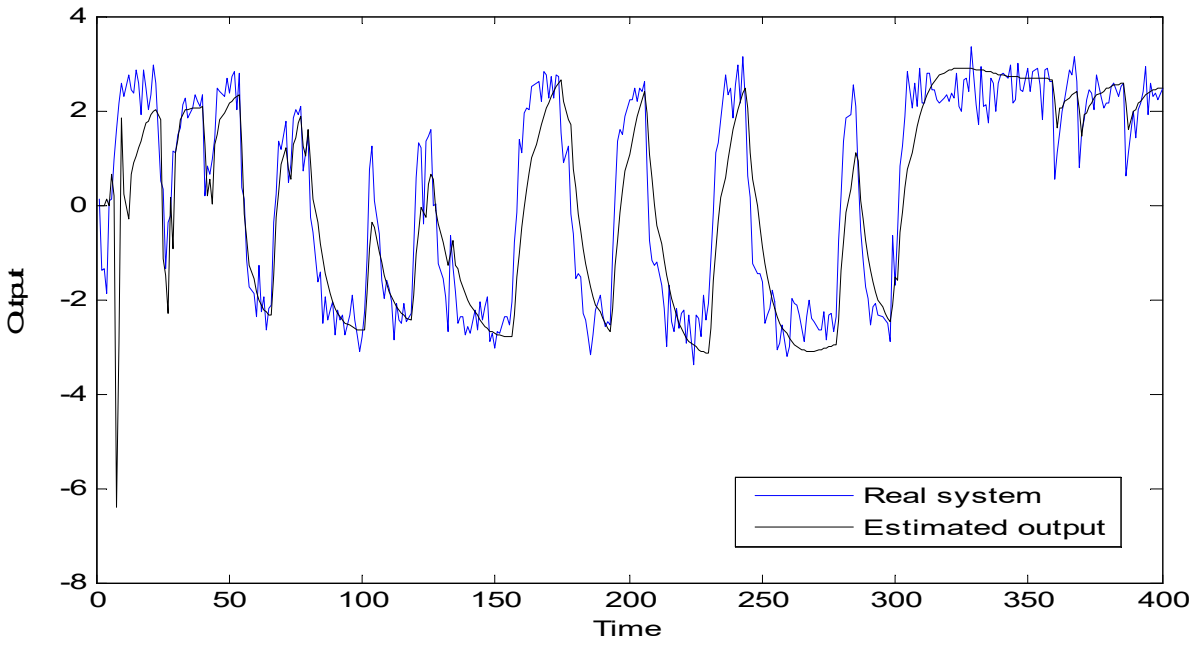


Figure (5): Real system responses versus estimated for the second order model 1.

