Piezoelectric Subordinate Oscillator Arrays: Performance Recovery and Robustness to Uncertainty

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Prior work demonstrates that an attached subordinate oscillator array (SOA) can attenuate vibration of a host structure. The distribution of masses and stiffnesses of the attached oscillators can craft a flat frequency response over a desired band. This response modification can be a significant improvement over classical dynamic vibration absorbers (DVA) that attenuate response at one target frequency while increasing the frequency response amplitude at nearby side frequencies. Performance of the SOA can be highly sensitive to the uncertainty or disorder in the mechanical properties of the system. This paper shows that use of piezolectric SOAs (PSOAs) has the potential to address and ameliorate such sensitivity to off-design situations. It is important to note that the design strategy is simple and effective: it can be carried out without optimization techniques by choosing simple or well-known distributions of electromechanical properties.

1 Introduction

The design of dynamic vibration absorbers (DVAs) is a classical topic of modern vibrations that has been well-studied in many texts. [1,2] In this work, the system to which a DVA is attached is called the host. The essential and well-known feature of DVA performance for a single degree of freedom (SDOF) system is that it is possible to exactly cancel the frequency response of the host at its natural frequency, if the mechanical properties of both host and DVA are known precisely. Even for multi-degree of freedom (MDOF) systems, this result holds approximately true. Such an exact cancellation, however, comes at a cost: the amplitude of the frequency response of the host structure can be substantially increased at frequencies close to the target frequency. A fundamental drawback in this approach is the need for exact knowledge of the mechanical properties of the host and DVA. The DVA designs can therefore be sensitive to uncertainty and temporal drift in the mechanical properties of the system.

A number of strategies have been studied over the years to enhance the response achieved by a classical DVA. These strategies include: active tuning or adaptation of DVAs; switching or semi-switching techniques; designs based on subordinate oscillator arrays (SOAs); or vibration attenuation via active structural components. We review many of

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these techniques in more detail in the next section (see Table 1), but for now we summarize the philosophy behind the use of *mechanical* SOAs.

An SOA generalizes the idea behind a DVA by attaching an array of oscillators that have different mechanical properties: the mass, stiffness, and damping of each oscillator can be different in the design. It has been shown [3] that SOAs are capable of approximately uniform vibration attenuation of the magnitude of the frequency response of the host over a band of frequencies. In principle, an SOA provides substantial improvement in the robustness of the performance over a classical DVA because the design can tolerate variations in the input frequency around the target frequency. However, as in the design of DVAs, achieving this goal in practical SOA implementations can be problematic. Response reduction over the desired flat band exhibits sensitivity to *unintended* variation in the mechanical properties of the primary or the attached masses. Fabrication errors or uncertainty can play havoc on the realized performance of systems fabricated with a given design strategy. Philosophically, this robustness issue can be expressed in terms of the sensitivity of a design to disorder. [4, 5]

In this paper we present *piezoelectric* SOAs, or PSOAs, as an approach to ameliorate the sensitivity to disorder. A PSOA is an SOA with piezoelectric substructures and associated shunt circuits for each attached oscillator. We focus our study on developing *direct and simple* design strategies that are robust with respect to property uncertainty or disorder. In this sense, the paper can be viewed as a synthesis of results from the literature on SOAs and piezolelectric composite structures. These two directions of research have surprisingly little direct cross-references, and we are careful to make clear the nuances, similarities, and dissimilarities among these references. This paper does not consider or discuss a problem-specific, optimization-based approach to design, although such an analysis of PSOAs certainly would be valuable. In effect, we 1) seek to obtain a characterization of a design strategy that is nearly as simple as that for a classical DVA, 2) examine how post-fabrication tuning of the shunt circuits can influence robustness, 3) define a metric - the performance recovery - that quantifies in what sense a resulting design is robust, 4) study performance recovery for redesign of nominal PSOA designs. We base our analysis on both theoretical considerations and experimental results.

2 Literature Review

As mentioned above, there is a large variety of references that study vibration attenuation that feature piezoelectric sensors and actuators to realize active composites. With this huge body of literature there are many nuanced publications investigate quite specific topics relevant to piezoelectric active composites. In this section we carefully distinguish how the current paper on PSOAs relates to the existing literature.

Of course, one essential observation regarding the difference between many of these studies of piezoelectric composite systems and investigations of PSOAs is that the latter constitute a family of distinct piezoelectric composites that are connected to a rigid or linearly elastic host structure. This creates a coupled linear system of ODEs for the PSOAs *together with the host* that contain subcomponents that resemble the structure of the governing equations for many piezoelectric composite systems. But the PSOA equations have an important coupling structure that enables relatively simple closed form expressions for the mapping from *inputs to host response.* The response of the piezoelectrics PSOA, in and of itself, is of little interest in an analysis of the PSOA and host system. This fact stands in stark contrast to general studies of active piezoelectric composites in which the dynamics of the piezoelectric materials is at the heart of the study.

