Subordinate Oscillator Arrays: Physical Design and Effects of Error

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ABSTRACT

Prior work demonstrates that an attached subordinate oscillator array (SOA) can attenuate vibration of a host structure over a frequency range of interest. A judicious choice of the distribution of masses and stiffnesses of the attached oscillators can result in a relatively flat frequency responseof for the host structure over a desired band. This response modification can be a significant improvement over classical dynamic vibration absorbers (DVA) that attenuate a structure's response at one target frequency while increasing the frequency response amplitude at nearby side frequencies. Performance of the SOA can be highly sensitive to the uncertainty or disorder in the mechanical properties of the system. This paper introduces a novel design strategy that can make use of either 3D-printing or piezoelectric SOAs (PSOAs). This strategy has the potential to address and ameliorate. It is important to note that the design strategy is simple and effective in that it can be carried out without computational optimization techniques by choosing simple or well-known distributions of properties.

1 Introduction

The design of dynamic vibration absorbers (DVAs) is a classical topic of modern vibrations that has been well-studied in many texts [1, 2]. In this work, the system to which a DVA is attached is called the host. The essential and well-known feature of DVA performance for a single degree of freedom (SDOF) host system is that it is possible to exactly cancel the frequency response of the host at its natural frequency, provided that the mechanical properties of both host and DVA are known precisely. Even for multi-degree of freedom (MDOF) systems, this result holds approximately true. Such an exact cancellation, however, comes at a cost: the amplitude of the frequency response of the host structure can be substantially increased at frequencies close to the target frequency. A fundamental drawback to an approach using a traditional DVA is the need for exact knowledge of the mechanical properties of both the host and the DVA. The DVA design can therefore be sensitive to uncertainty or temporal drift in the mechanical properties of the system.

An SOA generalizes the idea behind a DVA by expanding it into a distribution of multiple sequential DVAs whose effects overlap. Each oscillator in this array has varied dynamic properties and these properties are what define the distribution. The mass, stiffness, and damping of each individual oscillator can be different in a given SOA design. It has been shown [3] that SOAs are capable of attenuating the vibration of the host in an approximately uniform way over a band of frequencies. In principle, an SOA provides substantial improvement in the robustness of the performance over a classical DVA because the SOA design can tolerate variations in the input frequency around the target frequency. However, as with the design of DVAs, realizing this goal in a physical mechanical SOA can be problematic. Response reduction over the desired flat frequency band exhibits sensitivity to *unintended* variation in the mechanical properties of the primary or the attached masses. Fabrication errors or uncertainty can have detrimental effects on the realized performance of systems fabricated with a given design strategy. Philosophically, this robustness issue can be expressed in terms of the sensitivity of a design to error in the distributions of the oscillators [4, 5].

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In this paper, a set of design strategies that could be applied singly or in combination for a reduction of this sensitivity is introduced. The discussion begins with a step-by-step outline of a method to design mechanical SOAs. Section 2 summarizes each of the pertinent design variables, showing the effects of each on the resulting modulated system response if the values are not optimal. Section 3 presents an algorithm to prescribe a geometry for manufacture. In Section 4, a thorough analysis of the disorder problem is presented. This analysis demonstrates the need careful attention to error in the SOA and suggests two methods for reducing the effects. The analysis introduces metrics to quantify disorder levels and provide some insight into acceptable levels of disorder under differing conditions.

2 SOA Design Space

Since the primary contributions of this paper are definitions of a general design strategy, a careful description of all relevant parameters follows. The SOA is depicted schematically as a collection of oscillators represented in the lumped parameter system model shown in Fig. 1. The physical implementation of that model used in this work is shown in Fig. 2. The design of SOA must specify the number of oscillators and their length, width and height. However, there is no closed form prescription to solve this design problem. Determination of each of these values requires several steps.

In the design process, Eq. 1 governs the minimum required number of oscillators,

$$N = \eta \Delta Q \tag{1}$$

where N is the minimum number of oscillators, η is the modal overlap, Δ is the desired fractional bandwidth, and Q is the Quality Factor. Note that n will be used to functionally indicate individual oscillators where N is that total number of oscillators. All of these parameters are used to shape the frequency response function from input to host over a target frequency range. The fractional bandwidth is defined as the bandwidth of the effect as a fraction of the center frequency of the range that we aim to correct. Modal overlap is a measure of modal density of the SOA. It is a unitless $\left(\frac{Hz}{Hz}\right)$ value

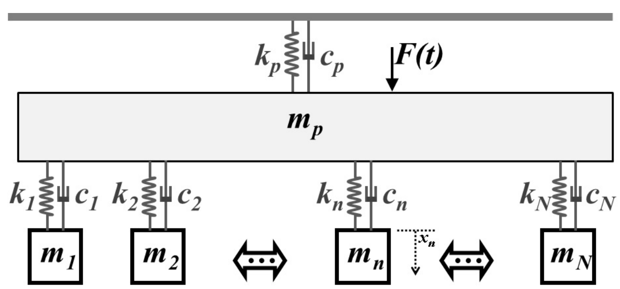

Fig. 1: The SOA has a primary mass and subordinate masses attached. The frequency band of suppression is defined by the values of mass and stiffness in the subordinate elements.