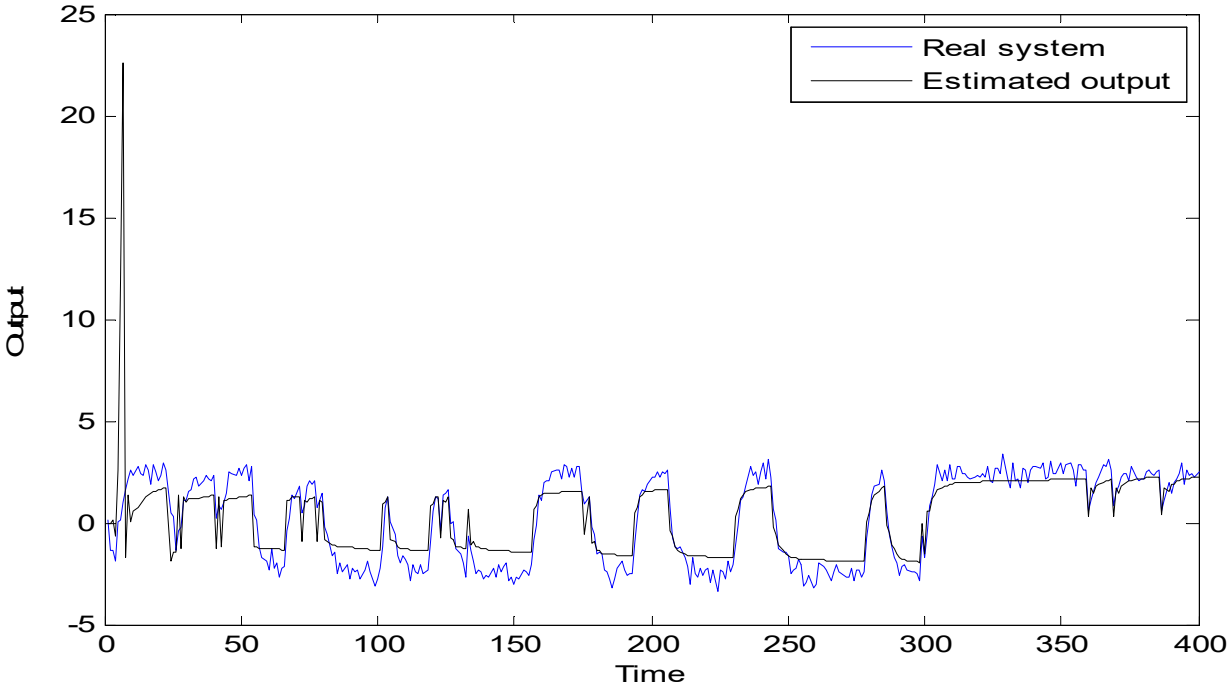


Figure (6): Real system responses versus estimated for the second order model 2.

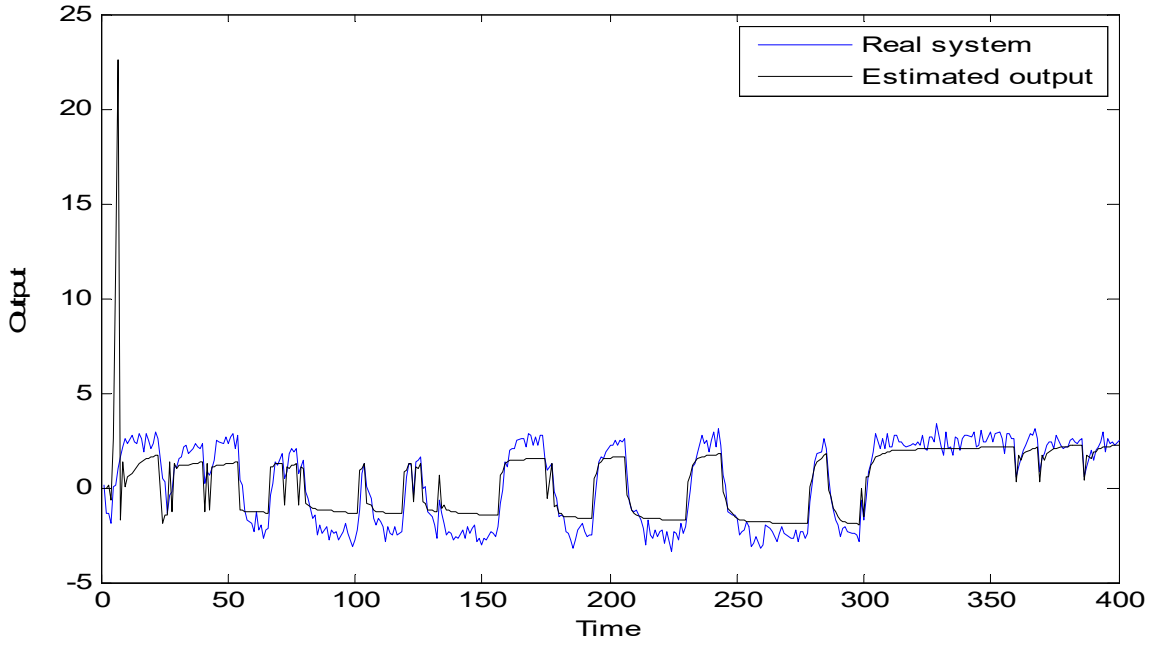


Figure (7): Real system responses versus estimated for the second order model 3.