In addition, we focus in this paper on a number of topics that are seldom addressed explicitly in many studies of piezoelectric composite systems. As we will see below, the references on active linear piezoelectric systems *generally or typically* do not (1) relate the studies of these piezoelectric systems to the existing body of literature on SOAs or PSOAs, (2) do not define or examine robustness as a design objective per se, (3) do not investigate the specific conclusion of this paper of developing a simple (non-optimization-based) design strategy, and (4) do not study the topic of *performance recovery* as an explanation of robustness that is afforded by PSOA redesign.

We make two more observations before considering Table 1 in detail. Note that many of the publications listed in the table below can actually be located in different "general type" categories. When more than one general type of designation is applicable, we have selected the general type to reflect the authors' overall assessment of the most salient features of the paper. Also, the table is largely limited to the last 20 years of research. There are a large number of influential and fundamental research efforts that predate the table. For instance, the well-cited paper [6] is the earliest paper in the table; general considerations of the theory of modern linear piezoelectric systems predate this paper by decades, see [7] for instance.

As shown in Table 1, perhaps the largest category in table are those papers that perform qualitative, numerical, and/or experimental studies of piezoelectric systems with shunt circuits. These electromechanical models generally have mechanical and electrical subsystems. This class includes models of mechanical systems that are single degree of freedom (SDOF), multi-degree of freedom (MDOF), and distributed or infinite dimensional systems. The electrical subsystems can include SDOF or MDOF electrical circuits also. The collection of papers in this category include studies of specific designs, general modeling principles, active piezoelectric systems, and passive piezoelectric systems. None of these papers discusses SOAs, or exploits known results from SOAs in their analysis, nor makes a systematic analysis of some aspect of robustness. On the other hand, the results in these papers, or even in textbooks such as [7], constitute the theoretical foundation for the analysis of the PSOAs in this paper. In effect, the theoretical starting point of this paper is demonstrating that concise design principles for SOAs can be "lifted" to the more general case of PSOAs, whose models are derived from such linear piezoelectric system theories and models.

A second important category of relevant studies of linear piezoelectric systems are those that use state switching to enhance vibration attenuation. This approach has been studied intently over the past two decades, but again, systematic investigation of sensitivity or robustness is infrequently treated in this category too. These techniques are sometimes referred to as semi-active piezoelectric systems. As opposed to active control of vibration that continuously modulates the voltages, these systems perform (relatively) low-energy switching among essentially passive shunt circuits to improve performance. Semi-active switching is not considered in our study of PSOAs, although again, it would be of interest to understand the interplay between PSOA designs switching strategies for the shunt circuits. Also, it is worth noting that this category of publications does not make systematic study of the robustness of piezoelectric system designs. For instance, many state-switched realizations make use of peak detection to realize state switches. Stated results are frequently limited to studies of (essentially) fixed, stead-state, harmonic inputs that are amenable to simple peak detection. To the authors knowledge these studies have not investigated the effect of these methods to system or controller parameters or properties of the *class* of input signals. For purposes of illustration, we can think of two cases. Peak detection is difficult to realize in practice for noisy signals. A time delay can be intentionally introduced in peak detection or the signal can be filtered to eliminate high frequency oscillations in peak detection due to local minima and maxima. These are just two parameters that can be associated with a particular design implementation. Intuitively, we expect performance to deteriorate as the harmonic content of a signal increases in bandwidth or the time-delay varies. It is expected that a near white noise input (or signal with high finite bandwidth) will not be amenable to peak switching, and also the level of noise would affect the performance. It would be valuable to know this relationship and study robustness of vibration attenuation performance with respect to parameters such as the bandwidth of the input, or the peak-detection time delay, for instance.

Another interesting class that has gained popularity just over the past few years is the important discipline of waveguide design and metamaterials. Essentially, by distributing an array of identical substructures periodically on the host structure, it is desired to design structures that have bandgaps in their frequency response over a specified frequency range. Many of these studies investigate optimization to achieve optimal or near optimal placement or patterning of piezoelectric layers. Again, as of yet, issues like robustness or sensitivity to disorder have not yet been studied theoretically in a systematic way. It is also noteworthy in the studies of metamaterials and waveguide analyses that that the prototypical systems that are studied in these papers consist of *monolithic* beam, plate, or shell structures with integrated piezoelectric lamina or patches. In theory the linear equations governing the dynamics of such systems are very close to that of the PSOAs studied in this paper. Still, there are significant differences between typical papers on metamaterial realizations and the problem of vibration attenuation addressed by PSOAs. The primary distinction among these fields of study is that metamaterials uses identical substructures and rely on spatial periodicity to achieve distributed regions of nonresonance. On the other hand, the problems addressed in the study of PSOAs seek to obtain vibration attenuation around a resonant frequency of a host that is connected to collection of distinct piezoelectric composites at very low mass ratios. The magnitude of the coupling terms in the mass matrix of a composite laminate can be quite large relative to the size of diagonal or block diagonal coefficients in case of metamaterials.

