Fixture-Resolved Water Monitoring and Leak Localization via Transmission-Line Modeling and Active Time-Domain Reflectometry of Residential Plumbing

A Technical Disclosure

Calvin Rose

calvinrose18938@gmail.com

Version 1.0 — July 18, 2026

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Abstract

We describe a method for inferring, from a single instrumented point on a dwelling's main water supply, (i) which plumbing fixture is drawing water and (ii) the presence and physical location of leaks. The dwelling's pipe network is modeled as a branched acoustic transmission line: pressure and volumetric flow play the roles of voltage and current, the distributed inertance and compliance of the water-filled pipe set a characteristic impedance and wave speed, viscous friction sets the attenuation, and a leak appears as a shunt admittance to a pressure sink at a specific location. Each fixture terminates a branch of distinct length and load impedance, so opening a fixture measurably changes the driving-point complex impedance $Z_{\text{in}}(\omega)$ observed at the main and produces a reflection whose time-of-flight localizes the fixture. We describe both *passive* inference (from the pressure transients fixtures generate when their valves actuate) and, as the distinguishing element of this disclosure, *active* interrogation, in which a controlled transducer injects a known excitation and the system measures $Z_{\text{in}}(\omega)$ or performs time-domain reflectometry (TDR) to obtain range to each active fixture or leak directly. We give the governing equations, the transfer-matrix (ABCD) and scattering-matrix network formulations, a complete and reproducible reference simulation of a model dwelling, and an enumeration of practical embodiments and variants, including methods addressing the dominant real-world error sources (trapped air, pipe-material variability, and simultaneous fixture use). The purpose of the disclosure is to make these methods and their variants freely available.

Keywords: water leak detection; leak localization; fixture disaggregation; acoustic transmission line; telegrapher's equations; time-domain reflectometry (TDR); driving-point impedance; impedance spectroscopy; non-intrusive load monitoring; active interrogation; smart water monitoring; defensive publication; prior art.

1 Introduction and motivation

Water damage is among the most frequent and costly causes of residential insurance loss, and much of it originates from slow leaks and pipe failures that go undetected until damage is extensive. Two broad classes of consumer device address this today. Spot ("puck") sensors placed on the floor detect a puddle by the conductivity bridging two electrodes; they are inexpensive but purely reactive and blind to anything that does not reach the sensor. Whole-home inline monitors installed on the supply line measure aggregate flow, pressure, and temperature and can close a motorized valve; they detect anomalous continuous flow and can run a static pressure-decay "health test," but in their common form they report a system-level anomaly rather than *which* fixture is responsible or *where* a leak is located.

The physical fact this disclosure exploits is that a dwelling's plumbing is a wave-bearing network. Pressure disturbances propagate as acoustic waves along the water column and reflect from every impedance discontinuity—junctions, elbows, valves, appliances, and leaks. Because each fixture sits at a distinct distance from a sensor on the main, through pipes of distinct diameter and length, and terminates in a distinct load, each fixture imprints a distinguishable signature on quantities measurable at that single sensor. The same is true of a leak, which introduces a reflector where none previously existed. We formalize this by modeling the network as a transmission line and show that both fixture identity and leak location are recoverable, in principle, from the driving-point complex impedance and its time-domain (reflectometry) dual.

2 Relationship to prior work

We situate this disclosure honestly against known work so that the specific elements offered here are unambiguous.

Whole-home flow monitors (e.g. commercially available inline devices with an integral shutoff valve) detect anomalous flow and perform static pressure-decay tests. They establish the value of single-point supply-line sensing and automatic shutoff but do not, in their standard form, resolve individual fixtures or localize leaks by distance.

Single-sensor fixture disaggregation was demonstrated in the HydroSense research program, which recovered fixture-level water activity from one pressure-based sensor by recognizing the pressure transients produced when valves open and close [1, 2]. A contemporary commercial system samples supply-line pressure at high rate and uses machine-learning pattern matching over large corpora of fixture events to classify fixtures and flag leaks [3]. Both exploit the same physical premise as this disclosure but treat the network empirically—as a black box whose signatures are *learned*—and both operate *passively*.

Transient- and impedance-based pipeline diagnostics form a mature literature in water engineering. Inverse transient analysis fits a hydraulic model to measured transients to infer leak size and location [4, 5]. Time-domain reflectometry combined with a scattering-parameter transmission-line model has been used to detect and locate multiple leaks in long-distance pipelines [6]. These works establish the transmission-line/TDR machinery in the pipeline setting.

Against that background, the elements this disclosure places in the public domain are the combination and residential application of: (a) an explicit, physics-based transmission-line *forward model* of a branched household network used for both fixture disaggregation and leak localization from a single main-line node; (b) *active* interrogation of that network by a controlled excitation transducer (impedance spectroscopy and active TDR) rather than passive listening alone; and (c) the specific calibration and inference methods of Section 6 that make (a) and (b) robust to trapped air, pipe material, and simultaneous fixture use. We make no proprietary claim to the underlying physics, which is classical.

3 Physical model

3.1 The hydraulic–acoustic transmission line

Consider a straight, uniform, liquid-filled pipe segment of cross-sectional area A. Let $p(x,t)$ be the acoustic pressure perturbation and $Q(x,t)$ the volumetric flow perturbation. For small perturbations the segment obeys the one-dimensional telegrapher's equations,

$$-\frac{\partial p}{\partial x} = R'\,Q + L'\,\frac{\partial Q}{\partial t}, \tag{1}$$

$$-\frac{\partial Q}{\partial x} = G'\,p + C'\,\frac{\partial p}{\partial t}, \tag{2}$$

which are identical in form to those of an electrical line under the correspondence

$$p \leftrightarrow V, \qquad Q \leftrightarrow I.$$

The distributed (per-unit-length) parameters are the fluid inertance $L' = \rho/A$, the compliance $C' = A/(\rho c^2)$, the viscous series resistance R', and—central to leak detection—the shunt conductance G', which represents fluid escaping the pipe. Here ρ is the liquid density and c the wave speed defined below. Table 1 summarizes the analogy.