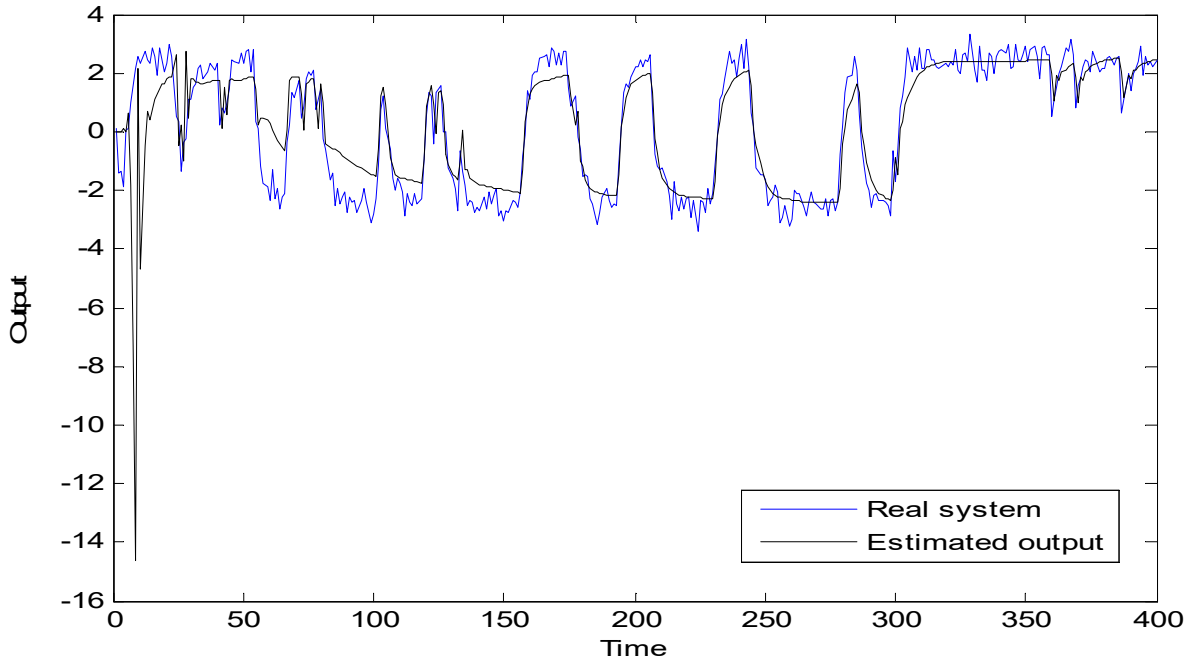


Figure (8): Real system responses versus estimated for the second order model 4.

Corresponding quadratic cost function (J) to the all models is shown in table (1)

Table (1): The quadratic cost function for all mathematical models.

Number of model structure	Quadratic cost function (J)
1	1.0963
2	1.2098
3	0.7365
4	2.8888
5	2.8888
6	1.8879

3.1 Model Validation:

Fit Percentage is a normalized metric used to compare between simulated and measured responses which is implemented in identification Toolbox in MATLAB and defined as

$$FIT = 100\left(1 - \frac{\|y - \hat{y}\|}{\|y - \bar{y}\|}\right) \ldots\ldots\ldots\ldots (22).$$

Where:

(y) represent the measured output data.

(ȳ) represent its (channel -wise) mean.

(ŷ) represent the simulated or predicted response.

‖.‖ represent the second norm of the model. [Mathworks, 2026]

3.2 System Identification results:

When the Simulink program is run the plots of input/output measured data is shown in **Figure (10).**

The step responses of the estimated models by using System Identification Apps are shown as in **Figure (11).**