In contrast, for PSOAs, the governing equations are (originally, as explained in reference [8] are only coupled via the mass matrix. Moreover, the mass coupling between the DOFS in the governing equations of a typical PSOA are exceptionally small. The added mass of the PSOA oscillators are usually just a few percent, or smaller, of the mass of the primary oscillator. There are therefore structural differences in the governing equations, and qualitative differences in the magnitude of the mass coupling in PSOA systems in comparison

The most important implication of this block structure in a PSOA enables the derivation of an analytical expression for the frequency response from input to host motion of the PSOAs. The frequency response function of the PSOAs can be identified directly as a generalization of the expression used for SOAs. For SOAs this frequency response function has the form [3]

$$\begin{aligned}\frac{x_p K_p}{f_p} &= \Bigg[1-\Omega^2+\frac{i\Omega}{Q_p} \\ &\quad +\sum_{n=1}^{N}\hat{\alpha}_n\left[\Omega^2+\frac{-\Omega^2\left(1+\frac{i\Omega}{\beta_n Q_n}\right)}{1-(\frac{\Omega}{\beta})^2+\frac{i\Omega}{\beta_n Q_n}}\right]\Bigg]^{-1},\end{aligned} \tag{1}$$

where

$$\begin{aligned}\Omega &= \omega\sqrt{\frac{M_{pp}}{K_p}}, & \tilde{\alpha}_n &= \frac{M_{nn}}{M_{pp}}, \\ \beta_n &= \sqrt{\frac{\gamma_n}{\tilde{\alpha}_n}}, & \gamma_n &= \frac{\hat{K}_{nn}}{K_p}, \\ Q_n &= \frac{\sqrt{M_{nn}\hat{K}_{nn}}}{C_{nn}}, & \hat{\alpha}_n &= \alpha_n^2\tilde{\alpha}_n.\end{aligned} \tag{2}$$

as shown in [8]. The above expression can be identified directly as a generalization of the expression used for SOAs which has the form

$$\frac{X_p(\Omega)k_p}{F_p(\Omega)} = \left(1-\Omega^2+\frac{i\Omega}{Q_p}+\sum_{n=1}^{N}\alpha_n\left(\frac{-\Omega^2\left(1+\frac{i\Omega}{\beta_n Q_n}\right)}{1-\left(\frac{\Omega}{\beta_n}\right)^2+\frac{i\Omega}{\beta_n Q_n}}\right)\right)^{-1}, \tag{3}$$

where

$$\begin{aligned} \Omega &:= \omega/\omega_p, & \alpha_n &:= \frac{m_n}{m_p}, & \beta_n &:= \sqrt{\frac{\gamma_n}{\alpha_n}}, \\ \gamma_n &:= \frac{k_n}{k_p}, & Q_n &:= \frac{\sqrt{m_n k_n}}{c_n}. \end{aligned} \tag{4}$$

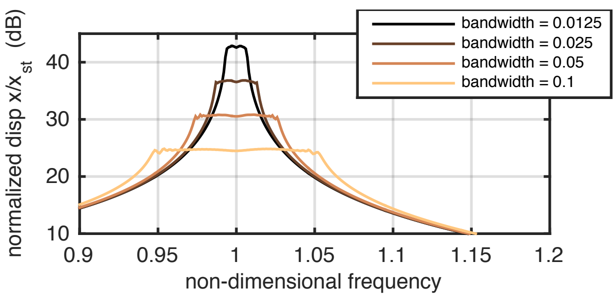

Fig. 1. Attenuation achieved by SOA for varying bandwidths. Ref: [9]

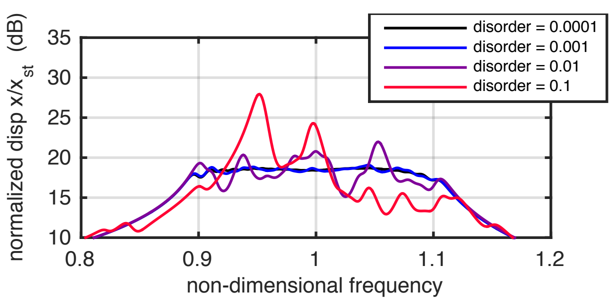

Fig. 2. SOA's performance degradation with disorder. Ref: [9]