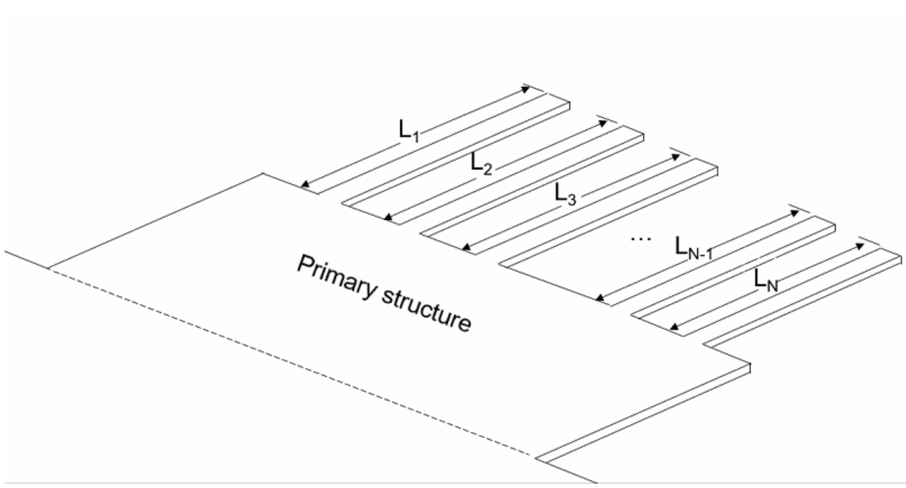

Fig. 2: A drawing of the physical layout used to realize the system model in Fig. 1

calculated on an individual basis by the half-power bandwidth divided by the frequency span between the fundamental frequency of the current oscillator to that of the next. Modal overlap is defined in more detail Section 2.1.

There are several additional factors that affect the design of an SOA. System properties of the host system or primary oscillator and material properties of the SOA to be designed must be considered. The relevant system properties are (1) the mass of the primary system and (2) the width and center frequency of the band to be suppressed. The relevant material properties are the Quality Factor, Young's modulus, and density. Also, if there are significant environmental changes, such as temperature variation, additional care must be taken to make sure the SOA will perform correctly throughout the temperature range.

In the next few sections, we explain each of the parameters in the design space and discuss how they can be interpreted. The goal is to build an intuitive notion of how each parameter can be used to alter the designed performance of the system. Section 2.1 details the definition and calculation of modal overlap, η. Section 2.2 explores the effects of Δ, the fractional bandwidth. Section 2.3 describes the effects of damping in the SOA system. Section 2.4 explores the effects of the mass ratio of the primary element to the subordinate elements, α. The generation of the frequency distribution, β, using the frequency distribution parameter, p, is outlined in Sec 2.5. Section 2.6 gives an explanation of several manufacturing constraints.

2.1 Modal Overlap Parameter, η

Modal overlap is defined as the half-power bandwidth divided by the difference in frequency successive peaks, Δf. Modal overlap has been studied in the literature [6, 7] For the system as a whole, modal overlap can be calculated as

$$\eta = \frac{N-1}{\Delta Q_{\text{SOA}}} \tag{2}$$

where $Q_{\text{SOA}} = Q_n$ is the Quality Factor of the nth resonance, and the frequency separation between adjacent resonances is given by $\Delta/(N-1)$. Modal overlap, η, is generally set to 2 when calculating the required number of oscillators [4]. Any value below two means that there is not enough modal density to properly create the desired effect. The effect shows up as ripple in the combined response and is visible in Fig. 3. η greater than 2 would be shown but they line up precisely in the response. Raising η above 2 has little benefit but is revisited in Sec. 4.2.

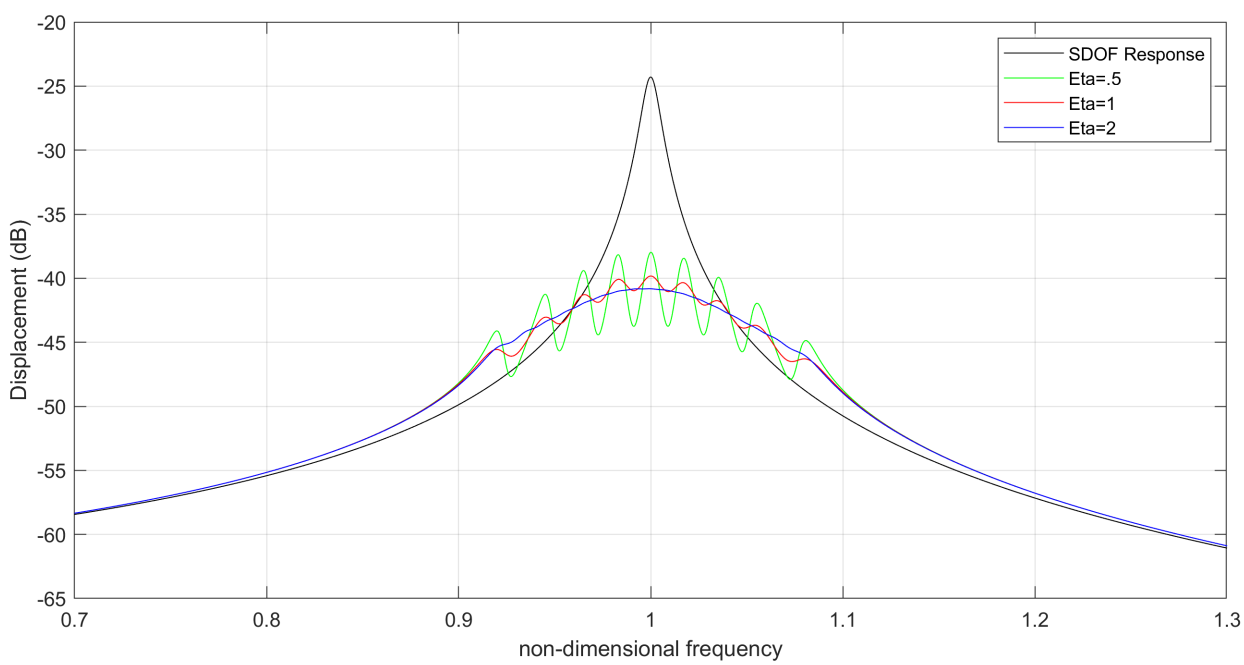

Fig. 3: An example of the effects of η on the SOA system. The ripple effect starts small but ramps up as η is decreased.