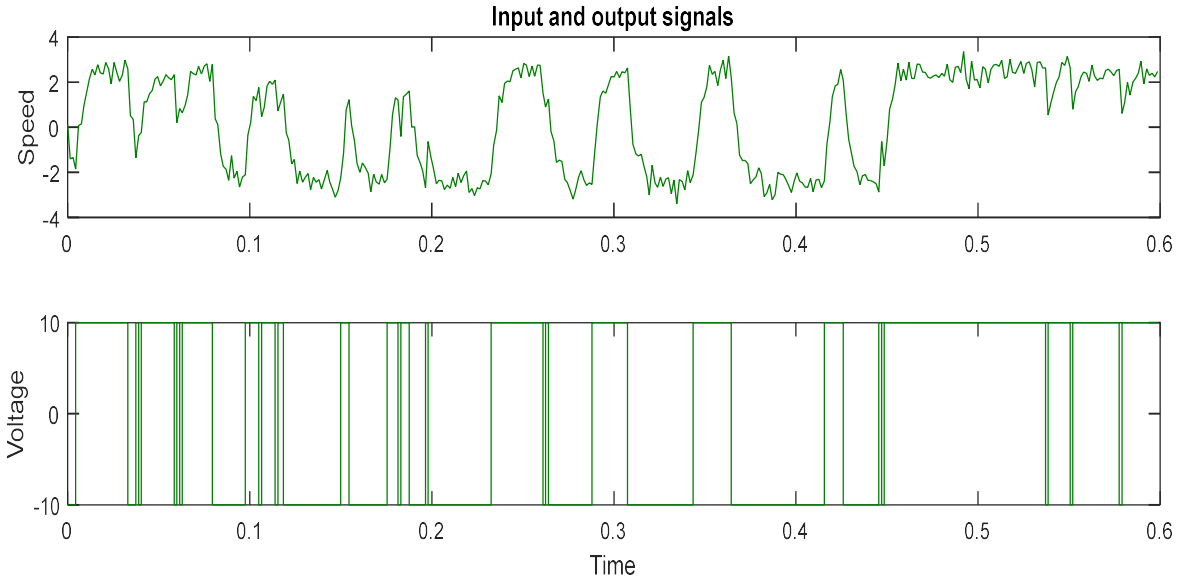


Figure (9): Input and output measured signals of DC motor.

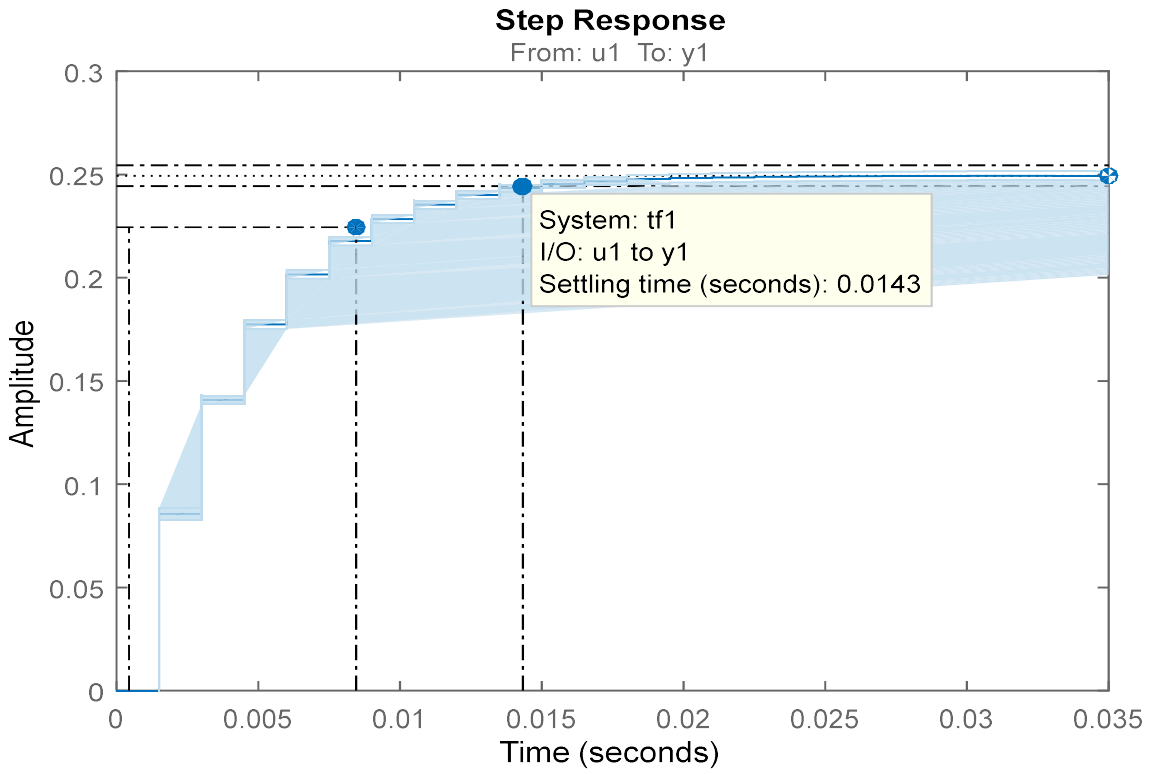


Figure (10): Step responses of the models identifying by system identification Apps.