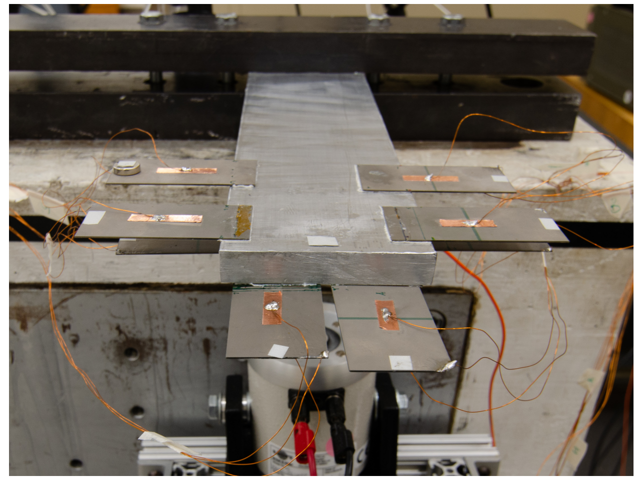

Fig. 3. Top view showing placement of bimorphs with differing overhang lengths

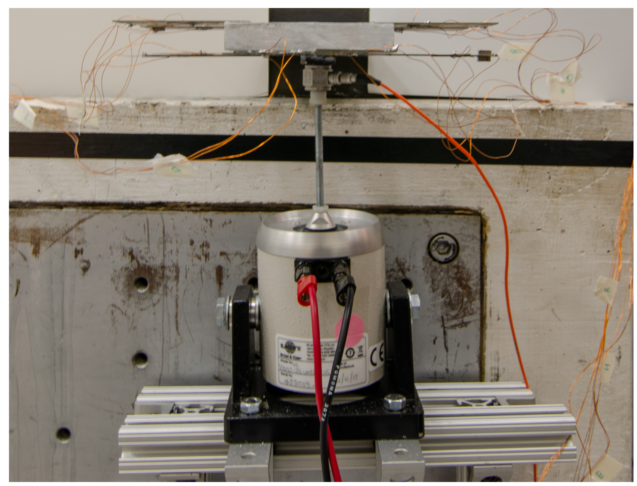

Fig. 4. Bottom view showing shaker force application

3 Experimental Setup

A 1kg beam was manufactured with geometry such that the first bending mode occurs at a frequency that matched the inherent natural frequency of the bimorphs (nominally 110Hz). These bimorphs were attached with differing overhanging lengths such that the bimorph frequencies fall in an appropriate distribution to create an SOA. The system was subjected to stepped-sine excitation and is shown in Figures 3 and 4. A laser Doppler vibrometer is used for measurement of system response.

4 Piezoelectric Tuning of Oscillators

An SOA, whether purely mechanical or piezoelectric, has a defined bandwidth and center frequency. Peak performance will occur at the center frequency and will degrade toward the edge of the band, at which point the primary system will respond as if there is no SOA at all. Environmental or application-based situations can shift the primary frequency which will deteriorate SOA performance. The piezoelectric bimorphs chosen for this application (Piezo.com Model T226-H4-503Y) were selected for maximum stiffness difference between open- and short-circuited condition. This bimorph characteristic maximizes the tuning authority of the PSOA. As the circuit is brought away from open circuit modes, either through a short circuit or introduction of capacitance, it has the effect of removing stiffness from the subordinate oscillators. Figure 5 shows the stiffness sensitivity from open to short circuit, with an intermediate stiffness achieved by adding 220 nF of capacitance in the circuit. This capacitive modification allows fine tuning of the PSOA. This configuration demonstrates a tuning authority of approximately 7%. This modification approach allows the PSOA to be tuned to follow a moving primary resonance.

The example in Figure 6 shows the experimental case in which mass is added to the primary resonator, reducing the primary frequency. This addition of mass pulls the primary resonant frequency toward the edge of the as-designed

General Type	Year Published & Reference
SOAs	[10] 2012, [3] 2009, [11] 2012, [9] 2016,
Qualitative, numerical, and experimental study of piezoelectric systems with shunt circuits	[6] 1990, [12] 2000, [13] 2000, [14] 2001, [15] 2001, [16] 2006, [17] 2010, [18] 2011, [19] 2011, [18] 2011, [20] 2011, [21] 2011, [22] 2012, [23] 2012, [24] 2012, [23] 2012, [24] 2012, [25] 2013, [26] 2014, [27] 2014, [28] 2016, [29] 2016 [30]
Mechanical SDOF or MDOF system, state switched or semi-active piezoelectric DVAs	[31] 1999, [32] 1999, [33] 2000, [34] 2000, [35] 2001, [36] 2001, [37] 2002, [38] 2004, [39] 2006, [40] 2006, [41] 2008, [42] 2009, [43] 2010 , [44] 2011, [45] 2012
Gain scheduled or operating mode switched piezoelectric composite DVAs	[46] 1997, [47] 1998, [48] 2000,
Optimization-Based DVA analysis and design	[49] 2003, [50] 2012, [51] 2014,
Piezoelectric energy harvesting, unswitched switched	[46] 1997, [47] 1998, [48] 2000, [52] 2010
Metamaterials and Wave propagation design and tailoring	[53] 2011, [54], [55] 2013, [56] 2013, [57] 2015, [58] 2016, [59] 2016, [60] 2017, [61] 2017, [62] 2017, [63] 2017