Table 1: Hydraulic–electrical transmission-line correspondence.

Electrical line	Hydraulic pipe	Physical origin
Voltage V	Pressure p	—
Current I	Volumetric flow Q	—
Series inductance L'	Inertance ρ/A	Inertia of the water column
Shunt capacitance C'	Compliance $A/(\rho c^2)$	Fluid + pipe-wall elasticity
Series resistance R'	Viscous friction	Unsteady wall shear
Shunt conductance G'	**Leakage**	**Water escaping the pipe**

3.2 Characteristic impedance and wave speed

In the frequency domain (time dependence $e^{j\omega t}$) the segment has characteristic impedance and propagation constant

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \xrightarrow[\text{low loss}]{} \sqrt{\frac{L'}{C'}} = \frac{\rho c}{A}, \qquad \gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta, \quad \beta = \frac{\omega}{c}. \tag{3}$$

The wave speed in a compliant pipe is reduced from its free-fluid value $c_0 = \sqrt{K/\rho}$ (about 1480 $\mathrm{m\,s^{-1}}$ for water, with K the fluid bulk modulus) by wall elasticity, per the Korteweg relation [7]

$$c = \frac{c_0}{\sqrt{1 + \dfrac{K}{E}\dfrac{D}{e}}}, \tag{4}$$

where E is the pipe-wall Young's modulus, D the diameter, and e the wall thickness. Equation (4) is why material matters: rigid copper ($E \approx 117\,\mathrm{GPa}$) yields $c \approx 1200$–$1400\ \mathrm{m\,s^{-1}}$, whereas compliant PEX ($E \sim 0.5$–$1\,\mathrm{GPa}$) can drop c to a few hundred $\mathrm{m\,s^{-1}}$, rescaling every resonance and time-of-flight.

3.3 The attenuation model

Viscous and viscothermal losses make α frequency dependent and damp the high-frequency resonances that would otherwise sharpen the signatures. A convenient parameterization capturing the characteristic $\sqrt{\omega}$ growth of unsteady friction is

$$\alpha(\omega) = \kappa\sqrt{\omega}, \tag{5}$$

with κ a lumped loss coefficient (units $\mathrm{s^{1/2}\,m^{-1}}$, since α is in $\mathrm{m^{-1}}$ and ω in $\mathrm{s^{-1}}$) identified per installation. Physically κ sets the quality factor of the branch resonances—how quickly the signatures blur as frequency rises—and is fit from the observed linewidths (bandwidths) of the measured resonances.

3.4 Segment two-ports: ABCD and scattering forms

A uniform segment of length ℓ relates its input and output port variables by the transmission (ABCD) matrix

$$\begin{bmatrix} p_{\mathrm{in}} \\ Q_{\mathrm{in}} \end{bmatrix} = \begin{bmatrix} \cosh\gamma\ell & Z_0\sinh\gamma\ell \\ \dfrac{1}{Z_0}\sinh\gamma\ell & \cosh\gamma\ell \end{bmatrix} \begin{bmatrix} p_{\mathrm{out}} \\ Q_{\mathrm{out}} \end{bmatrix}. \tag{6}$$

The input impedance looking into a segment terminated by a load Z_L follows directly:

$$Z_{\mathrm{in}} = Z_0\,\frac{Z_L + Z_0\tanh\gamma\ell}{Z_0 + Z_L\tanh\gamma\ell}. \tag{7}$$

Equivalently one may work with scattering (S) parameters referenced to Z_0, in which incident and reflected wave amplitudes are propagated and cascaded via wave (T-) matrices; this form is numerically convenient for long cascades and multiple leaks [6]. The two formulations are interchangeable.

3.5 Network assembly and the driving-point impedance

A dwelling is a tree of segments. Terminations model fixtures: a closed valve is an open circuit, $Z_L \to \infty$ (no flow); an open fixture discharging to atmosphere presents a finite, largely resistive orifice load $Z_L \approx R_{\mathrm{orifice}}$, smaller for a wide tub filler and larger for a restrictive showerhead. To obtain the quantity measured at the sensor, one (i) transforms each terminating load back to its branch junction with (7), (ii) combines all branches meeting at a junction in parallel,

$$Y_{\mathrm{junction}} = \sum_i \frac{1}{Z_i}, \tag{8}$$

and (iii) transforms the combined junction impedance through the trunk to the sensor, yielding the driving-point impedance $Z_{\mathrm{in}}(\omega)$. A leak at a point along a branch is inserted by splitting that branch at the leak and adding a shunt leak admittance Y_{leak} (a small orifice to atmosphere, i.e. a high resistance and weak reflector) in parallel at that node before transforming onward.

4 Interrogation and inference

4.1 Passive transient signatures

When any valve opens or closes it launches a pressure transient (mild water hammer) that reverberates through the network's reflections. Because the reverberation pattern is fixed by the branch that produced it, the transient carries a fixture-specific signature observable at the main. Passive operation requires no actuator and is the mode used by prior single-sensor systems [1, 3].