2.2 Fractional Bandwidth, Δ

The SOA is designed to suppress a band of frequencies over a bandwidth measured as a fraction of the center frequency. The center frequency, F_c, and fractional bandwidth, Δ must be chosen ahead of time in order to design an SOA. The required width of the fractional bandwidth is dependent on the nature of the peak to be suppressed. For example, a fractional bandwidth of 10% would suppress magnitudes between 95% and 105% of the center frequency The SOA is effective at suppressing peaks of both high and low quality factor, but the fractional bandwidth needs to contain the entire width to be suppressed. The fractional bandwidth is easily visualized and shown in Figure 4. Looking at the figure, it may be tempting to increase the bandwidth artificially to increase the total magnitude reduction. However, this can lead to additional design issues such as: increased attention to Q in the SOA material, increased SOA physical size, and violating material constraints listed in Sec. 2.6.

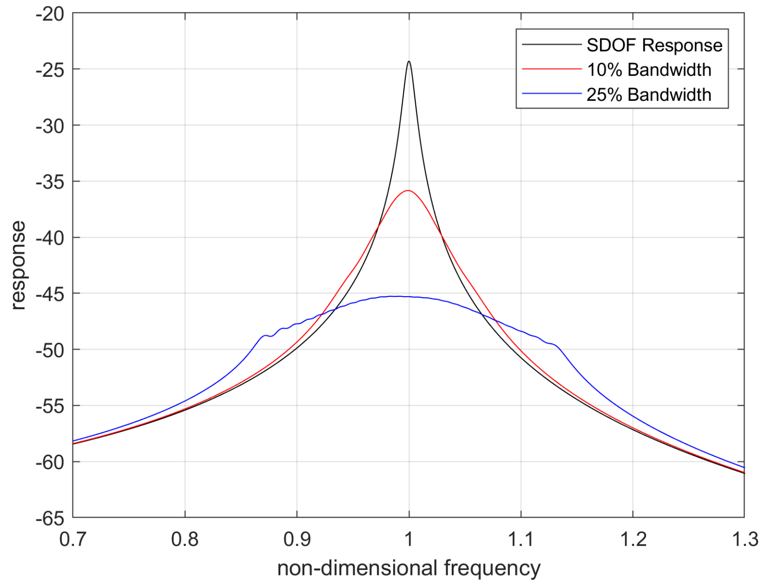

Fig. 4: This shows 10% vs 25% bandwidth. Note the non-dimensional frequencies where each curve begins to diverge from the SDOF are at their respective fractional bandwidth levels.

2.3 Quality Factor, Q

When considering Eq. 1 with respect to SOA design, modal overlap, η, is generally set to two and fractional bandwidth, Δ, is the bandwidth of the frequency range to be suppressed. Those two constraints leave the Quality Factor, Q, as the only material property involved in determination of the number of oscillators, N, required for the SOA. Higher Quality Factor will marginally increase the peak suppression but also increase the number of oscillators required. Using a high-Q material in the SOA also requires strict attention to the fractional bandwidth that is chosen. If the fractional bandwidth is too high, the high-Q SOA will create artifacts at the edges of the band whereas a low-Q SOA is less sensitive to this effect. These effects are visible in Fig. 5.

2.4 Mass Distribution, α, and Mass Ratio, μ

With the number of oscillators required in the array, N, determined, the mass distribution α can be defined. The required total mass of the SOA elements is expressed as the sum of the elements of the mass distribution $\alpha(n)$. For the bandpass configuration, the optimal mass ratio between the total mass of the SOA and the host structure, μ, has been determined to be $\frac{\Delta^2}{4}$ [4]. This requirement dictates that more mass will be required to suppress a wider bandwidth.

For the example presented in Fig. 6, a desired response maximizes the peak reduction while minimizing side artifacts. Increasing the mass ratio too much gives the SOA a significant mass ratio compared to the host. This high mass ratio will create resonant peaks of its own to the sides of the primary peak. The SOA will still achieve vibration suppression, but will introduce side peaks that can offset the benefits of the initial peak reduction. This behavior essentially returns the

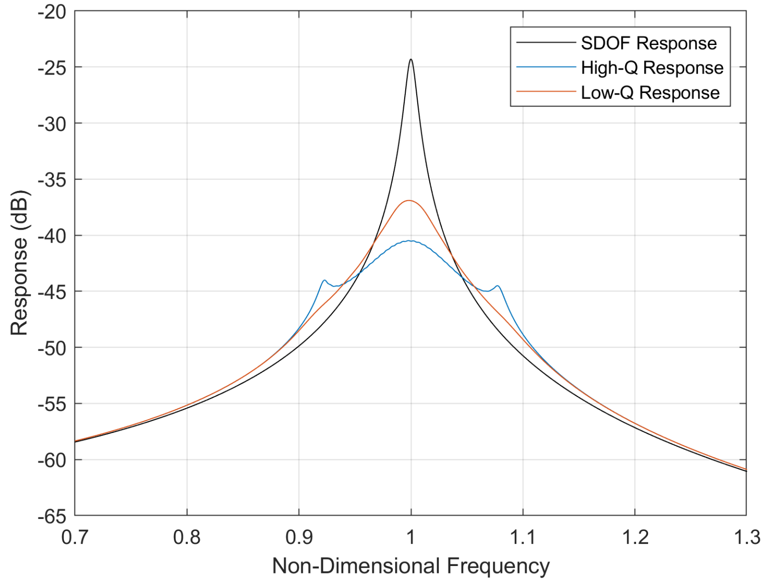

Fig. 5: An example of the effect of choosing fractional bandwidth that is too large. Note that the low-Q response does not express the edge artifacts but also achieves less peak suppression. Both curves display a 15% fractional bandwidth

performance profile of the system to that of a classical DVA. In contrast, decreasing the mass ratio too far reduces the amount of energy tranferrable into the SOA system, reducing the suppression capabilities of the system.