Table 1. Summary of Relevant SOA and Piezoelectric Systems Literature

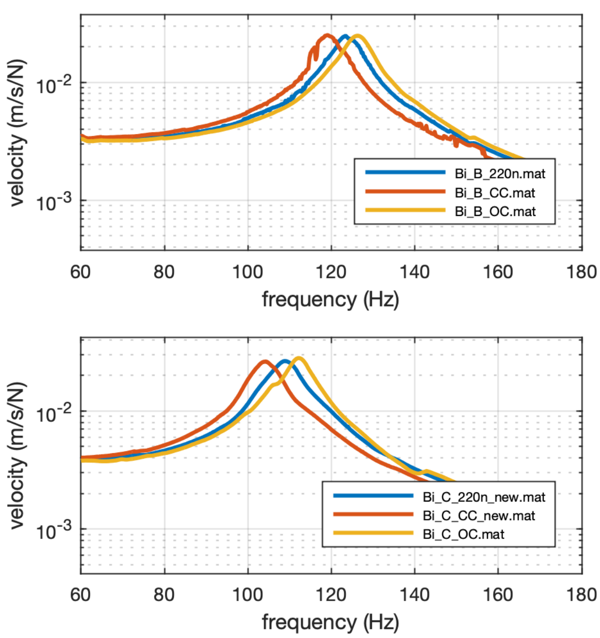

Fig. 5. The response of a selected bimorphs subjected to differing electrical boundary conditions. OC means open circuit, 220n represents an intermediate stiffness achieved with capacitance, and CC means closed (short) circuit

PSOA band, reducing PSOA effectiveness at the lower frequencies. Following this series of curves, each curve represents an addition of mass to the primary oscillator and resulting response. As the open circuit curves(solid lines) move into lower frequencies, the energy transfer out of the primary oscillator and into the PSOA decreases. This has the effect of increasing the measured magnitude of the primary. Therefore, as the primary frequency moves further away from the design frequency of the PSOA, the performance of the PSOA decreases.

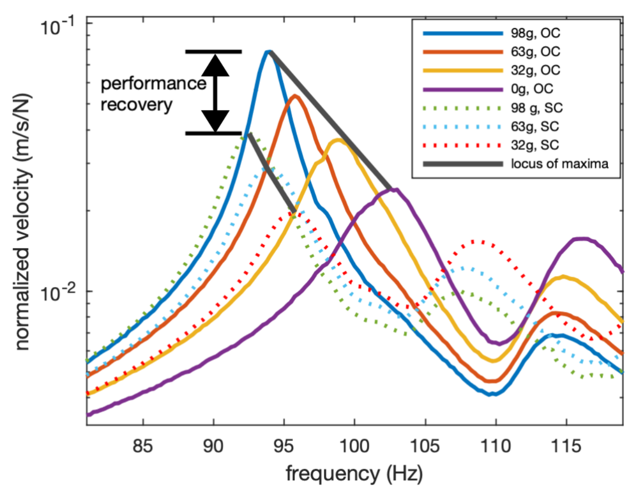

Fig. 6. A measurement of the response of the primary oscillator with a PSOA attached. Solid lines represent open circuit modes and dotted lines represent short circuit modes. The “locus of maxima” line serves to visually connect the associated families of curves.

It is possible to affect a “performance recovery” by reducing the center frequency of the PSOA by short-circuiting the elements. The dotted lines represent the system response after the short-circuit is applied causing the corresponding lowering of the PSOA band. Since the band is now better centered around the peak, the energy transfer to the PSOA is increased and the measured velocity of the primary mass is decreased. The magnitudes of the modified system (dotted lines) do still increase as the frequency is lowered, but that is the nature of the system. The important feature to note is the decrease in magnitude when compared to the open-circuit curve with the same added mass.

5 Conclusions

The implications of this tuning capability are novel and important for vibration suppression in any system with time-varying frequency characteristics. A standard DVA shows significant degradation of performance and may even amplify signal amplitude in off-design cases. A purely mechanical SOA has limited resilience to target frequency shifts, but lacks the ability to dynamically respond to changing characteristics of the primary system. In effect, the capability of the mechanical SOA is restricted to the upper family of curves from Figure 6. The PSOA exhibits significant gains in vibration suppression ability through the use of its electromechanical properties.

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References

[1] Inman, D. J., 2001. "Engineering Vibration". *Vibration Fundamentals and Practice*, p. 560.

[2] Ginsberg, J. H., 2001. *Mechanical and Structural Vibrations*. John Wiley & Sons, New York.