4.2 Active impedance spectroscopy

The distinguishing mode of this disclosure adds a controlled excitation. A transducer coupled to the supply line—a piezoelectric stack, a voice-coil or solenoid-driven small piston, or a rapidly modulated pilot valve—injects a known small-signal waveform (swept sine or multitone chirp) while a co-located high-bandwidth pressure sensor measures the response. Dividing measured pressure by the known injected flow yields the driving-point impedance $Z_{\text{in}}(\omega)$ directly, from which the network state is inferred by comparison against the forward model of the previous section. Active operation does not wait for a user to actuate a fixture and can interrogate the network on demand, including when the dwelling is quiescent (the ideal condition for leak search).

4.3 Active time-domain reflectometry (TDR)

The time-domain dual is the more powerful framing for localization because it returns *range* directly. The reflection coefficient at the sensor, referenced to the trunk impedance, is

$$\Gamma(\omega) = \frac{Z_{\text{in}}(\omega) - Z_0}{Z_{\text{in}}(\omega) + Z_0}. \tag{9}$$

Its band-limited inverse transform is the impulse reflection response,

$$h(t) = \mathcal{F}^{-1}\big\{\, \Gamma(\omega)\, W(\omega) \,\big\}, \tag{10}$$

where $W(\omega)$ is an interrogation-pulse window. Each impedance discontinuity returns an echo at delay $t = 2d/c$, so mapping delay to distance by

$$d = \frac{c\,t}{2} \tag{11}$$

places every reflector—junction, active fixture, or leak—at its physical distance from the sensor. Equation (11) is the core localization relation.

4.4 Fixture disaggregation

Opening a fixture flips its termination from an open circuit to a finite load, which (i) shifts the standing-wave resonances of $Z_{\text{in}}(\omega)$ and (ii) changes the echo at that fixture's time-of-flight. Differencing the measured trace against a quiescent reference isolates the change and yields a peak at the active fixture's distance. Because distinct fixtures occupy distinct distances and present distinct load impedances, the difference signature identifies which fixture is active. Simultaneous fixtures are treated in Section 6.

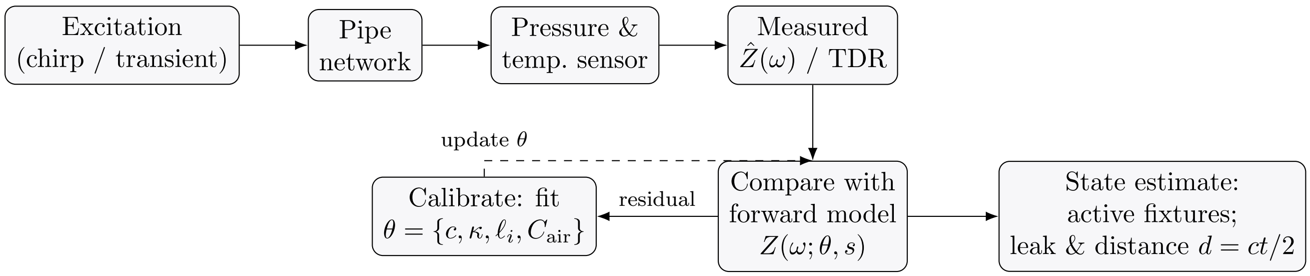

Figure 1: Two-mode inference/calibration loop. Solid path: a measurement is compared against the per-dwelling forward model to estimate state. Feedback path (dashed): during quiescence the residual drives re-identification of the model parameters θ, keeping the model matched to the installation as conditions (e.g. temperature, trapped air) drift.

4.5 Leak detection and localization

A leak manifests in three complementary ways, any or all of which may be combined. Statically, it slowly bleeds pressure, detectable by sealing the line with the shutoff valve and watching for decay (the “health test”). Dynamically, it is a shunt admittance that returns an echo at its own time-of-flight, appearing in a reference-differenced TDR trace as a new reflector at a location where no fixture exists—so even with every valve shut, (11) yields the leak’s distance. Broadband turbulent noise (leak “hiss”) provides a third channel; cross-correlating two spatially separated sensors localizes the leak from the inter-sensor time delay, as in utility practice. A leak also perturbs every echo beyond it, so the *onset* of the disturbance in a differenced trace marks the leak position.

4.6 Frequency band, bandwidth, and range resolution

The useful signatures occupy a low, accessible band. A branch of length ℓ has quarter-wave resonances near $f \approx c/(4\ell)$, so multi-metre branches resonate at tens of hertz (e.g. $c = 1300\ \mathrm{m\,s^{-1}}$, $\ell = 14\,\mathrm{m}$ gives $\approx 23\,\mathrm{Hz}$), with informative structure up to the low kilohertz before the attenuation of (5) dominates. For active TDR the range resolution is set by the interrogation bandwidth B,

$$\Delta d \approx \frac{c}{2B}, \tag{12}$$

so with $c \approx 1300\ \mathrm{m\,s^{-1}}$ and $B \approx 4\,\mathrm{kHz}$ (as used in the reference implementation of Section 5) the resolution is $\Delta d \approx 0.16\,\mathrm{m}$—sub-metre, ample to separate fixtures spaced metres apart. The unambiguous range follows from the sweep’s frequency spacing Δf as $d_{\max} = c/(2\Delta f)$; the reference code’s $\Delta f = 5\,\mathrm{Hz}$ gives $d_{\max} \approx 130\,\mathrm{m}$, far beyond any dwelling. The pressure sensor is sampled at no less than twice the highest interrogation frequency (a few kilohertz) to satisfy the Nyquist criterion over this band.

4.7 Inference and calibration procedure

The forward model is exercised in a two-mode loop (Figure 1). A *calibration* mode identifies the per-dwelling parameters θ by fitting the model to a quiescent measurement; an *inference* mode then estimates the network state s—which fixtures are open, and whether and where a leak is present—by comparing each new measurement against the calibrated model. Listing 1 states both procedures. Temperature is read on each inference and used to correct the wave speed c (Section 6).