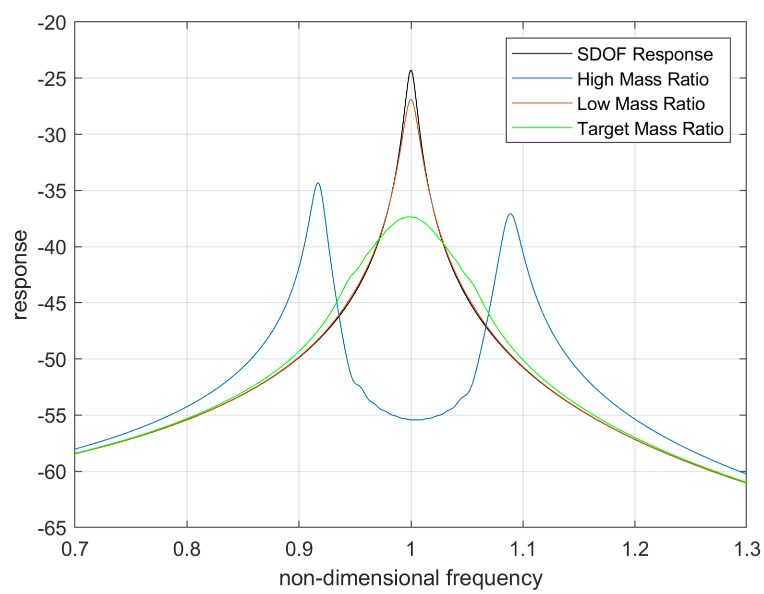

Fig. 6: An example of the effects of mass ratio on the SOA-adorned system. The high mass ratio returns the performance to that of a classical DVA. The low mass ratio decreases the performance of the system.

2.5 Frequency Distribution β and Frequency Distribution Parameter *p*

Many of the properties of an SOA design are defined in terms of a discrete distribution of a particular property over the set of oscillators. The discrete distribution of frequencies across the fractional bandwidth, Δ, is represented by the set β. Equation 3 defines β for the n^{th} oscillator in terms of the number of oscillators, N, the fractional bandwidth, Δ, and the frequency distribution parameter, p. As presented, Eq. 3 generates a β that has a non-dimensional center frequency of 1. The actual frequency distribution is created by multiplying the set β by the center frequency of the host structure. The β distribution represents the target or as-designed isolated natural frequencies of each of the individual oscillators that comprise the SOA.

Figure 7 shows the effect of distribution parameter p on the frequency distribution β and Fig. 8 shows the effect of p on the actual SOA response. Note that $p > 1$ packs more oscillator frequencies on the edges of the SOA target band. Therefore, the SOA would be less effective on the at modifying the response at the host structure's center frequency, diminishing the performance of the SOA as a whole for an SDOF system. Such a distribution may be of use on a host structure with a bimodal frequency response. A p of approximately 0.75 produces a flat response near the center of the target band(color), generally best suited for an SDOF host structure. As p approaches 0.6 and below, a dimple will form in the flat response(color). By $p = 0.25$, the SOA system response becomes essentially equivalent to that of a classical DVA with an equivalent mass that is simply the total SOA mass.

$$\beta_n = \begin{cases} \frac{\Delta}{2}\left(\left(\frac{2(n-1)}{N-1}\right)^p - 1\right) + 1 & \text{for } n \leq \frac{N}{2} \\ \frac{\Delta}{2}\left(1 - \left(\frac{2(n-1)}{N-1}\right)^p\right) + 1 & \text{for } n > \frac{N}{2} \end{cases} \quad (3)$$

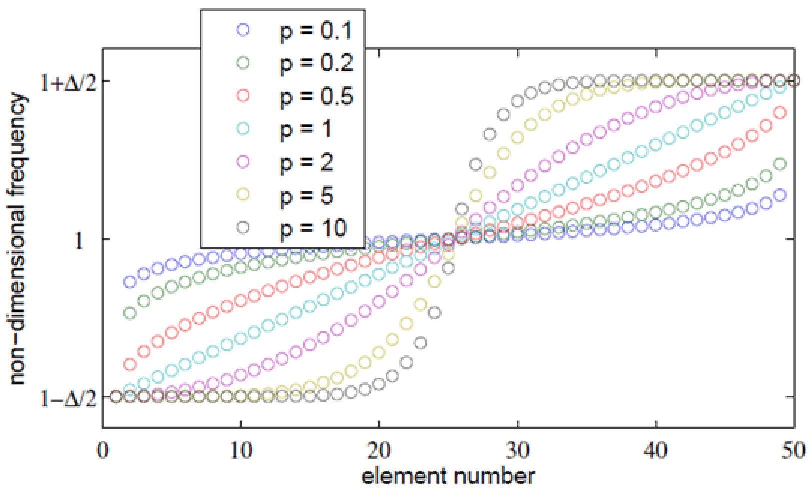

Fig. 7: The p value affects the density of the frequency distribution about the center frequency. A p value of 1 provides a linear distribution of frequencies throughout the bandwidth, δ.