[3] Vignola, J. F., Judge, J. A., and Kurdila, A. J., 2009. "Shaping of a system's frequency response using an array of subordinate oscillators". *The Journal of the Acoustical Society of America,* **126**(1), jul, pp. 129–139.

[4] Vignola, J., Glean, A., Judge, J., and Ryan, T., 2013. "Optimal apparent damping as a function of the bandwidth of an array of vibration absorbers". *The Journal of the Acoustical Society of America,* **134**(2), Aug., pp. 1067–1070.

[5] Ryan, T. J., Judge, J. A., Glean, A. A. J., and Vignola, J. F., 2012. "Noise sensitivity of a mass detection method using vibration modes of coupled microcantilever arrays". *Applied Physics Letters,* **101**(4), July, pp. 043104–1–4.

[6] Hagood, N. W., Chung, W. H., and Von Flotow, A., 1990. "Modelling of Piezoelectric Actuator Dynamics for Active Structural Control". *Journal of Intelligent Material Systems and Structures,* **1**(3), jul, pp. 327–354.

[7] Tiersten, H. F., 1969. *Linear Piezoelectric Plate Vibrations*. Springer US, Boston, MA.

[8] Paruchuri, S. T., Sterling, J., Kurdila, A., and Vignola, J., 2017. "Piezoelectric composite subordinate oscillator arrays and frequency response shaping for passive vibration attenuation". In 2017 IEEE Conference on Control Technology and Applications (CCTA), IEEE, pp. 702–707.

[9] Vignola, J., Judge, J., Sterling, J., Ryan, T., Kurdila, A., Paruchuri, S. T., and Glean, A., 2016. "On the Use of Shunted Piezo Actuators for Mitigation of Distribution Errors in Resonator Arrays". In Proceedings of the 22nd International Congress on Acoustics.

[10] Carcaterra, A., Akay, A., and Bernardini, C., 2012. "Trapping of vibration energy into a set of resonators: Theory and application to aerospace structures". *Mechanical Systems and Signal Processing,* **26**(1), pp. 1–14.

[11] Vignola, J., Glean, A., Judge, J., and Ryan, T., 2013. "Optimal Apparent Damping as a Function of the Bandwidth of an Array of Vibration Absorbers". *The Journal of the Acoustical Society of America,* **134**, pp. 1067–1070.

[12] Fleming, A. J., Behrens, S., and Moheimani, S. O. R., 2000. "Synthetic impedance for implementation of piezoelectric shunt-damping circuits". *Electronics Letters,* **36**(18), pp. 1525–1526.

[13] Tylikowski, A., 2000. "Distributed Piezoelectric Vibration Absorbers". *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik,* **80**(S2), jan, pp. 305–306.

[14] Tylikowski, A., 2001. "Control of circular plate vibrations via piezoelectric actuators shunted with a capacitive circuit". *Thin-Walled Structures,* **39**(1), pp. 83–94.

[15] Granier, J. J. J. J., Haundhausen, R. J. R. J., and Gaytan, G. E. G. E., 2001. "Passive modal damping with piezoelectric shunts".

[16] Al-Ahmad, M., Rolland, N., and Rolland, P. ., 2006. "Piezoelectric-based tunable microstrip shunt resonator". In 2006 Asia-Pacific Microwave Conference, pp. 653–656.

[17] Casadei, F., Dozio, L., Ruzzene, M., and Cunefare, K. A., 2010. "Periodic shunted arrays for the control of noise radiation in an enclosure". *Journal of Sound and Vibration,* **329**(18), pp. 3632–3646.

[18] Airoldi, L., and Ruzzene, M., 2011. "Wave Propagation Control in Beams Through Periodic Multi-Branch Shunts". *Journal of Intelligent Material Systems and Structures,* **22**(14), sep, pp. 1567–1579.

[19] Ruzzene, L. A., and M, 2011. "Design of tunable acoustic metamaterials through periodic arrays of resonant shunted piezos". *New Journal of Physics,* **13**(11), p. 113010.

[20] Wen, G. W., Chen, S., and Jihong, 2011. "Low-frequency locally resonant band gaps induced by arrays of resonant shunts with Antoniou's circuit: experimental investigation on beams". *Smart Materials and Structures,* **20**(1), p. 15026.

[21] Beck, B. S., Cunefare, K. A., Collet, M., and Ruzzene, M., 2011. "Active vibration control of a stiffened panel

through application of negative capacitance shunts". In Proc.SPIE, Vol. 7977.

[22] Deü, O. T., Ducarne, J., and J-F, 2012. "Performance of piezoelectric shunts for vibration reduction". *Smart Materials and Structures,* **21**(1), p. 15008.

[23] Lu, Y., and Tang, J., 2012. "Electromechanical tailoring of structure with periodic piezoelectric circuitry". *Journal of Sound and Vibration,* **331**(14), pp. 3371–3385.