Listing 1: Calibration and inference procedures.

```
procedure CALIBRATE(quiescent):          # per install; repeat periodically
    inject known excitation x(t); record pressure p(t)
    form measured driving-point Ẑ(ω)  (or TDR ĥ)
    fit θ = {c, κ, {ℓ_i}, topology, C_air}
        minimizing ‖ Ẑ(ω) − Z_model(ω; θ, all closed) ‖
    (optional) refine per-fixture loads R_i from labeled fixture events
    store θ* and the reference response

procedure INFER(observation):            # continuous or on-demand
    acquire response (passive transient or active sweep)
    read temperature T;  correct c ← c(T)
    residual ← observation − reference
    s* = arg min_s ‖ observation − Z_model(ω; θ*, s) ‖
        over combinations s of open-fixture loads
    detect reflection peaks; map delay to distance d = c t/2
    if a reflector appears where no fixture exists:
        report a leak at that distance
    return active fixtures and leak state
```

5 Reference implementation and simulated results

To demonstrate that the signatures are computable and separable, we implemented the full forward model of a model dwelling: a sensor at the main, a 2 m trunk to a junction, and three branches—a kitchen sink at 4 m (medium orifice), a guest tub at 9 m (wide, low-resistance faucet), and a master shower at 14 m (restrictive head)—with an optional small leak 11 m out on the master branch. Wave speed is $c = 1300\ \mathrm{m\,s^{-1}}$ (copper). Each pipe segment uses (6); branches combine in parallel and fold to the sensor by the recursion of the previous section. The complete source code is given in Appendix A and reproduces Figure 2.

The model recovers each feature at its correct location. Table 2 reports the detected time-of-flight peak for each fixture and for the leak, against the known ground-truth distance.

Table 2: Time-of-flight localization in the reference simulation: detected TDR peak vs. ground-truth distance.

Feature	Ground truth (m)	Detected (m)
Kitchen sink	4.0	4.0
Guest tub	9.0	9.0
Master shower	14.0	14.0
Leak (all valves shut)	11.0	11.0

6 Practical embodiments and non-obvious elements

The idealized model of Section 5 does not by itself survive a real dwelling. The following methods, disclosed here, address the dominant error sources; each is offered as part of the public-domain disclosure.

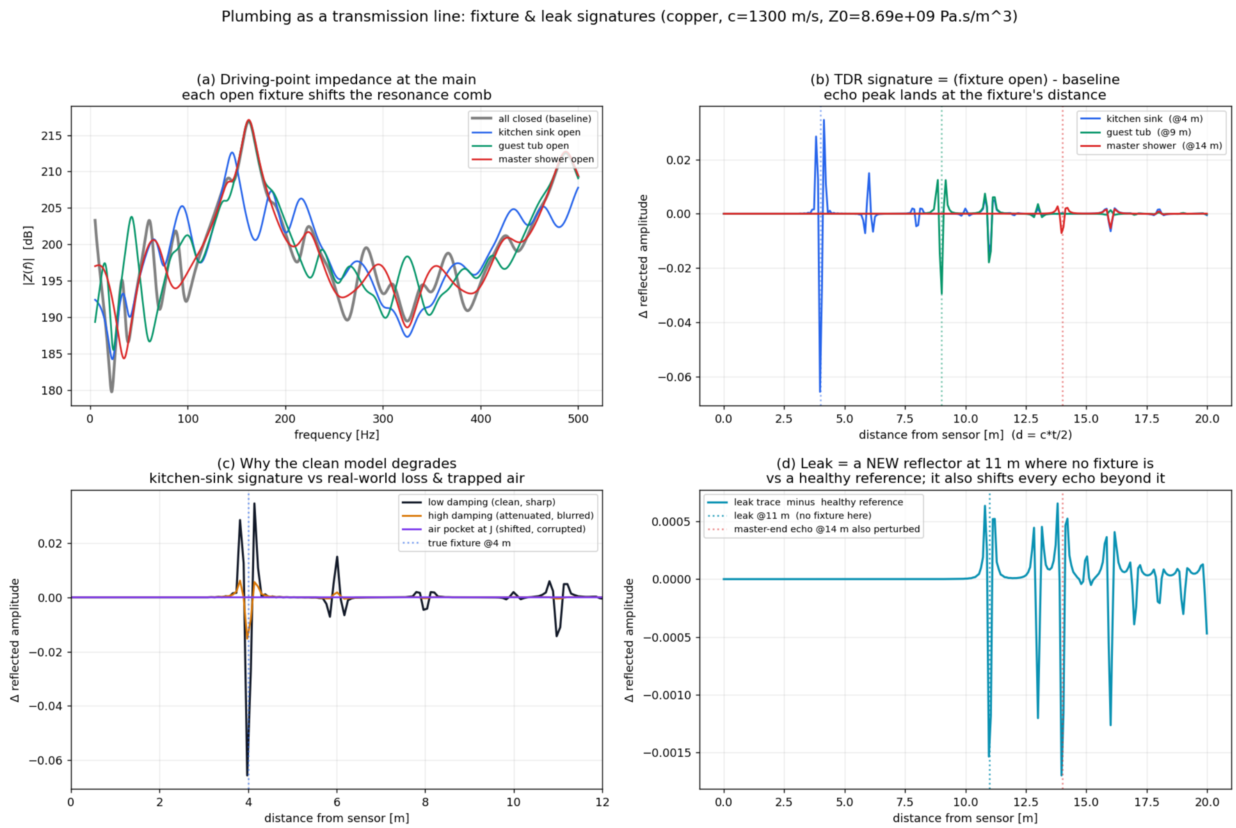

Figure 2: Simulated signatures for the model dwelling. **(a)** Driving-point impedance $|Z_{\text{in}}(f)|$: the "all closed" baseline is a resonance comb set by branch lengths, which each open fixture detunes. **(b)** Reference-differenced TDR traces: each open fixture produces an echo peak at its true distance (4/9/14 m). **(c)** Degradation of the (clean) kitchen-sink signature under higher damping and a trapped air pocket at the junction. **(d)** With every valve shut, a small leak appears as a new reflector at 11 m—where no fixture exists—against a healthy reference, while also perturbing the 14 m echo beyond it.