The step responses of the models estimating by the RLS method and the model used System Identification method which all are took the same sampling time = 0.0015 are shown in the Figure (10).

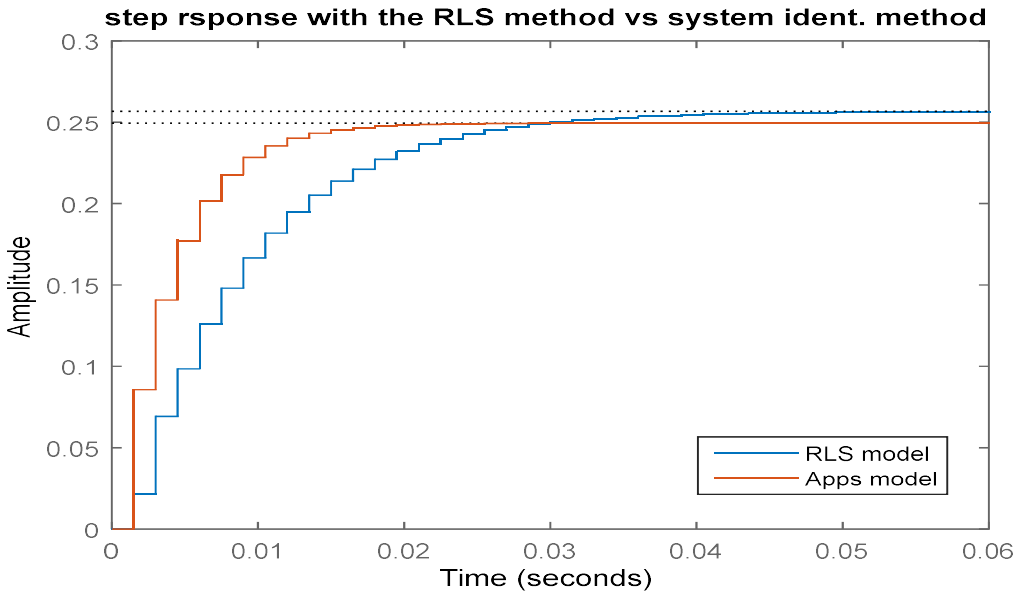


Figure (11): Step response of the model RLS method vs. system identification Apps.

4. Discussion

In this section the validation of the model is observed from the graphs. RLS algorithm made estimated system behaved as a tracker. On the other hand it would be easy to know what was the valid model of the system according to their quadratic cost function for each one J. And it could be concluded that J=0.7365 gives the minimum quadratic cost function related to the following model $G(z) = \frac{cz + d}{z^2 - az - b}$.

The parameters a, b, c and d have deduced From MATLAB program to be $a = 0.4855,\ b = -0.2862,\ c = 0.0218$ and $d = 0.0368.$ After, the parameters have been substituted the valid model is given by the formula $G(z) = \frac{0.0218z + 0.0368}{z^2 - 0.4855z - 0.2862}$

5. Conclusions

This paper presents offline identification of DC motor. Recursive least square algorithm was used to identify the model. The data was collected to view the speed measure values and MATLAB codes were written to perform the validation process. The identification results show that the valid model is the second order model of the system which was compared to the model obtained using identification Toolbox in MATLAB/ Simulink program. The comparison showed best a fit percentage of 84.5% and that was indicated a good agreement between estimated model and the measured system response. The step responses are not exactly the same inspite of the the sampling time as noticed in the simulation and that is due to the differences in the parameter estimation method and the resulting model parameters. Nevertheless, the model is suitable basis for designing robust and adaptive control system. It is recommended to use another identification tool and applied intelligent control methods like artificial neural network to find the valid model of the system and compare it with system identification techniques.

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