[24] Casadei, F., Beck, B. S., Cunefare, K. A., and Ruzzene, M., 2012. "Vibration control of plates through hybrid configurations of periodic piezoelectric shunts". *Journal of Intelligent Material Systems and Structures,* **23**(10), may, pp. 1169–1177.

[25] Chen, S., Wang, G., Wen, J., and Wen, X., 2013. "Wave propagation and attenuation in plates with periodic arrays of shunted piezo-patches". *Journal of Sound and Vibration,* **332**(6), pp. 1520–1532.

[26] Casadei, F., and Bertoldi, K., 2014. "Wave propagation in beams with periodic arrays of airfoil-shaped resonating units". *Journal of Sound and Vibration,* **333**(24), pp. 6532–6547.

[27] Guo, K. M., and Jiang, J., 2014. "Independent modal resonant shunt for multimode vibration control of a truss-cored sandwich panel". *International Journal of Dynamics and Control,* **2**(3), pp. 326–334.

[28] Lossouarn, B., Deü, J.-F., Aucejo, M., and Cunefare, K. A., 2016. "Multimodal vibration damping of a plate by piezoelectric coupling to its analogous electrical network". *Smart Materials and Structures,* **25**(11), p. 115042.

[29] Larbi, W., Deü, J.-F., and Ohayon, R., 2016. "Finite element reduced order model for noise and vibration reduction of double sandwich panels using shunted piezoelectric patches". *Applied Acoustics,* **108**, pp. 40–49.

[30] Lossouarn, B., Aucejo, M., Deü, J.-F., and Multon, B., 2017. "Design of inductors with high inductance values for resonant piezoelectric damping". *Sensors and Actuators A: Physical,* **259**, pp. 68–76.

[31] Clark, W. W., 1999. "Semi-active vibration control with piezoelectric materials as variable-stiffness actuators". Vol. 3672, pp. 3672–3678.

[32] Richard, C., Guyomar, D., Audigier, D., and Ching, G., 1999. "Semi-passive damping using continuous switching of a piezoelectric device". Vol. 3672, pp. 3672–3678.

[33] Clark, W. W., 2000. "Vibration Control with State-Switched Piezoelectric Materials". *Journal of Intelligent Material Systems and Structures,* **11**(4), apr, pp. 263–271.

[34] Cunefare, K. A., de Rosa, S., Sadegh, N., and Larson, G., 2000. "State-Switched Absorber for Semi-Active Structural Control". *Journal of Intelligent Material Systems and Structures,* **11**(4), apr, pp. 300–310.

[35] Morgan, R. A., and Wang, K.-W., 2001. "Multi-frequency piezoelectric vibration absorber for variable frequency harmonic excitations". Vol. 4331, pp. 4311–4331.

[36] Holdhusen, M., Cunefare, K., and Larson, G. D., 2001. "Role of damping in state-switched absorber for vibration control". Vol. 4326, pp. 4311–4326.

[37] Cunefare, K. A., 2002. "State-Switched Absorber for Vibration Control of Point-Excited Beams". *Journal of Intelligent Material Systems and Structures,* **13**(2-3), feb, pp. 97–105.

[38] Holdhusen, M. H., and Cunefare, K. A., 2004. "Optimization of a state-switched absorber applied to a continuous vibrating system". Vol. 5386, pp. 5310–5386.

[39] Badel, A., Sebald, G., Guyomar, D., Lallart, M., Lefeuvre, E., Richard, C., and Qiu, J., 2006. "Piezoelectric vibration control by synchronized switching on adaptive voltage sources: Towards wideband semi-active damping". *The Journal of the Acoustical Society of America,* **119**(5), apr, pp. 2815–2825.

[40] Niederberger, D., and Morari, M., 2006. "An autonomous shunt circuit for vibration damping". *Smart Materials and Structures,* **15**(2), pp. 359–364.

[41] Ji, H., Qiu, J., Badel, A., and Zhu, K., 2008. "Semi-active Vibration Control of a Composite Beam using an Adaptive SSDV Approach". *Journal of Intelligent Material Systems and Structures,* **20**(4), sep, pp. 401–412.

[42] Qiu, J., Ji, H., and Zhu, K., 2009. "Semi-active vibration control using piezoelectric actuators in smart structures". *Frontiers of Mechanical Engineering in China,* **4**(3), pp. 242–251.

[43] Ji, H., 2010. "Semi-Active Vibration Control Based on Switched Piezoelectric Transducers". IntechOpen, Rijeka, p. Ch. 10.

[44] Cheng, J., Ji, H., Qiu, J., and Takagi, T., 2011. "Semi-active vibration suppression by a novel synchronized switch circuit with negative capacitance.". *International Journal of Applied Electromagnetics & Mechanics,* **37**(4), dec, pp. 291–308.