Per-installation system identification. Rather than assume textbook constants, the effective wave speed c, loss coefficient κ, branch lengths, and junction topology are *estimated* for each dwelling by fitting the forward model to a calibration record—either a quiescent active sweep or a period of labeled fixture use. Fitting removes dependence on unknown pipe material and layout and yields a dwelling-specific forward model.

Trapped-air detection and compensation. Entrained air is the dominant corruptor: even a small void collapses the effective wave speed and adds a large local compliance, detuning every resonance (Figure 2(c)). A trapped pocket is disclosed as detectable by its signature as a lumped shunt compliance $Z_{\text{air}} = 1/(j\omega C_{\text{air}})$ whose $1/\omega$ frequency dependence and location are estimable from the same active sweep, after which the fitted forward model is corrected for it. Periodic re-identification tracks air that accumulates or clears over time.

Material-agnostic self-calibration. Because c and Z_0 are fitted rather than assumed, the same device operates on copper, PEX, CPVC, or mixed systems; the fitted c is itself a diagnostic of pipe material and condition.

Simultaneous-fixture disaggregation. When several fixtures draw at once the network state is a joint boundary condition, not a superposition of single-fixture signatures. Disclosed approaches include: model-based state estimation that searches over combinations of terminating loads to best explain the measured $Z_{\text{in}}(\omega)$; exploitation of the distinct time-of-flight of each open branch in the TDR trace, which remains separable when distances differ; and a learned classifier trained on forward-model-simulated combinations to initialize the search.

Active–passive fusion. Passive transient capture (free, triggered by ordinary use) and on-demand active interrogation (available during quiescence, ideal for leak search) are combined, the passive events also serving as opportunistic re-calibration excitations.

Multi-sensor and multi-modal fusion. A pressure sensor at the main may be augmented by a second pressure sensor or a pipe-clamped accelerometer/ hydrophone; two sensors enable cross-correlation leak localization from turbulent hiss, complementing the reflectometric range estimate, and improve observability of far branches.

Excitation hardware variants. The active source may be a piezoelectric stack bonded to the pipe, a voice-coil or solenoid-driven piston, a modulated pilot valve that briefly perturbs flow to launch a known chirp, or controlled actuation of the main shutoff valve itself as a reference transient generator.

Transducer coupling. Two coupling classes are disclosed. A *non-intrusive* (clamp-on) transducer—a piezoelectric element bonded to the outside of the pipe, or a pipe-clamped accelerometer/hydrophone—avoids any contact with the potable water and introduces no flow obstruction, at the cost of weaker, wall-mediated coupling. A *wetted* transducer—a piezo diaphragm or small piston mounted in a tee in direct contact with the water—couples far more efficiently but must be a sealed, potable-safe fitting. The same device may use a clamp-on sensor for passive listening and a wetted actuator for active interrogation, or vice versa.

Drive level, bandwidth, and signal-to-noise. The active excitation is small-signal—pressure perturbations of order a kilopascal, well below cavitation and water-hammer thresholds—and is confined to the band of Section 4.6 (tens of hertz to a few kilohertz). Signal-to-noise is raised by coherent averaging over many repeated chirps and by matched filtering against the known excitation, so that even a weak reflector (a small leak) is detectable despite flow and appliance noise.

Temperature compensation. Both the density ρ and the wave speed c depend on water temperature (over the domestic range c rises with temperature to roughly 74°C and then falls; ρ decreases monotonically). Because every resonance and time-of-flight scales with c, an uncompensated warm-water event would shift the apparent distances. The device reads temperature at the sensor and either applies a known $c(T)$ correction to the forward model or continuously re-identifies c, so that localization by $d = ct/2$ remains referenced correctly.

Potable-water and regulatory compliance. Any wetted component is specified in lead-free, potable-contact-rated materials (e.g. conforming to NSF/ANSI 61 and applicable lead-free plumbing rules); the non-intrusive coupling option sidesteps water contact entirely. Installation follows the same practice as existing inline monitors and shutoff valves.

7 Enumerated variants

For breadth of disclosure, the following non-exhaustive variants are expressly included. Any workable combination of them is also disclosed.

1. Passive-only, active-only, or fused passive/active operation.
2. Frequency-domain ($Z_{\mathrm{in}}(\omega)$, impedance spectroscopy) or time-domain (TDR) inference, or joint use of both.
3. ABCD/transfer-matrix or scattering-parameter/wave-matrix forward-model formulations.
4. Single-sensor operation, or multi-sensor operation with cross-correlation localization.
5. Excitation by piezoelectric, electrodynamic (voice-coil), solenoid/piston, modulated pilot valve, or main-valve actuation.
6. Model-based inversion, learned (machine-learning) inference, or hybrid physics-informed learning trained on forward-model-simulated data.
7. Per-installation system identification of c, κ, lengths, topology, and air-pocket parameters, with periodic re-identification.
8. Application to any liquid-carrying building network, including potable supply, recirculation, radiant heating, and fire-suppression lines.
9. Localization of, and discrimination among, leaks, blockages/scale, trapped air, frozen/burst sections, and running or faulty fixtures, by their distinct impedance and time-of-flight signatures.
10. Coupling of any of the above to an automatic shutoff actuator and to remote notification.