2.6 Manufacturing Constraints

Successful design of an SOA must consider the practical constraints on the ranges of each of the aforementioned physical parameters. There are two categories of constraints in the SOA design methodology. Geometric constraints involve ensuring that certain design assumptions are held true. For example, this formulation relies on beam aspect ratios that satisfy Euler-Bernoulli conditions. Material constraints will differ between materials and chosen manufacturing processes, but must be considered thoroughly during SOA design. For example, steel will have a much smaller practical minimum thickness than 3D-printed materials. As another example, steel has been successfully used at a thickness of 0.01 inches, while the recommended minimum thickness for ABS, a common 3D-printed polymer, is nearly 5 times greater at 0.047 inches (1.2mm). Typical constraints to consider are: minimum and maximum length, minimum width, minimum thickness, thickness to length ratio, and width to length ratio. The values of these constraints are not absolute and, especially for 3D-printed materials, must be determined independently for each design situation.

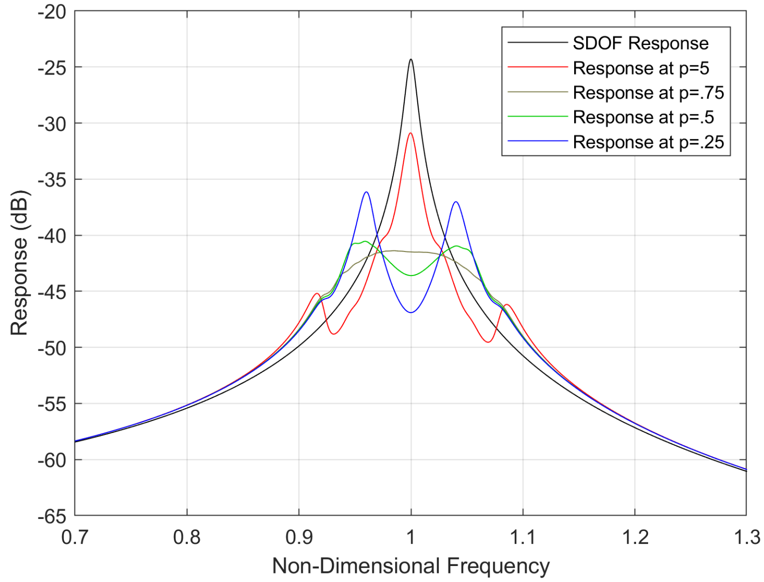

Fig. 8: The p value affects the density of the frequency distribution about the center frequency. A p-value of 1 will return a balanced curve with equal density of frequency distribution throughout the target band. A p-value of 0.75 will offer more of a flat-band effect as it has more elements of the frequency distribution closer to the center frequency JS: [match colors to fig 7]

3 Physical Design of SOAs

The preceding section has introduced all of the theoretical parameters required for SOA design. At its heart, the purely mathematical SOA is simply a set of property distributions that can be applied to any physical second order system, such as electrical, acoustic, or mechanical. The following sections will show the steps for the design a planar mechanical SOA. Section 3.1 will discuss mechanical design of an SOA.

3.1 Design of Mechanical SOA

As the manufacture of a mechanical SOA is attempted, one must carefully consider the properties of the primary system that is to be damped. Specifically, the mass and frequency bandwidth of the target system must be determined accurately. That frequency band is defined on two parameters, center frequency, F_c and fractional bandwidth, Δ. Additionally, the material from which the SOA is constructed is also important. The material stiffness and density must be known accurately.

Starting from the classical fundamental beam frequency estimate equation from Kinsler [8], the uniform rectangular cross-section this becomes

$$F_n = 1.875^2 \frac{1}{2\pi} \sqrt{\frac{Et^2}{12\rho L^4}} \tag{4}$$

where E is the material stiffness, t is thickness, ρ is material density, and L is beam length. Equation 4 can be rearranged into an equation with length on the left. Then, realizing that $\beta(n)F_c$ is equivalent to F_N, an expression for the length of each element is generated

$$L(n) = \frac{1}{2\pi} \sqrt[4]{\frac{Et^2}{12\rho} \left(\frac{1.875^2}{\beta(n)F_c}\right)^2} \tag{5}$$

Recall from Sec. 2.4, it was stated that the optimum mass ratio for the flat-band response, μ, equal to $\frac{\Delta^2}{4}$. The mass of each oscillator is calculated knowing the total SOA mass and distributing it across the array. For this system, the mass ratio distribution will have a negative linear distribution parameter, optimized in [4] to be -1. The lowest frequency element will be the most massive one and the highest frequency will be the least massive. The mass distribution, α, is defined as

$$\alpha(n) = -\left(\frac{\mu}{N}\right)\alpha^*(n) \tag{6}$$

where $\alpha(n)$ represents the distributions of the masses of each oscillator, N is the total number of oscillators and α^* is a linear distribution from 1-$\frac{\Delta}{2}$ to 1+$\frac{\Delta}{2}$ with N^* elements.

Once element masses are defined, geometry determination can proceed using a given material's density and material constraints and iterating the thickness. Knowing that

$$m = \rho LWt,$$

one can define the width distribution as

$$W(n) = \frac{\alpha(n)}{\rho L(n)t_i}. \tag{7}$$

where t_i is the iterated thickness level. Assuming thickness to be constant for all elements, the length and width distributions are adjusted to follow the prescribed $\alpha(n)$. As such, there are an infinite number of solutions to the SOA design problem. This infinite set is bounded by eliminating violations of previously discussed material limits, which, again, will vary depending on material and manufacturing method.