[45] Mokrani, B., Rodrigues, G., Ioan, B., Bastaits, R., and Preumont, A., 2012. "Synchronized switch damping on inductor and negative capacitance". *Journal of Intelligent Material Systems and Structures,* **23**(18), jan, pp. 2065–2075.

[46] Davis, C. L., Lesieutre, G. A., and Dosch, J. J., 1997. "Tunable electrically shunted piezoceramic vibration absorber". In Proceedings of SPIE, Vol. 3045, pp. 3045–3049.

[47] Davis, C. L., and Lesieutre, G. A., 1998. "Actively tuned solid state piezoelectric vibration absorber". Vol. 3327, pp. 3314–3327.

[48] Davis, C. L., and Lesieutre, G. A., 2000. "An Actively Tuned Solid-State Vibration Absorber Using Capacitive Shunting of Piezoelectric Stiffness". *Journal of Sound and Vibration,* **232**(3), pp. 601–617.

[49] Holdhusen, M., and Cunefare, K. A., 2003. "Optimization of location and tuning of a state-switched absorber for controlling beam vibration". Vol. 5049, pp. 5010–5049.

[50] Ducarne, J., Thomas, O., and Deü, J.-F., 2012. "Placement and dimension optimization of shunted piezoelec-

tric patches for vibration reduction". *Journal of Sound and Vibration,* **331**(14), pp. 3286–3303.

[51] Pereira da Silva, L., Larbi, W., and Deü, J.-F., 2014. "Topology optimization of shunted piezoelectric elements for structural vibration reduction". *Journal of Intelligent Material Systems and Structures,* **26**(10), jun, pp. 1219–1235.

[52] Kong, N., Ha, D. S., Erturk, A., and Inman, D. J., 2010. "Resistive Impedance Matching Circuit for Piezoelectric Energy Harvesting". *Journal of Intelligent Material Systems and Structures,* **21**(13), jan, pp. 1293–1302.

[53] Chen, S.-B., Wen, J.-H., Wang, G., Han, X.-Y., and Wen, X.-S., 2011. "Locally Resonant Gaps of Phononic Beams Induced by Periodic Arrays of Resonant Shunts". *Chinese Physics Letters,* **28**(9), p. 94301.

[54] Wang, J.-W., Wang, G., Chen, S.-B., and Wen, J.-H., 2012. "Broadband Attenuation in Phononic Beams Induced by Periodic Arrays of Feedback Shunted Piezoelectric Patches". *Chinese Physics Letters,* **29**(6), p. 64302.

[55] Chen, S.-B., Wen, J.-H., Wang, G., and Wen, X.-S., 2013. "Tunable band gaps in acoustic metamaterials with periodic arrays of resonant shunted piezos". *Chinese Physics B,* **22**(7), p. 74301.

[56] Huang, T. L., Ichchou, M. N., Bareille, O. A., Collet, M., and Ouisse, M., 2013. "Multi-modal wave propagation in smart structures with shunted piezoelectric patches". *Computational Mechanics,* **52**(3), pp. 721–739.

[57] Zhang, H., Wen, J.-H., Chen, S.-B., Wang, G., and Wen, X.-S., 2015. "Flexural wave band-gaps in phononic metamaterial beam with hybrid shunting circuits". *Chinese Physics B,* **24**(3), p. 36201.

[58] Lian, Z., Jiang, S., Hu, H., Dai, L., Chen, X., and Jiang, W., 2016. "An Enhanced Plane Wave Expansion Method to Solve Piezoelectric Phononic Crystal with Resonant Shunting Circuits". *Shock and Vibration,* **2016**.

[59] Yi, K., Collet, M., Ichchou, M., and Li, L., 2016. "Flexural waves focusing through shunted piezoelectric patches". *Smart Materials and Structures,* **25**(7), p. 75007.

[60] Sugino, C., Leadenham, S., Ruzzene, M., and Erturk, A., 2017. "An investigation of electroelastic bandgap formation in locally resonant piezoelectric metastructures". *Smart Materials and Structures,* **26**(5), p. 55029.

[61] Jiang, S., Dai, L., Chen, H., Hu, H., Jiang, W., and Chen, X., 2017. "Folding beam-type piezoelectric phononic crystal with low-frequency and broad band gap". *Applied Mathematics and Mechanics,* **38**(3), pp. 411–422.

[62] Sugino, C., Ruzzene, M., and Erturk, A., 2017. Dynamics of Hybrid Mechanical-Electromechanical Locally Resonant Piezoelectric Metastructures.

[63] Wang, G., Cheng, J., Chen, J., and He, Y., 2017. "Multi-resonant piezoelectric shunting induced by digital controllers for subwavelength elastic wave attenuation in smart metamaterial". *Smart Materials and Structures,* **26**(2), p. 25031.