8 Reproducibility

Appendix A contains the complete reference implementation (`numpy/matplotlib`, no other dependencies). Running it reproduces Figure 2 and prints the localization check of Table 2. Parameters (wave speed, branch lengths and loads, damping, air-pocket compliance, leak strength) are exposed at the top of the relevant functions; setting the wave speed to a PEX value, adding fixtures to the "open" set, or sweeping the air-pocket compliance reproduces the corresponding qualitative effects discussed above.

9 Publication status and establishing priority

This document is published as a defensive publication. Its authoritative date of public disclosure is the timestamp of the archived record in which it appears (for example, a DOI-bearing repository deposit), not any date printed on this page. For the material to function as prior art that bars later patents, the disclosure must be public, dated, and discoverable; to that end it is deposited in venues that provide an authoritative timestamp and indexing that patent examiners search, including a DOI-issuing archive (e.g. Zenodo), a preprint server (e.g. TechRxiv or arXiv), and dedicated defensive-publication venues (e.g. the Prior Art Archive and a technical disclosure commons). A defensive publication is effective only to the extent it is *enabling*: the equations, parameters, and reference code included here are intended to meet that bar for the disclosed methods and their enumerated variants. Publication is deliberate and, by design, places the described methods in the public domain; it is generally irreversible with respect to the author's own patent rights, which the author knowingly forgoes.

This draft was prepared with computational assistance. Technical content is the author's; it is not legal advice.

References

[1] J. Froehlich, E. Larson, T. Campbell, C. Haggerty, J. Fogarty, S. N. Patel. *HydroSense: Infrastructure-Mediated Single-Point Sensing of Whole-Home Water Activity.* Proc. UbiComp, 2009.

[2] J. Froehlich et al. *Disaggregated Water Sensing from a Single, Pressure-Based Sensor: An Extended Analysis of HydroSense Using Staged Experiments.* Pervasive and Mobile Computing, 8(1), 2012.

[3] Phyn. *Technology.* `https://www.phyn.com/technology/` (accessed 2026).

[4] *Leak Detection in Pipe Systems Using Transients: A Statistical and Methodological Review.* Water, MDPI, 2025.

[5] A. F. Colombo, P. Lee, B. W. Karney. *A selective literature review of transient-based leak detection methods.* J. Hydro-environment Research, 2009.

[6] *TDR-based Multiple Leak Detection System using an S-parameter Transmission Line Model for Long-Distance Pipelines.* Int. J. Prognostics and Health Management (PHM Society).

[7] E. B. Wylie, V. L. Streeter. *Fluid Transients in Systems.* Prentice Hall, 1993.