4 Disorder

One of the most important aspects in the design of a subordinate oscillator array is the sensitivity of performance to error in the distributions of mass and frequency. The physical realization of an SOA design will have unavoidable error, deviation, or drift from the specified design parameters. There are several sources of such unanticipated disorder. These include, but are not limited to variation due to: manufacturing tolerances, corrosion, fatigue, and imprecise knowledge of boundary conditions. Even though the initial design may perform well, life cycle and environmental factors must be considered. As shown in Fig. 9, error in the α and β distributions on the order of 1 part in 1000 is detectable and 1 in 100 has a significant effect on the results [4, 9].

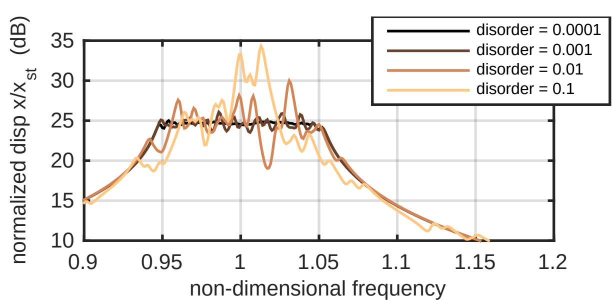

Fig. 9: A flat-band response was designed to have a 10% bandwidth. The performance degrades as error in the property distribution increases from $d = .0001$ where the bandpass performance is unaffected to $d = 0.1$ where the bandpass is response is not apparent.

Figure 9 includes four different distributions of the masses, m_n, and stiffnesses, k_n, and constant low damping. In this figure, disorder is randomly introduced into the β distribution. Disorder is introduced in a statistical manner according to

$$\beta_d(n) = \beta_0(n) + d\varepsilon(n), \tag{8}$$

where $\beta_d(n)$ is the disordered β distribution, β_0 is the original, zero-disorder distribution, d is the disorder level, and $\varepsilon(n)$ is the Gaussian random distribution with a standard deviation of 1. This formulation may produce values for β_d that are negative, setting a maximum disorder such that $d < 0.25$ drastically reduces this chance. At $d = 0.25$, the chance of this occurrence is 4 standard deviations from the mean, less than 0.0001. Systems with disorder at or above this level will perform so poorly that analysis of the system becomes unimportant. As the disorder multiplier is reduced to levels that are relevant to SOA system analysis, on the order of $d = 0.1$, this chance becomes vanishingly small. As an error trap, any negative values of β_n are eliminated by using the absolute value. Figure 9 also illustrates how poorly systems with $d > .1$ perform and why disorder levels above that are of little interest.

4.1 Performance metrics

Two methods have been developed to distill the performance of a complex structure like an SOA into a single number that apply to different situations. We begin with Fig. 10 which shows the basic SDOF response of the primary system. Also shown is an ideal, zero-disorder response as well as a response with 10% disorder.

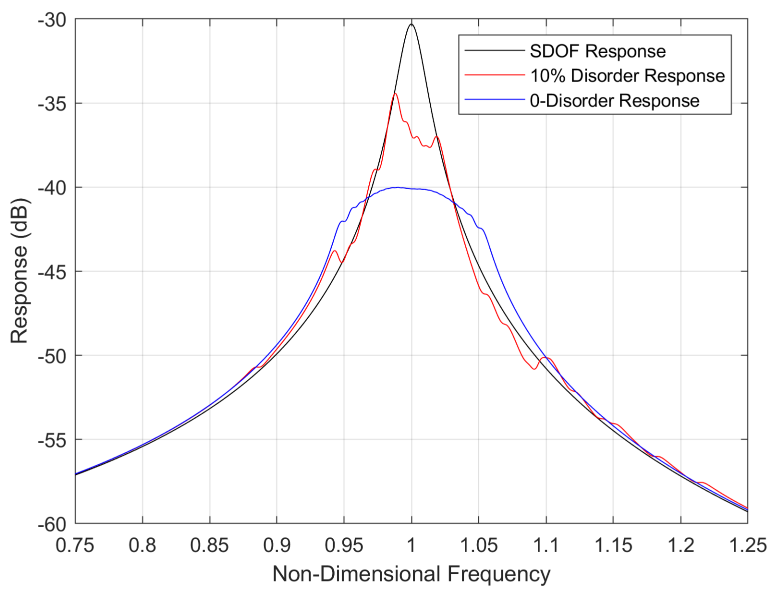

Fig. 10: The difference between the 0-disorder and disordered curves form the basis for the set used to calculate the RMS and MDIB metrics

As stated before, the response is simulated from the disordered distributions by adding or subtracting randomized amounts to each element in the β distribution. Although there are a discrete number of oscillators, the response of the SOA system is continuous. This continuous response can be calculated at any arbitrary point to provide better resolution for the metric than simply N oscillators worth of data points. The difference between the two curves at some number of frequency points that is $>> N$ is calculated, similar to a standard transfer function. This calculation is shown in Eq. 9.

$$G_M(\Omega_j) = G_0(\Omega_j) - G_d(\Omega_j) \tag{9}$$

where G_M is the set of difference between the zero-disorder G_0 and the disordered curve G_d and j marks the individual sample in the frequency domain. With G_M created, we can calculate a metric for this individual pair of FRFs.

The root-mean-square of the set has been labeled the RMS metric and is calculated as shown in Eq. 10.

$$\hat{M}_{rms} = \left(\frac{1}{J} \sum_{j=1}^{J} |G_M(\Omega_j)|^2 \right)^{1/2} \tag{10}$$

The greatest absolute value in this set has been labelled the Maximum-Difference-In-Band (MDIB) metric. This is measured simply in Eq. 11 as

$$\hat{M}_{MDIB} = \underset{j=0\to J}{MAX}\left(|G_M(\Omega_j)|\right). \tag{11}$$

When measuring the performance of an SOA, it is important to consider both metrics each individual application of the SOA system and whether the MDIB or RMS is appropriate. RMS will be used as the in analysis for the rest of this paper so as not to unnecessarily double calculations.