A Reference implementation (complete source)

```
# SPDX-License-Identifier: Apache-2.0
# Copyright (c) 2026 Calvin Rose.
# Reference implementation for the technical disclosure "Fixture-Resolved Water
# Monitoring and Leak Localization via Transmission-Line Modeling and Active
# Time-Domain Reflectometry of Residential Plumbing." Published as prior art;
# provided WITHOUT WARRANTY OF ANY KIND. See LICENSE-CODE-Apache-2.0.txt.

"""
Plumbing-as-transmission-line simulator
=======================================

Models a small house's water pipes as an acoustic transmission-line network and
shows how different fixtures (and a leak) imprint distinct signatures on

  (1) the driving-point complex impedance Z(f) measured at the main, and
  (2) the Time-Domain Reflectometry (TDR) trace -- a pulse launched at the main,
      whose echoes locate each feature by round-trip time-of-flight (t = 2d/c).

Physics mapping (hydraulic <-> electrical transmission line):
    pressure p <-> voltage V
    volumetric flow Q <-> current I
    fluid inertance <-> series inductance
    fluid+wall compliance<-> shunt capacitance
    viscous friction <-> series resistance
    a LEAK <-> a shunt admittance to a pressure sink

Each uniform pipe segment is a 2x2 ABCD (transmission) two-port. We fold the
network into the sensor with the standard line-input-impedance recursion and
combine branches in parallel at junctions.

Author: built for Calvin
"""

import numpy as np
import matplotlib
matplotlib.use("Agg")
import matplotlib.pyplot as plt

# ---------------------------------------------------------------------------
# Physical constants and default pipe properties
# ---------------------------------------------------------------------------
RHO = 1000.0 # water density [kg/m^3]
C_COPPER = 1300.0 # Korteweg-corrected wave speed in 1/2" copper [m/s]
C_PEX = 450.0 # compliant PEX slows waves dramatically [m/s]
D_PIPE = 0.0138 # inner diameter, ~1/2" copper [m]
A_PIPE = np.pi * (D_PIPE / 2) ** 2 # cross-sectional area [m^2]

def Z0_of(c):
    """Acoustic characteristic impedance Z0 = rho*c/A (pressure/flow)."""
    return RHO * c / A_PIPE

Z0 = Z0_of(C_COPPER) # reference characteristic impedance
Z_CLOSED = 1e6 * Z0 # a shut valve: (near) open circuit, Q ~ 0

# ---------------------------------------------------------------------------
# Transmission-line primitives
# ---------------------------------------------------------------------------
def gamma(omega, c, damping):
    """Complex propagation constant gamma = alpha + j*beta.
    beta = omega/c (phase). alpha ~ damping*sqrt(omega) mimics the
    frequency-dependent (unsteady-friction / viscothermal) attenuation that
    smears high-frequency resonances."""
    beta = omega / c
    alpha = damping * np.sqrt(np.abs(omega))
    return alpha + 1j * beta
```

```python

def transform_through_pipe(ZL, L, omega, c, damping):
    """Input impedance looking into a pipe of length L terminated by load ZL.
        Zin = Z0 * (ZL + Z0 tanh(gL)) / (Z0 + ZL tanh(gL))
    Handles ZL -> inf (closed valve) gracefully via the finite Z_CLOSED."""
    g = gamma(omega, c, damping)
    z0 = Z0_of(c)
    t = np.tanh(g * L)
    return z0 * (ZL + z0 * t) / (z0 + ZL * t)

def parallel(*Zs):
    """Parallel combination of impedances at a common-pressure node."""
    Y = np.zeros_like(Zs[0], dtype=complex)
    for Z in Zs:
        Y = Y + 1.0 / Z
    return 1.0 / Y

# ---------------------------------------------------------------------------
# House topology
# ---------------------------------------------------------------------------
# Sensor at the main -> trunk -> junction J -> three branches.
# Distances below are TOTAL path length from the sensor (so echoes land there).
L_TRUNK = 2.0 # sensor -> junction J [m]

FIXTURES = {
    # name L_from_J R_open (as fraction of Z0) total distance
    "kitchen sink": dict(L=2.0, Ropen=0.50), # 4 m out, medium orifice
    "guest tub": dict(L=7.0, Ropen=0.20), # 9 m out, wide (low R) faucet
    "master shower":dict(L=12.0, Ropen=0.80), # 14 m out, restrictive head
}
# Leak sits on the master-shower branch, 9 m along it from J => 11 m from sensor.
LEAK_L_FROM_J = 9.0 # so fixture (12 m) is 3 m beyond the leak
LEAK_R = 8.0 # small hole -> high resistance -> weak reflector (xZ0)

def branch_impedance_at_J(name, is_open, omega, c, damping,
                          leak_here=False, leak_R=LEAK_R):
    """Impedance of one branch transformed back to the junction J."""
    f = FIXTURES[name]
    z0 = Z0_of(c)
    ZL = (f["Ropen"] * z0) if is_open else Z_CLOSED # fixture termination

    if leak_here:
        # split branch at the leak: [J --9m-- leak --3m-- fixture]
        L_far = f["L"] - LEAK_L_FROM_J
        Z_at_leak = transform_through_pipe(ZL, L_far, omega, c, damping)
        Z_leak = leak_R * z0 # shunt to atmosphere
        Z_node = parallel(Z_at_leak, np.full_like(omega, Z_leak, dtype=complex))
        return transform_through_pipe(Z_node, LEAK_L_FROM_J, omega, c, damping)
    else:
        return transform_through_pipe(ZL, f["L"], omega, c, damping)

def driving_point_Z(omega, open_set, c=C_COPPER, damping=0.0010,
                    leak_on=False, air_C=0.0, leak_R=LEAK_R):
    """Driving-point impedance at the sensor for a given set of open fixtures.
    open_set : set of fixture names currently flowing.
    air_C : lumped air-pocket compliance at J [m^3/Pa]; 0 = none.
    leak_R : leak-hole resistance as a multiple of Z0 (higher = smaller hole).
    """
    branch_Zs = []
    for name in FIXTURES:
        leak_here = leak_on and (name == "master shower")
        branch_Zs.append(
            branch_impedance_at_J(name, name in open_set, omega, c, damping,
                                  leak_here=leak_here, leak_R=leak_R)
        )
    if air_C > 0:
        Z_air = 1.0 / (1j * omega * air_C) # trapped-air compliance
        branch_Zs.append(Z_air)
```

```

    Z_J = parallel(*branch_Zs) # combine at junction
    return transform_through_pipe(Z_J, L_TRUNK, omega, c, damping)