These metrics generate a single value that represents the performance of the SOA. However, as it depends partly on a random distribution, it is prone to inaccuracy and does not lead to any insights on its own. In order to reduce the variation, the disordered curve G_d is regenerated at the same disorder level and a new $\hat{M}$ is calculated. This is repeated until the set of $\hat{M}$ has stabilized. Experience has shown that this occurs after approximately 250 iterations. In this work, 2500 calculations were made to ensure this stability. This mean is designated as M.

4.2 Influence of Varying SOA Parameters on Metrics

Now that we have a single accurate RMS metric M_{RMS}, we can begin to vary parameter values. The following analysis shows the effect of multiple levels of modal overlap and quality factor as disorder increases. The RMS metric will be shown, but the MDIB, results follow the same trends, only with higher values of the metric.

In the following figures, the x-axis has a resolution of 24 disorder levels ranged from 10^{-4} to 1. For each modal overlap, η, or quality factor, Q, M_{RMS} is generated at the given disorder level. It is important to note that the curves in these figures have 3 regions. There is a region where $d < 0.001$ where the results are defined by the ability of the parameters to generate a response that meets the zero-disorder reference curve. In other words, this low disorder zone is dominated by the ability of the disordered curve to match the reference curve from Fig. 10. There is a transition zone, $0.001 < d < 0.1$ where the effects of of disorder and dynamic properties of the system are in conflict, and a zone, $d > 0.1$ where the effects of disorder completely take over the results. In the disorder-dominated zone, there is essentially nothing that can be done to recover the system.

Figure 11 shows an example of the effect of increasing disorder at different levels of η. There are two important points to take away from these curves. The first observation is that for $\eta < 2$, there exists levels of M_{RMS} that do not vanish no matter how low the disorder goes. This effect is due to the intrinsic lack of coverage of the frequency band that having a modal overlap below 2 causes as discussed in Sec. 2.1. The second observation is that $\eta > 2$ has very little effect in the low-disorder regime and only moderate effectiveness in the transition zone. Once modal overlap reaches a value of 2, the requirements for a fully covered frequency band are met and increasing from there is simply adding additional unnecessary oscillators to the system. The additional oscillators add some resistance to the effects of disorder in the mid-range, but while the disorder is low there is negligible effect.

Figure 12 depicts the effect of varying damping levels in the SOA. Two characteristics are immediately visible. Much like having $\eta < 2$, if the damping is too high, there is static M_{RMS}, even in the zero-disorder case, as the highly-damped oscillators are not able to create the reference response. The more highly damped system simply cannot accurately recreate the chosen reference. On the other hand, if the damping level is too low, M_{RMS} begins to increase more quickly. This effect is due to the fact that with lower damping, any individual error can have a bigger effect on the system as a whole. Like with η in Fig. 11, there is a transition point between disorder and damping which determines the dominant factor in the metric. This region occurs in both figures at the same levels of disorder.

One must notice there is a particular Q, termed $\hat{Q}$, below which the M_{RMS} curve is only concave up. When $Q < \hat{Q}$, the static M_{RMS} caused by the inability to precisely recreate the reference response increases. Conversely, $Q > \hat{Q}$ increases the system's disorder sensitivity in the transition region. In Fig. 12, $\hat{Q}$ is approximately 50. Numerical simulation indicates that $\hat{Q}$ depends on the primary structure's damping and is generally less than the Quality Factor of the primary structure, which in this case is $Q = 100$.

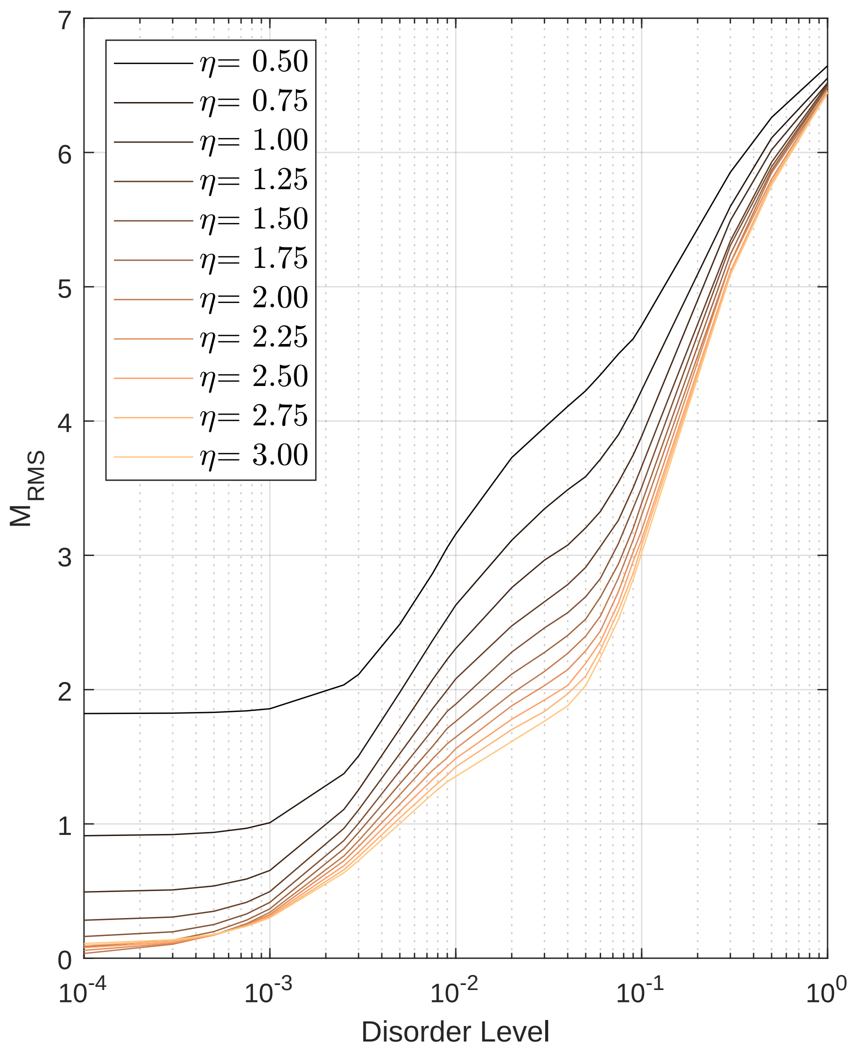

Fig. 11: RMS metric showing η as a function of averaged random disorder. Note that even at very low disorder, $\eta < 2$ has base levels of error sensitivity. The parameter $\eta > 2$ has very little advantage. By 1%, the errors begin to line up.

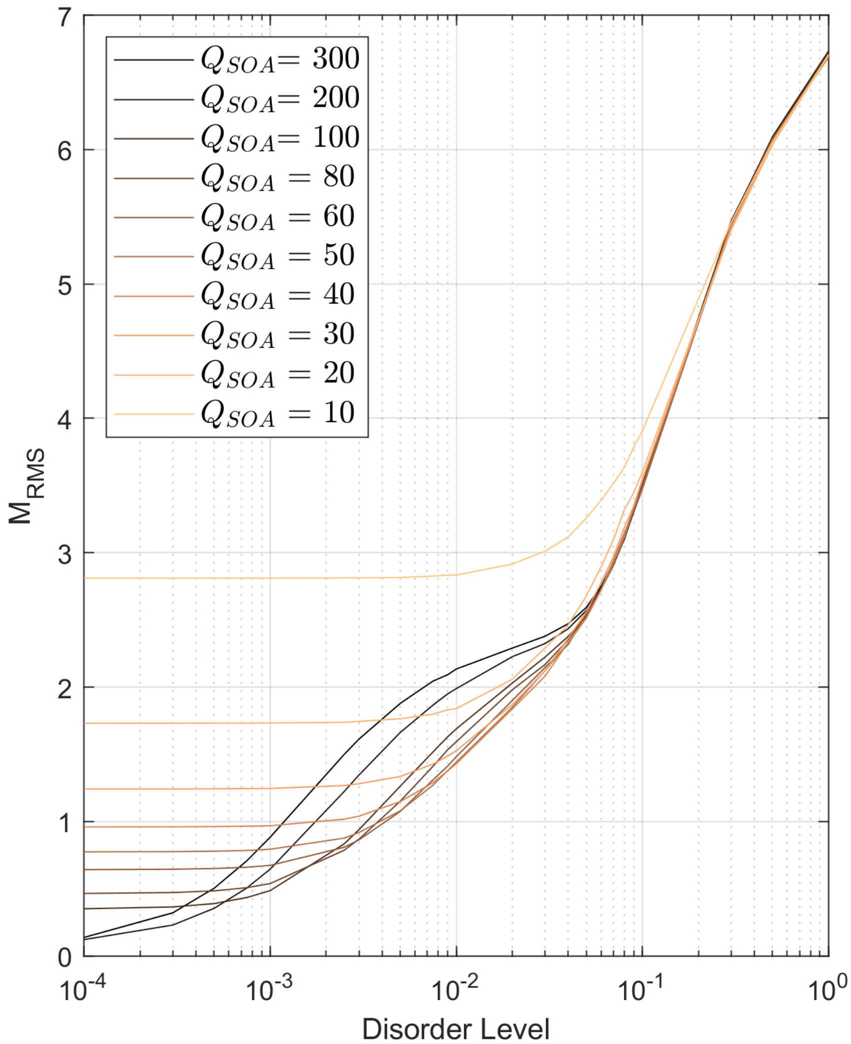

Fig. 12: This is a set of curves detailing RMS of several values of Q_{SOA} with primary damping set to 100

5 Conclusions

This work has presented the underlying principles and implementation guidelines for creating a set of attachments that allows the tailoring of the frequency response of a host structure. The effects of the sensitivity of these attachments to error in the distributions of key properties was analyzed. Such error may be caused by several factors, such as manufacturing defects, environmental effects, and design methodology problems.

In cases where temporal drift of host structure properties is an issue, a classical vibration absorber shows significant degradation of performance and may even increase response amplitude in nearby frequencies. The subordinate oscillator array offers improved host structure response modification performance. If the range of the anticipated frequency drift can be quantified ahead of time, an increased fractional bandwidth can be used to accommodate the drift at the cost of more stringent manufacturing constraints.

Sources of disorder are manifold. It is not possible to eliminate any of the listed sources of disorder but the magnitude of these effects may be small. In a normally distributed addition of error to the distributions of SOA parameters, there is a highly nonlinear response. SOA design parameters can be altered to accommodate sources of disorder.

In cases where the mass of the host structure is large, reasonable manufacturing techniques allow for increasingly tight tolerances which keeps the error levels low, even at low damping levels. As such, the larger the host structure, the less of an issue disorder of this sort presents. In systems that require less stringent response reduction, constructing an SOA with more highly-damped materials lowers this sensitivity to error. This sensitivity reduction comes with a moderately decreased ability for the SOA to pull energy from the host structure, but the advantage of reduced sensitivity to disorder may often be worth the compromise.

Acknowledgements

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