# ---------------------------------------------------------------------------
# TDR: synthesize the impulse-reflection response from Gamma(omega)
# ---------------------------------------------------------------------------
def tdr(open_set, c=C_COPPER, damping=0.0010, leak_on=False, air_C=0.0,
        leak_R=LEAK_R, fmax=4000.0, N=801):
    """Return (distance_axis, reflected_amplitude) for a band-limited pulse."""
    freqs = np.linspace(0.0, fmax, N)
    omega = 2 * np.pi * freqs
    omega[0] = 1e-3 # avoid divide-by-zero at DC
    Zin = driving_point_Z(omega, open_set, c, damping, leak_on, air_C, leak_R)
    Gamma = (Zin - Z0_of(c)) / (Zin + Z0_of(c)) # reflection vs the line
    window = np.hanning(N) # band-limit the pulse
    signal = np.fft.irfft(Gamma * window)
    dt = 1.0 / (2 * fmax)
    dist = c * (np.arange(len(signal)) * dt) / 2.0 # d = c*t/2
    return dist, signal

# ---------------------------------------------------------------------------
# Build the figure
# ---------------------------------------------------------------------------
def main():
    fig, ax = plt.subplots(2, 2, figsize=(15, 10))
    colors = {"kitchen sink": "#2563eb", "guest tub": "#059669",
              "master shower": "#dc2626"}
    total_dist = {"kitchen sink": 4.0, "guest tub": 9.0, "master shower": 14.0}

    # ---- (a) Driving-point impedance |Z(f)| ------------------------------
    f_imp = np.linspace(5, 500, 3000)
    w_imp = 2 * np.pi * f_imp
    base = driving_point_Z(w_imp, set())
    ax[0, 0].plot(f_imp, 20 * np.log10(np.abs(base)), color="0.5", lw=2.4,
                  label="all closed (baseline)")
    for name in FIXTURES:
        Z = driving_point_Z(w_imp, {name})
        ax[0, 0].plot(f_imp, 20 * np.log10(np.abs(Z)), color=colors[name],
                      lw=1.5, label=f"{name} open")
    ax[0, 0].set_title("(a) Driving-point impedance at the main\n"
                       "each open fixture shifts the resonance comb")
    ax[0, 0].set_xlabel("frequency [Hz]")
    ax[0, 0].set_ylabel(r"$|Z(f)|$ [dB]")
    ax[0, 0].legend(fontsize=8, loc="upper right")
    ax[0, 0].grid(alpha=0.25)

    # ---- (b) TDR difference traces: fixture location pops out ------------
    d0, s0 = tdr(set())
    for name in FIXTURES:
        d, s = tdr({name})
        diff = s - s0
        m = d <= 20
        ax[0, 1].plot(d[m], diff[m], color=colors[name], lw=1.6,
                      label=f"{name} (@{total_dist[name]:.0f} m)")
        ax[0, 1].axvline(total_dist[name], color=colors[name], ls=":", alpha=0.5)
    ax[0, 1].set_title("(b) TDR signature = (fixture open) - baseline\n"
                       "echo peak lands at the fixture's distance")
    ax[0, 1].set_xlabel("distance from sensor [m] (d = c*t/2)")
    ax[0, 1].set_ylabel(r"$\Delta$ reflected amplitude")
    ax[0, 1].legend(fontsize=8)
    ax[0, 1].grid(alpha=0.25)

    # ---- (c) Degradation: damping and an air pocket ---------------------
    # Each condition is differenced against ITS OWN matched all-closed baseline,
    # so the curve isolates the fixture signature under that condition.
    target = {"kitchen sink"} # strong, clean 4 m peak
    def signature(damping, air_C):
```

```
        d, s = tdr(target, damping=damping, air_C=air_C)
        _, b = tdr(set(), damping=damping, air_C=air_C)
        return d, s - b
    d, sig_lo = signature(0.0010, 0.0)
    _, sig_hi = signature(0.0028, 0.0)
    _, sig_air = signature(0.0010, 2.5e-11)
    m = d <= 20
    ax[1, 0].plot(d[m], sig_lo[m], color="#111827", lw=1.8,
                  label="low damping (clean, sharp)")
    ax[1, 0].plot(d[m], sig_hi[m], color="#d97706", lw=1.6,
                  label="high damping (attenuated, blurred)")
    ax[1, 0].plot(d[m], sig_air[m], color="#7c3aed", lw=1.6,
                  label="air pocket at J (shifted, corrupted)")
    ax[1, 0].axvline(4.0, color="#2563eb", ls=":", alpha=0.6,
                     label="true fixture @4 m")
    ax[1, 0].set_xlim(0, 12)
    ax[1, 0].set_title("(c) Why the clean model degrades\n"
                       "kitchen-sink signature vs real-world loss & trapped air")
    ax[1, 0].set_xlabel("distance from sensor [m]")
    ax[1, 0].set_ylabel(r"$\Delta$ reflected amplitude")
    ax[1, 0].legend(fontsize=8)
    ax[1, 0].grid(alpha=0.25)

    # ---- (d) Leak localization -------------------------------------------
    d, s_noleak = tdr(set()) # all closed, no leak
    _, s_leak = tdr(set(), leak_on=True, leak_R=3.0) # all closed + small leak
    m = d <= 20
    ax[1, 1].plot(d[m], (s_leak - s_noleak)[m], color="#0891b2", lw=1.8,
                  label="leak trace minus healthy reference")
    ax[1, 1].axvline(11.0, color="#0891b2", ls=":", alpha=0.8,
                     label="leak @11 m (no fixture here)")
    ax[1, 1].axvline(14.0, color="#dc2626", ls=":", alpha=0.5,
                     label="master-end echo @14 m also perturbed")
    ax[1, 1].set_title("(d) Leak = a NEW reflector at 11 m where no fixture is\n"
                       "vs a healthy reference; it also shifts every echo beyond it")
    ax[1, 1].set_xlabel("distance from sensor [m]")
    ax[1, 1].set_ylabel(r"$\Delta$ reflected amplitude")
    ax[1, 1].legend(fontsize=8)
    ax[1, 1].grid(alpha=0.25)

    fig.suptitle("Plumbing as a transmission line: fixture & leak signatures "
                 f"(copper, c={C_COPPER:.0f} m/s, Z0={Z0:.2e} Pa.s/m^3)",
                 fontsize=13, y=0.995)
    fig.tight_layout(rect=[0, 0, 1, 0.97])
    fig.savefig("plumbing_signatures.png", dpi=130)
    print("saved plumbing_signatures.png")

    # ---- quick numerical sanity check: peak locations --------------------
    print("\nTime-of-flight sanity check (expected vs detected peak):")
    for name in FIXTURES:
        d, s = tdr({name})
        diff = np.abs(s - s0)
        m = (d > 1) & (d <= 20)
        peak_d = d[m][np.argmax(diff[m])]
        print(f" {name:14s} expected {total_dist[name]:5.1f} m "
              f"detected {peak_d:5.1f} m")
    d, s = tdr(set(), leak_on=True, leak_R=3.0)
    diff = np.abs(s - s_noleak)
    m = (d > 8) & (d < 13) # search near the leak, below the fixture
    print(f" {'leak':14s} expected 11.0 m detected "
          f"{d[m][np.argmax(diff[m])]:5.1f} m (nearest new reflector)")

if __name__ == "__main__":
    main()
```